Name ______________________________ Period __________ Date ______________

UNIT 10
STUDENT PACKET

MathLinks GRADE 8

SIMILARITY

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Materials | Grouping | Reproducibles | Slide Deck | Journal Idea

Parent (or Guardian) signature ______________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

dilation
scale factor
similar figures

A RUBBER BAND INVESTIGATION

Follow your teacher's directions for (1).

(1)

Compare $\triangle ABC$ and $\triangle A'B'C'$.

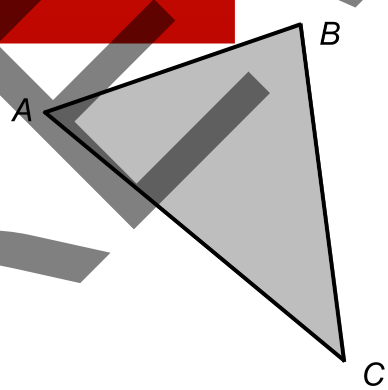
2. How do the areas relate?

3. How do corresponding angles relate?

4. How do corresponding sides relate?

5. Find the values of each ratio:
   a. $|A'C'| : |AC|$
   b. $|A'B'| : |AB|$
   c. $|B'C'| : |BC|$

6. On $\overline{PA'}$, where is A in relation to P and A'?



7. Does the same relationship hold for B along $\overline{PB'}$?

   For C along $\overline{PC'}$?

P •

We refer to P as a "center point."

A RUBBER BAND INVESTIGATION

Continued

Using the linked rubber band from the previous page, tie a knot in the middle of each rubber band to create four sections of equal length.

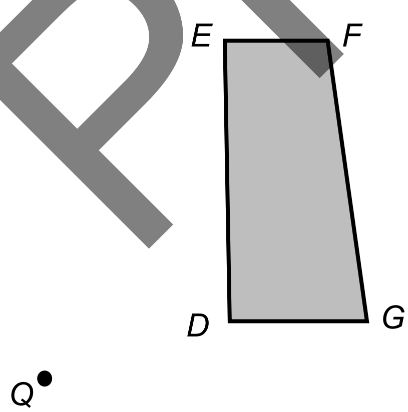
8. Use pre-image ⏢ *DEFG* to create image ⏢ *D′E′F′G′* with center point *Q*.
   - Anchor one end of the band at point Q with one pencil. Put a second pencil into the other end of the band.
   - Move the second pencil so that the knot closest to the first pencil traces over the trapezoid.
   - Clean up your image by drawing segments with a ruler and label the vertices appropriately.

9. How do the areas relate?

10. How do the corresponding angles relate?

11. How do corresponding sides relate?

12. On $\overline{QE'}$, where is *E* in relation to *Q* and *E′*?



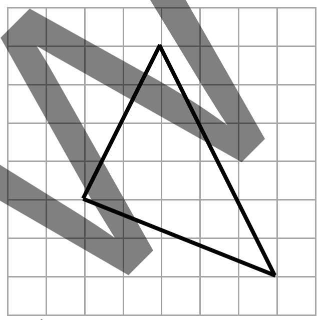
DILATIONS

We will experiment with dilations and explore properties of dilations.

[8.G.3, 8.G.7, 8.EE.2; SMP1, 5, 6, 7, 8]

GETTING STARTED

1. On the grid to the right, small grid squares are one square unit of area. Find the perimeter of the triangle. Write the result in square root form and as an approximated decimal value.



Simplify each expression below.

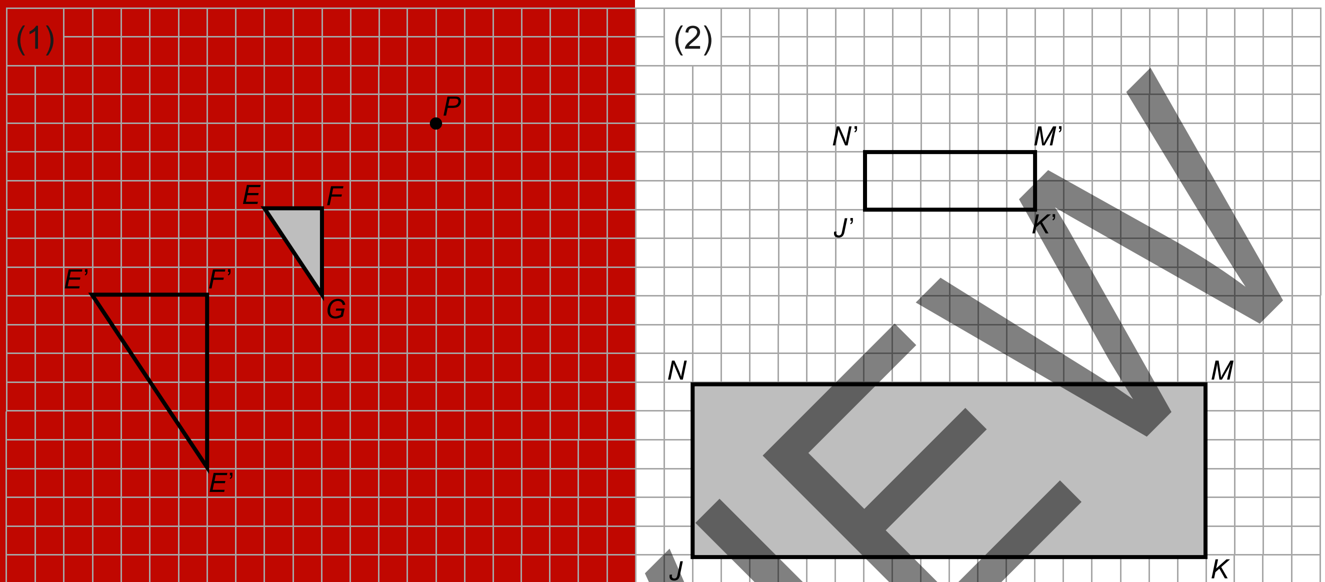
2a. $\frac{\sqrt{36}}{\sqrt{9}}$	3a. $\frac{\sqrt{81}}{\sqrt{9}}$	4a. $\frac{\sqrt{64}}{\sqrt{4}}$	5a. $\frac{\sqrt{100}}{\sqrt{4}}$
2b. $\sqrt{\frac{36}{9}}$	3b. $\sqrt{\frac{81}{9}}$	4b. $\sqrt{\frac{64}{4}}$	5b. $\sqrt{\frac{100}{4}}$

6. Based on problems 2 – 5 above, make a conjecture by filling in the blanks below. Then explain the conjecture in words.

$$\frac{\sqrt{m}}{\sqrt{n}} = \sqrt{\frac{\square}{\square}}$$

ABOUT DILATIONS

Follow your teacher's directions for (1) and (2).



3. Record the meanings of <u>dilation</u> and <u>scale factor</u> in **My Word Bank**.

Compare pre-image figures (shaded) above to their corresponding image figures (unshaded).

	triangles	**rectangles**
4. How do corresponding angles relate?		
5. How do corresponding sides relate?		
6. How do the areas relate?		
7. Circle the correct description.	This dilation is a(n): enlargement reduction	This dilation is a(n): enlargement reduction
8. Why is the image figure NOT congruent to its corresponding pre-image?		

ABOUT DILATIONS

Continued

9. Calculate the scale factor for the dilation of the triangles.

$\frac{\lvert PE' \rvert}{\lvert PE \rvert} \rightarrow$	$\frac{\lvert PF' \rvert}{\lvert PF \rvert} \rightarrow$	$\frac{\lvert PG' \rvert}{\lvert PG \rvert} \rightarrow$

10. For the rectangles, you drew four lines that went through corresponding vertices of the pre-image and image and coincided at point *L*. What do we call point *L*? What is its importance?

11. Calculate the scale factor for dilation of the rectangles.

$\frac{\lvert LN' \rvert}{\lvert LN \rvert} = \frac{\lvert LM' \rvert}{\lvert LM \rvert} \rightarrow$	$\frac{\lvert LJ' \rvert}{\lvert LJ \rvert} = \frac{\lvert LK' \rvert}{\lvert LK \rvert} \rightarrow$

For problems 12 – 14, if the answer is yes, provide one supporting example for each dilation. If the answer is no, provide a counterexample.

12. Are line segments taken to line segments of the same length for either the triangle or the rectangle?

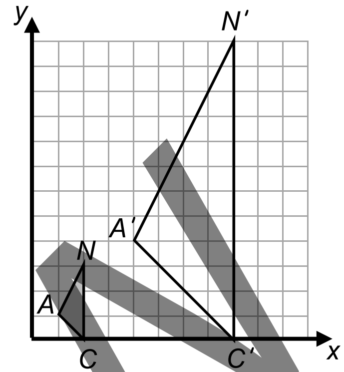
13. Are angles taken to angles of the same measure for either the triangle or the rectangle?

14. Are parallel lines taken to parallel lines for either the triangle or the rectangle?

15. How can you tell if a dilation is an enlargement or a reduction from the scale factor?

DILATIONS IN THE COORDINATE PLANE

In the grid to the right, small grid squares are one square unit of area. Under the dilation $\triangle C'A'N'$ is the image of $\triangle CAN$.



1. Use a ruler to draw rays through corresponding vertices. Label the intersection as point *O*.

   The coordinates of *O* are (____, ____).

   Point *O* is the _________ _________ of the dilation.

2. Find each side length below.

$\overline{CA}$	$\overline{CN}$	$\overline{AN}$	$\overline{C'A'}$	$\overline{C'N'}$	$\overline{A'N'}$

3. What is the value of the ratio (the scale factor) of corresponding side lengths (image : pre-image)?

4. For this dilation, it must be true that $\frac{|A'O|}{|AO|} = \frac{|N'O|}{|NO|} = \frac{|C'O|}{|CO|}$.

   What must this value be? _____ Verify by finding the value of any one of these ratios.

5. Write the coordinates of the vertices.

C (___, ___)	*A* (___, ___)	*N* (___, ___)	*C′* (___, ___)	*A′* (___, ___)	*N′* (___, ___)

6. What is the value of the ratio of corresponding vertex coordinates (image : preimage)?

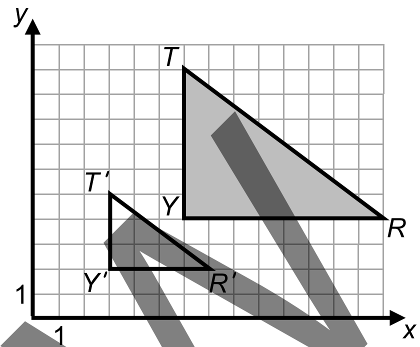
7. Finish the mapping rule for the transformation (from the pre-image to the image):

   $(x, y) \rightarrow$ (_____, _____)

8. Explain why this dilation represents a proportional increase (enlargement).

DILATIONS IN THE COORDINATE PLANE
Continued

In the grid to the right, small grid squares are one square unit of area. Under the dilation $\triangle T'R'Y'$ is the image of $\triangle TRY$.



9. Use a ruler to draw rays through corresponding vertices. Label the intersection as point *O*.

   The coordinates of *O* are (____, ____).

   Point *O* is the ________ ________ of the dilation.

10. Find each side length below.

$\overline{TY}$	$\overline{YR}$	$\overline{TR}$	$\overline{T'Y'}$	$\overline{Y'R'}$	$\overline{T'R'}$

11. What is the value of the ratio (the scale factor) of corresponding side lengths (image : pre-image)?

12. For this dilation, it must be true that $\frac{|T'O|}{|TO|} = \frac{|Y'O|}{|YO|} = \frac{|R'O|}{|RO|}$.

    What must this value be? _____ Verify by finding the value of any one of these ratios.

13. Write the coordinates of the vertices.

T (___, ___)	*R* (___, ___)	*Y* (___, ___)	*T′* (___, ___)	*R′* (___, ___)	*Y′* (___, ___)

14. What is the value of the ratio of corresponding vertex coordinates (image : preimage)?

15. Finish the mapping rule for the transformation (from the pre-image to the image):

    $(x, y) \rightarrow$ (_____, _____)

16. Explain why this dilation represents a proportional decrease (reduction).

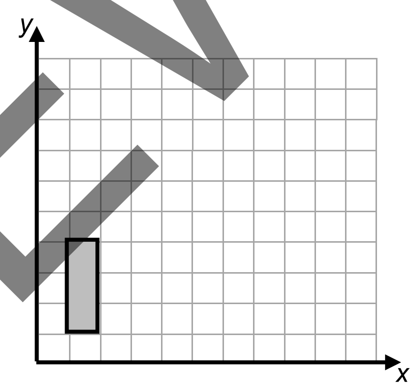
PRACTICE 1

1. To dilate a figure in the coordinate plane with the center of dilation at the origin, multiply the coordinates of its points by the __________ __________.

Use the graphs below for problems 2 – 4. In the grids below, small grid squares are one square unit of area.

- For each figure, dilate it using the given scale factor using the technique described above.
- Check that the center of the dilation is the origin by drawing rays from the origin through corresponding points on the figure and its image.
- Fill in the coordinates and rules in the table.

2. Scale factor = 2



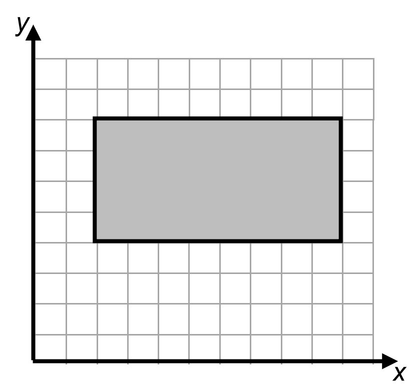
Coordinates (pre-image):

Coordinates (image):

Mapping rule:

$(x, y) \rightarrow$ (_____, _____)

3. Scale factor = $\frac{1}{4}$



Coordinates (pre-image):

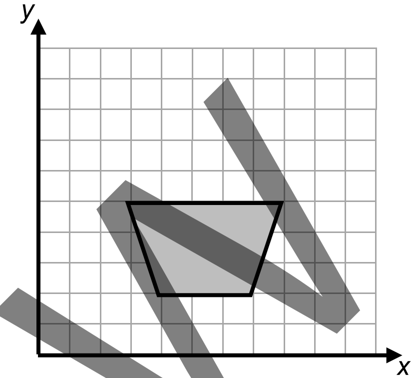
Coordinates (image):

Mapping rule:

$(x, y) \rightarrow$ (_____, _____)

PRACTICE 1
Continued

4. Scale factor = 1



Coordinates (pre-image):

Coordinates (image):

Mapping rule:

$(x, y) \rightarrow$ (_____, _____)

Refer to dilations in problems 2 – 4 to complete this table.

		Problem 2	Problem 3	Problem 4
5.	Are angles taken to angles of the same measure?			
6.	Are line segments taken to line segments of the same length?			
7.	Are parallel lines taken to parallel lines?			
8.	Is it an enlargement, reduction, or neither?			
9.	Is the scale factor greater than 1, equal to 1, or between 0 and 1?			

SIMILAR FIGURES

We will define similar figures and explore their properties. We will compare properties of similarity to properties of congruence.

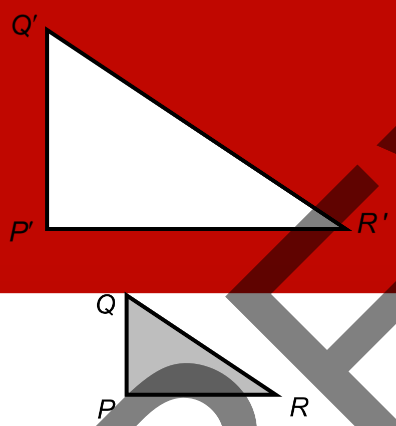
[8.G.4, 8.G.7, 8.EE.2; SMP1, 3, 5, 7, 8]

GETTING STARTED

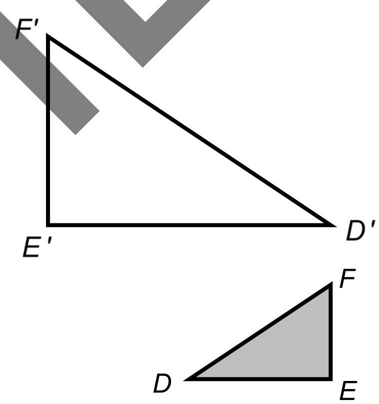
For problems 1 – 2:

- Use tools (e.g., ruler, straight edge, patty paper) to determine if the unshaded figure is a dilation of the shaded one.
- If it is a dilation, find the center point and scale factor.
- If it is not a dilation, describe one or more rigid motions that could be performed on the image so that it is a dilation of the pre-image.

1.



2.

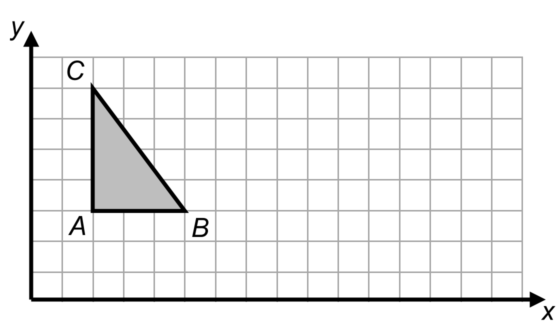


3. Rotate $\triangle CAB$ clockwise 90° about point B. Label the new triangle $C'A'B'$.

4. Reflect $\triangle C'A'B'$ around a vertical line through C'. Label the new triangle $C''A''B''$.

5. What point is in the same position as B?

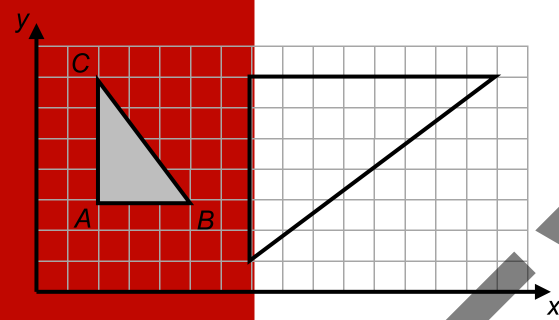
6. What point is in the same position as C'?



ABOUT SIMILARITY

Follow your teacher's directions. Use patty paper as needed. Small grid squares are one square unit of area. $\triangle ABC$ and its image are shown below.

(1)



(2)

(3)

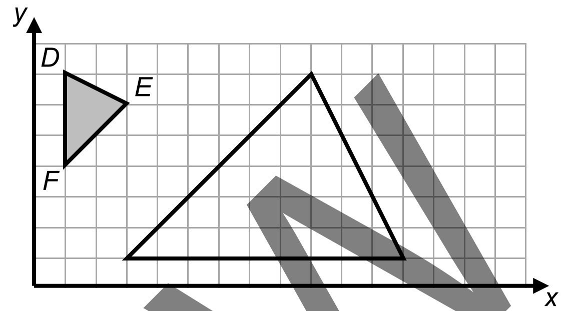
(4)

(5)

PRACTICE 2

1. Record the meaning of similar figures in **My Word Bank**.

2. Label vertices of the image $\triangle D'E'F'$ to the right.



3. Explain the relationship of pre-image side lengths compared to image side lengths. Include the value of the scale factor.

4. Does the image represent an enlargement or a reduction of the pre-image? Explain.

5. Write some steps to show that $\triangle DEF \sim \triangle D'E'F'$.

6. Write a different set of steps than problem 5 to show that $\triangle DEF \sim \triangle D'E'F'$.

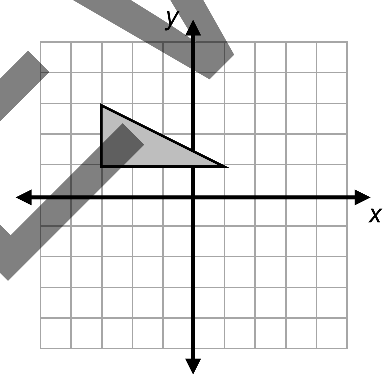
TALKING TRANSFORMATIONS: SIMILARITY

Small grid squares are one square unit of area.

1. This is an activity for two students. Cut up the cards and place them face down.
   - For Round 1, Student 1 chooses a card, but does not show it to Student 2.
   - Student 1 explains how to create the image through a sequence of rigid motions and a dilation.
   - Student 2 creates the image on this page.
   - Students compare to see if directions are clear. Then BOTH students copy the image and write a sequence of steps to show the figures are similar.
   - Students switch roles for each round.

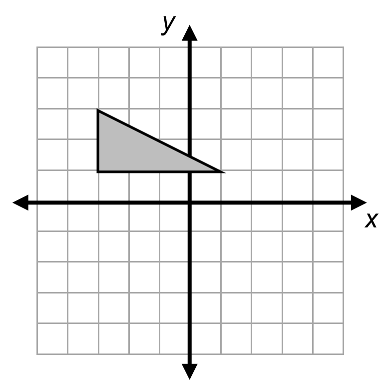
Round 1 (Card ___)

Steps:



Round 2 (Card ___)

Steps:

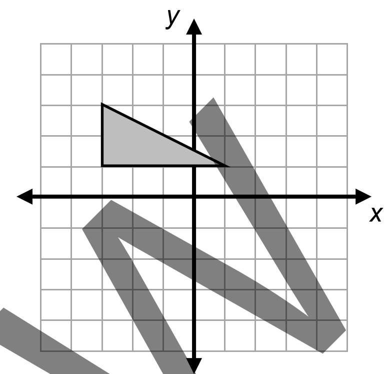


TALKING TRANSFORMATIONS: SIMILARITY

Continued

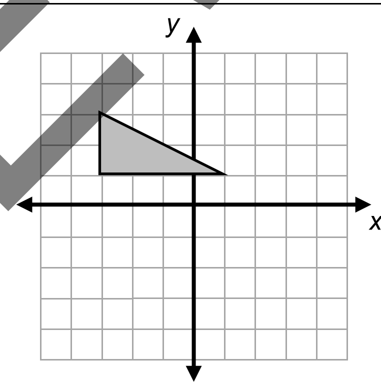
Round 1 (Card ___)

Steps:



Round 2 (Card ___)

Steps:



2. How do you know that the figures and images in this activity are similar?

3. Which card(s) produced enlargements? Explain.

4. Which card(s) produced reductions? Explain.

5. Which card(s) produced congruent figures? Explain.

TRUE-FALSE-EXPLAIN

Your teacher will give pairs or small groups of students a set of 10 cards.

1. Without anything in front of you for reference, discuss each card and sort them into a "true" group and a "false" group. Put cards you are unsure about into a third group for now.

We think these are true:	We think these are false:	We are unsure about these:

2. Use any resources available to you, including your packet, the internet, or other pairs/groups of students, to try to correctly place ALL of the cards ONLY in the first two boxes above. As a check:

- The letters for the true cards spell this number: _______________
- The letters for the false cards spell this number: _______________

3. Cards H and R seem very alike. Why are they different? In other words, what makes one true and the other false?

4. Choose two other false cards (other than the one from problem 3) and explain why they are false. Use counterexamples to support explanations.

____ is false. Explanation:	____ is false. Explanation:

PRACTICE 3

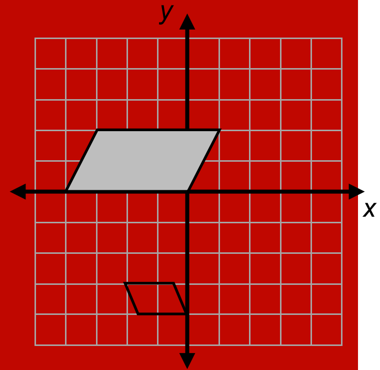
Recall that the pre-image is shaded. The pairs of figures in problems 1 and 2 are similar. Each student's incorrect steps to explain the similarity is listed. Critique their reasoning and suggest changes as needed. Small grid squares are one square unit of area.

1. Robin

Step 1:
Reflection over the y-axis

Step 2:
$(x, y) \rightarrow (x - 4, y + 2)$

Step 3:
Dilate with scale factor $\frac{1}{2}$ from the shared vertex.

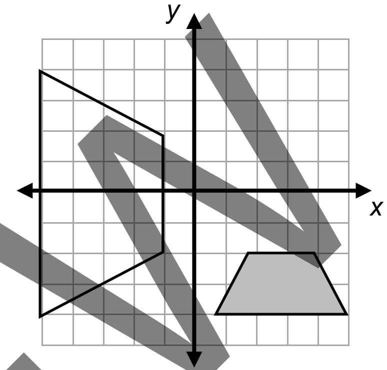


2. Dion

Step 1:
Rotate 90° clockwise about (2, -2)

Step 2:
$(x, y) \rightarrow (x - 5, y - 2)$

Step 3:
Dilate with scale factor 2 from the shared vertex.

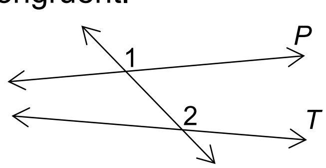


For problems 3 – 6, critique each student's statement.

3. Triangle 1 has a 45° angle and a 50° angle. Triangle 2 also has a 45° and a 50° angle. Jacob thinks that these triangles must have congruent third angles.

4. Maya dilates a pre-image figure to obtain a similar image figure. Maya thinks that corresponding parallel sides from the pre-image figure remain parallel in the similar figure.

5. Drew thinks that corresponding angles 1 and 2 must be congruent.



6. Ayla thinks that a dilation of a pre-image alone may result in a similar image figure.

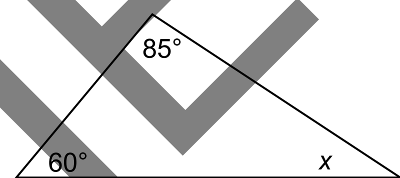
SIMILAR TRIANGLE RELATIONSHIPS

We will establish the angle-angle criterion for similar triangles, and use it to show that triangles are similar. We will find missing side lengths of similar triangles. We will link properties of similarity to the slope of a line.

[8.G.5, 8.G.7, 8.EE.2, 8.EE.6, 8.F.3; SMP1, 2, 3, 4, 5]

GETTING STARTED

Solve for *x*.

1. $\frac{x}{5} = \frac{18}{15}$	2. $\frac{10}{12.5} = \frac{4}{x}$
3. $\frac{3}{4} = \frac{x}{3}$	4. 

Use the figure below for problems 5 – 8. Review vocabulary from Unit 1, **Plane and Solid Figures**, as needed.

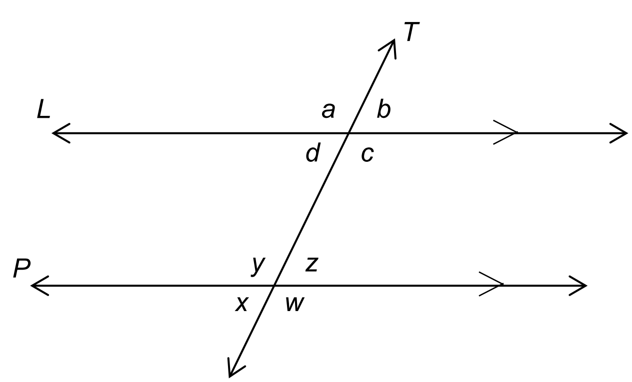
corresponding angles	**vertical angles**	**transversal**	**parallel**

5. The arrows on lines *L* and *P* means that they are _______________.

6. Since line *T* intersects *L* and *P*, it is called a _______________.

7. List the angle that corresponds to:

$\angle a \rightarrow$	$\angle c \rightarrow$
$\angle x \rightarrow$	$\angle z \rightarrow$



8. List the angle that is vertical to:

$\angle a \rightarrow$	$\angle w \rightarrow$
$\angle x \rightarrow$	$\angle d \rightarrow$

9. If $|\angle a| = 115°$, find the following:

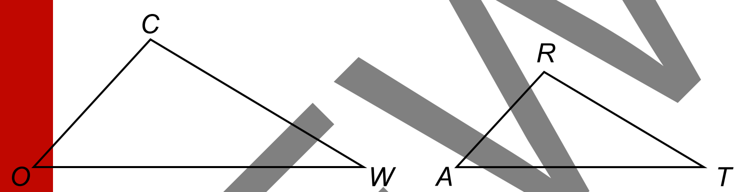
$|\angle y| \rightarrow$	$|\angle c| \rightarrow$	$|\angle b| \rightarrow$	$|\angle w| \rightarrow$

ANGLE-ANGLE SIMILARITY

1. Name a fact about corresponding sides of two similar triangles.

2. Name a fact about corresponding angles of two similar triangles.

3. Given: $\angle C \cong \angle R$ and $\angle O \cong \angle A$.
   Make arc markings on the triangles to the right to show these facts.



4. Write a fact about the relationship between $\angle W$ and $\angle T$.
   Explain how you know you are correct. Use a numerical example if it is helpful.

5. Why must it be true that $\triangle COW \sim \triangle RAT$?

> We have established the
> **angle-angle criterion for similarity of triangles (A-A criterion)**:
> If *two* angles of one triangle are congruent to *two* angles of another triangle, then the triangles are similar
> (since we know that the third pair of angles must be congruent too).

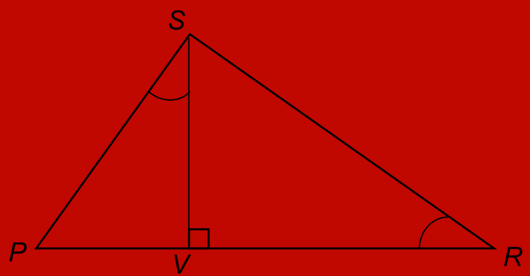
6. Two other triangles have corresponding congruent angles. How do we know that they are not necessarily congruent triangles?

7. Two quadrilaterals have three pairs of corresponding congruent angles. How do we know that they are not necessarily similar quadrilaterals? Use an example if it is helpful.

PRACTICE 4

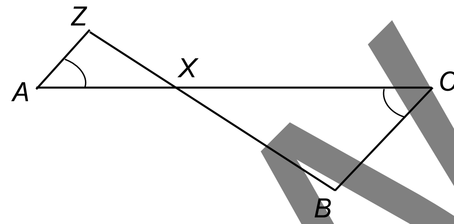
Use patty paper as needed.

Figure 1



Use Figure 1 for problems 1 – 3.
(Problems 2 and 3 are not related to each other.)

Figure 2



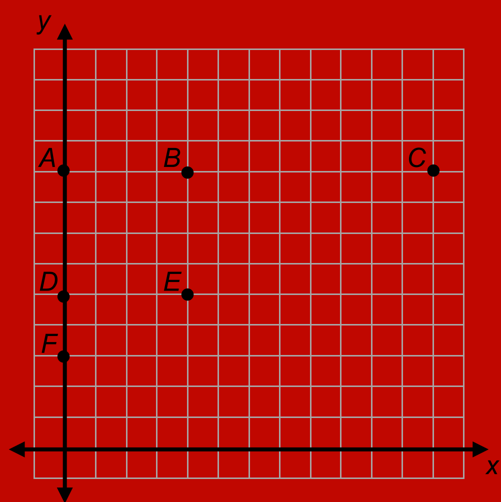
Use Figure 2 for problems 4 – 6.
(Problems 5 and 6 are not related to each other.)

1. $\triangle PVS \sim \triangle$ _____ because:	4. $\triangle AXZ \sim \triangle$ _____ because:
2. If $\left\|\angle PSV\right\| = 35°$, find: a. $\left\|\angle SPV\right\| =$ _____ b. $\left\|\angle SRV\right\| =$ _____ c. $\left\|\angle VSR\right\| =$ _____	5. If $\left\|\angle ZAX\right\| = 55°$, and $\left\|\angle AXZ\right\| = 45°$, find: a. $\left\|\angle AZX\right\| =$ _____ b. $\left\|\angle CXB\right\| =$ _____ c. $\left\|\angle CBX\right\| =$ _____
3. If $\left\|\overline{PV}\right\| = 6$ cm and $\left\|\overline{VS}\right\| = 8$ cm, find: a. $\left\|\overline{PS}\right\| =$ __________ b. $\left\|\overline{VR}\right\| =$ __________	6. If $\left\|\overline{AZ}\right\| = 10$ mm, $\left\|\overline{ZX}\right\| = 15$ mm, $\left\|\overline{BC}\right\| = 20$ mm, and $\left\|\overline{AX}\right\| = 18$ mm, find a. $\left\|\overline{BX}\right\| =$ _______ b. $\left\|\overline{CX}\right\| =$ ________

SIMILARITY AND SLOPE

Follow your teacher's directions for (1) – (6). In the graph below, small grid squares are one square unit of area.

(1)



(2)

(3)

(4)

(5)

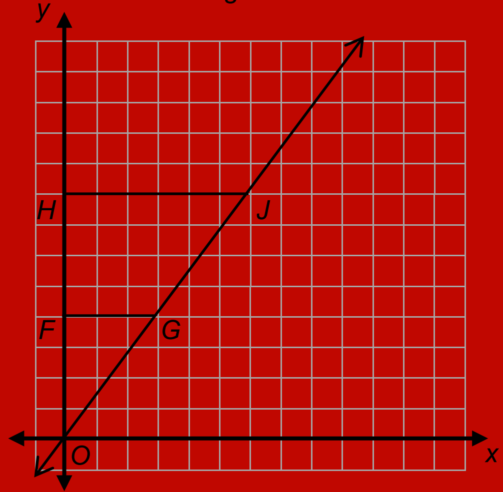
(6)

7. Explain the significance of the fraction in the results of problems 4 – 6 and how they relate.

8. List topics/skills/concepts you learned in this course that were applied to this page.

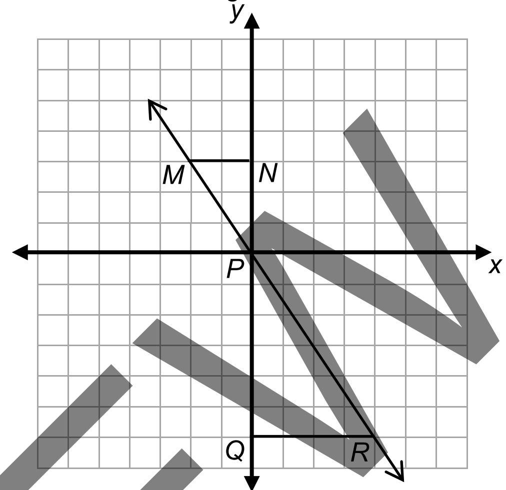
PRACTICE 5

Figure 1



Small grid squares are one square unit of area.

Figure 2



1. Name a pair of similar triangles in each figure above. Then explain why the pair are similar using the angle-angle criterion for similarity of triangles.

Figure 1: ___________ ~ ___________	Figure 2: ___________ ~ ___________

2. Find the values of the ratios of corresponding legs within the similar triangles.

Figure 1: $\frac{\lvert\overline{OF}\rvert}{\lvert\overline{FG}\rvert} \rightarrow$ $\frac{\lvert\overline{OH}\rvert}{\lvert\overline{HJ}\rvert} \rightarrow$	Figure 2: $\frac{\lvert\overline{PN}\rvert}{\lvert\overline{NM}\rvert} \rightarrow$ $\frac{\lvert\overline{PQ}\rvert}{\lvert\overline{QR}\rvert} \rightarrow$

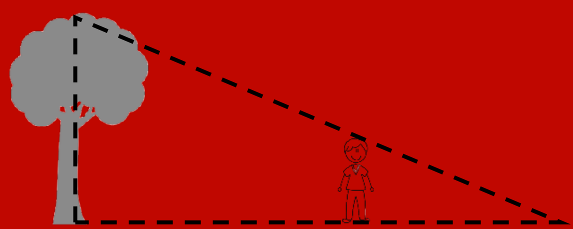
3. Write the equation of each line in slope-intercept form.

Figure 1: $\overleftrightarrow{OJ} \rightarrow$	Figure 2: $\overleftrightarrow{MR} \rightarrow$

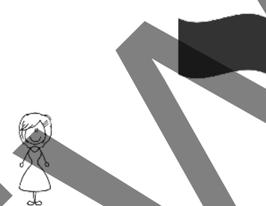
4. Look at problems 2 and 3 above. For each figure, how do the values of the ratios relate to the slopes of the lines?

PRACTICE 6: EXTEND YOUR THINKING

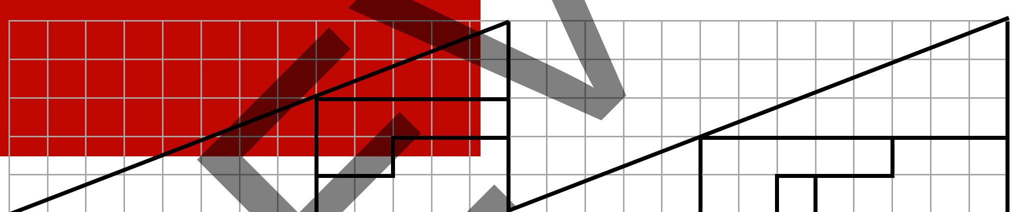
1. Marcellus is 5 feet tall. He casts a 7-foot shadow. At the same time, the shadow of a tree is 21 feet. Approximately how tall is the tree?



2. Nadia is 4 feet 8 inches tall. She casts a shadow that is 2 feet 4 inches. At the same time, the shadow of the flagpole is 6 feet 8 inches. Find the approximate height of the flagpole in feet and inches.



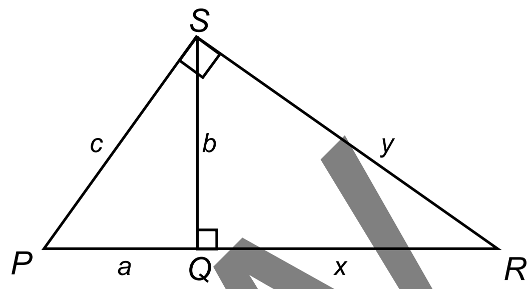
3. For this "missing square puzzle," it appears that when the four figures within the large triangle are rearranged, one square unit of area is lost.

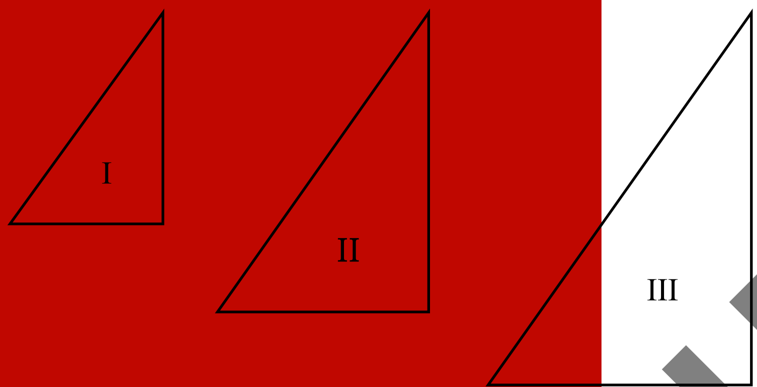


- Use four different colored pencils to lightly shade each pair of congruent figures in the same color so you can clearly see where they have moved.
- Explain in one or more ways why this puzzle is an optical illusion. In other words, explain why it is impossible to lose the square unit just by rearranging figures.

AND FINALLY… A MATHEMATICAL SURPRISE!

1. Within the diagram to the right are three right triangles. These same three triangles are drawn below and labeled I, II, and III. Label angles and side lengths of these three triangles so that corresponding segments are easily identified.





2. Establish that the triangles are similar using the A-A criterion.

   $\triangle I \sim \triangle III$ because…

   $\triangle II \sim \triangle III$ because…

   $\triangle I \sim \triangle II$ because…

When triangles are similar, their sides are proportional.

3. Write an equation that states that $\dfrac{\text{length of shorter leg}}{\text{length of longer leg}}$ in triangles I and II are proportional. This proportion tells us that ax = _____.	4. Write an equation that states that $\dfrac{\text{length of hypotenuse}}{\text{length of shorter leg}}$ in triangles I and III are proportional. This proportion tells us that ax = _____.

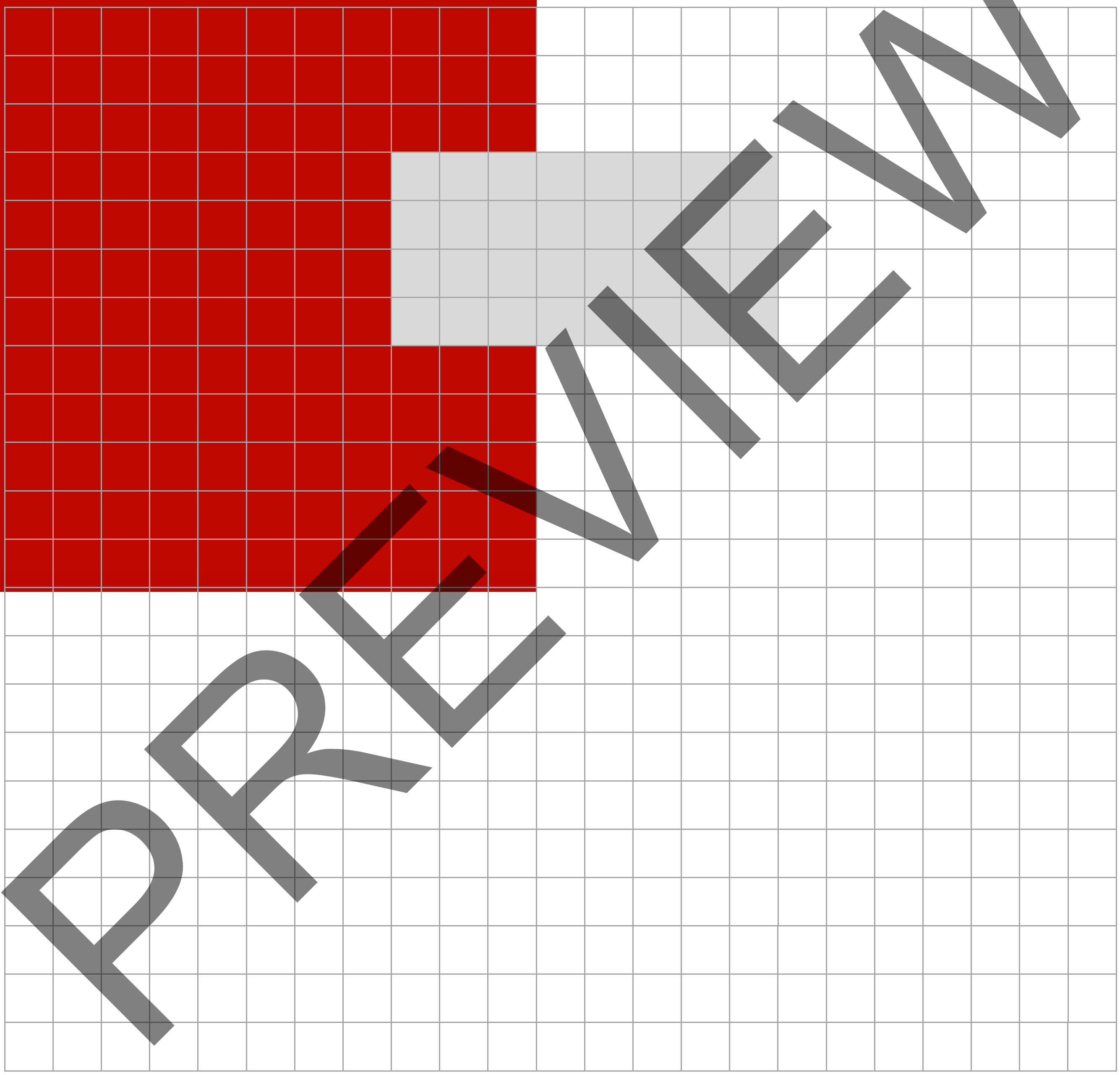
5. Since two expressions are equal to *ax*, they are equal to each other. Write the equality. Then rewrite it so there are no negative coefficients (no minus signs).

   What did you just prove?

REVIEW

SCALE UP!

Draw a simple picture, emoji, or your initials in the shaded space on the grid. Dilate your figure to fill the page as much as possible. Locate the center point of the dilation.



What is your scale factor?

POSTER PROBLEMS: SIMILARITY

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Follow your teacher's directions. Do the problems on posters.

Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
Use the rule: $(x, y) \longrightarrow (-x, y - 2)$	Use the rule: $(x, y) \longrightarrow (2x, 2y)$	Use the rule: $(x, y) \longrightarrow (x, x + y)$	Use the rule: $(x, y) \longrightarrow (2y, 2x)$

A. On grid paper, graph triangle *DEF* with vertices at *D* (3, 1), *E* (3, 3), and *F* (0, 1). Shade the triangle to show that it's the pre-image.

B. Copy the transformation rule. Write the image coordinates, graph the image, and label it.

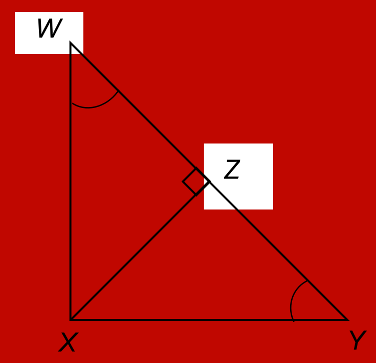
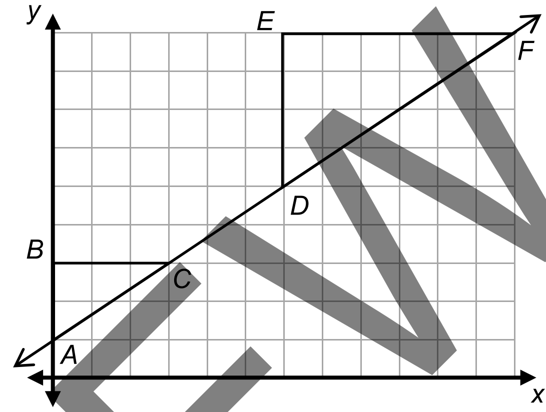
C. Explain why the pre-image and image are congruent, similar, or neither.

D. Describe properties of the transformation in terms of side lengths and angle measures.

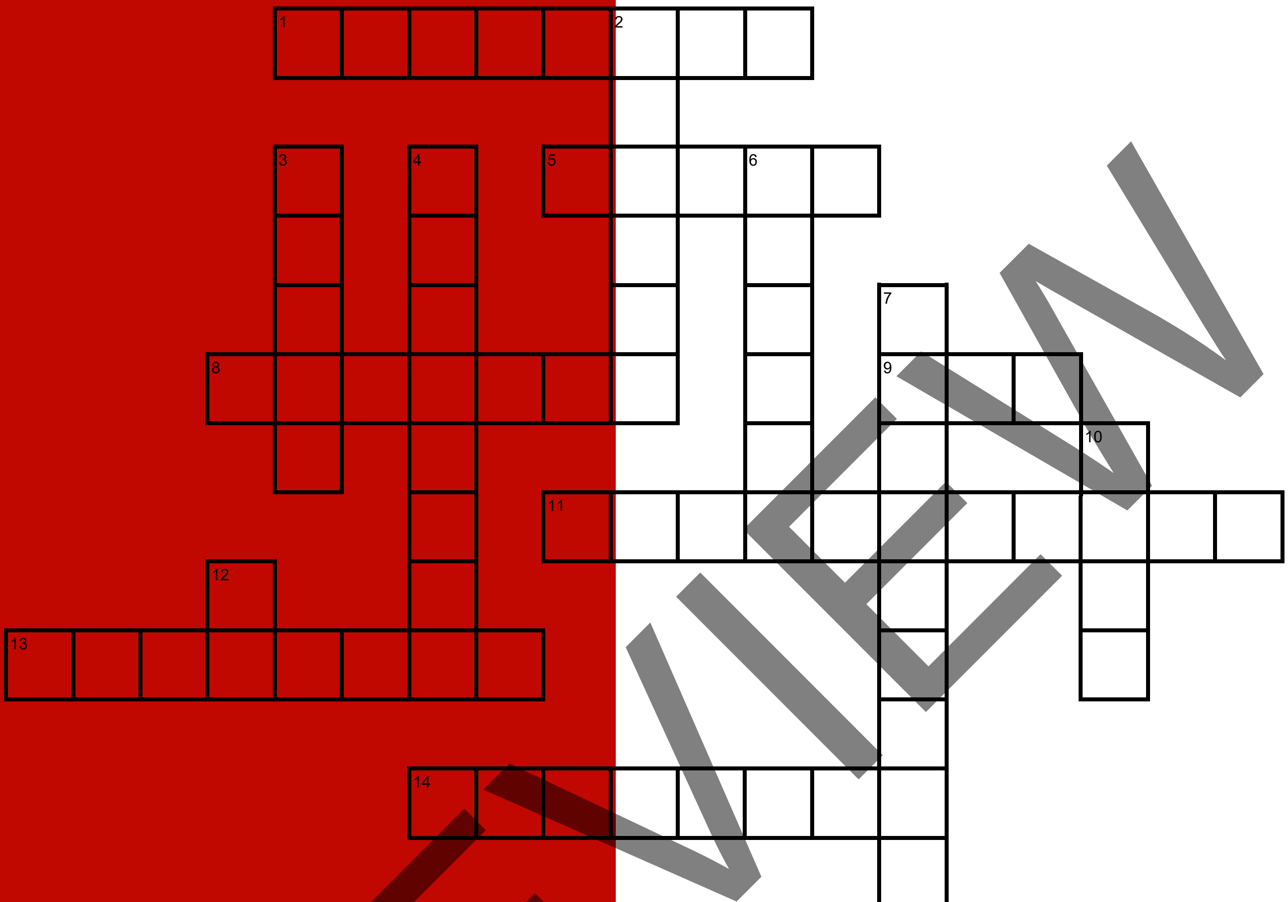
Part 3: Return to your seats and work with your group to critique your original poster. Review step C. Rewrite the explanation if it is not clear and concise. If it is, then explain using a different strategy.

MAKE LOGICAL ARGUMENTS

Discuss each statement below with your partner. Write an explanation to support the statement. Use abbreviations and symbols.

Use this diagram for problems 1 – 3. 	Use this diagram for problems 4 – 6.  (*A*, *C*, *D*, and *F* all lie on the same line.)
1. $\angle W \cong \angle Y$	4. $\triangle ABC \sim \triangle DEF$
2. If $\lvert\angle W\rvert$ = 45°, then $\lvert\angle WXY\rvert$ = 90°	5. $\frac{\lvert AB\rvert}{\lvert BC\rvert} = \frac{\lvert DE\rvert}{\lvert EF\rvert}$
3. If $\lvert\angle WXY\rvert$ = 90° then $\triangle WXY \sim \triangle XZY$	6. The slope of $\overline{AC}$ = The slope of $\overline{DF}$

FOCUS ON VOCABULARY



Across

1 opposite angles formed when two lines cross

5 ______measures do not change in rigid motions or dilations

8 figures that are a result of rigid motions and a dilation

9 scale factor of congruent figures

11 theorem to establish right triangles

13 horizontal lines are examples

14 image created using a scale factor

Down

2 origin of a dilation

3 motion that leads to congruent figures

4 The name of a figure prior to a dilation (usually hyphenated)

6 a measurement that is unchanged under a rigid motion

7 figures created from rigid motions

10 reductions have scale factor between _____ and one

12 criterion to establish similar triangles

SPIRAL REVIEW

1. **Alge-Grid: What's the *a*?** Each clue gives the value of a corresponding cell. Use clues to find *a*, which has the same value in all cells. Once evaluated, the cells will contain the whole numbers 1 – 9, exactly once each.

The Alge-Grid

$6a$	$(a + 1)^3$	$(a + 2)^2$
$a(a^2)(a^3)$	$10a \div 2$	$2a$
$(a + 1)^2$	$2a + 1$	$(a + 2)^2 - 2$

The Clues

Product of two different prime numbers	Number of faces on an octahedron
	Third prime number

2. Solve each system.

a. $\begin{cases} 3y = 15x + 15 \\ 5x = y \end{cases}$	b. $\begin{cases} -30 = -y - 5x \\ 4x + y = 24 \end{cases}$
c. $\begin{cases} -26 = -3x - 2y \\ -5y = 4x - 37 \end{cases}$	d. $\begin{cases} x = 7 + y \\ -4 = 3x - 8y \end{cases}$

SPIRAL REVIEW

Continued

3. Solve each equation, if possible.

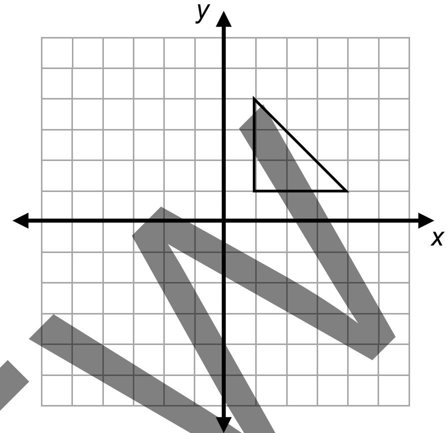
a. $x + 5 = 2x + 2$	b. $3(x + 2) = 2(x + 4)$	c. $-x - 5 = 4(x + 2) - 2$
d. $-8(x - 2) = 4(-2x + 4)$	e. $-x - 4 - x = -3(-2x - 2)$	f. $4x + 6 = 4(x + 6)$
g. $-(3x - 4) = -5(x + 2)$	h. $\frac{1}{4}x - 6 = \frac{1}{2}x + 2$	i. $-0.2(x - 2) = -0.4x + 1.6$

4. A linear equation in one variable may have:
- ______________________ solution(s)
- ______________________ solution(s)
- ______________________ solution(s)

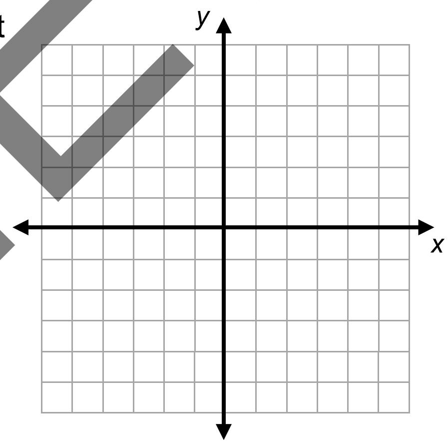
SPIRAL REVIEW

Continued

5. On the graph to right, translate the triangle (pre-image) 5 units left, and then reflect across the *x*-axis to get its image. Explain why the image and pre-image must be congruent.



6. On the graph to the right, graph triangle *BAT* with vertices at *B* (2, -1), *A* (6, -1), and *T* (6, 4). Shade this pre-image triangle.



   a. Translate the pre-image using this mapping rule: $(x, y) \rightarrow (x - 3, -y + 1)$. Graph the image, label its vertices, and write its vertex coordinates.

   b. Explain why the image and pre-image must be congruent.

   c. Why is it irrelevant to consider parallel lines being taken to parallel lines?

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- Use transformational geometry to investigate congruence and similarity
- Extend applications of volume to cylinders, cones, and spheres
- Explore bivariate data
- Complete the real number system
- Solve linear equations in one variable and linear systems in two variables
- Discover and apply properties of lines, angles, and triangles, including the Pythagorean Theorem
- Create, analyze, and use linear functions in problem solving
- Explore exponents and roots, and very large and very small quantities

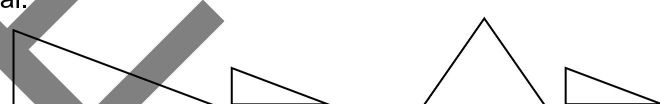
Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Why are transformations considered to be a type of function [SMP7]? Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **More Connections.** Describe something new that you learned about how rigid motions, dilations, and similarity are related.

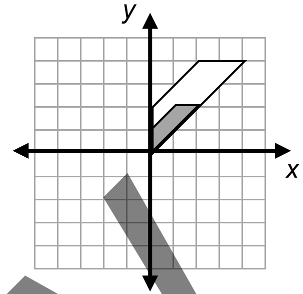
STUDENT RESOURCES

Word or Phrase	Definition
congruent figures	Two figures in the plane are <u>congruent figures</u> if the second can be obtained from the first by a sequence of one or more translations, rotations, and reflections. Two squares are congruent if they have the same side-length.  congruent not congruent
dilation	A <u>dilation</u> is a transformation that moves each point along the ray through the point emanating from a fixed center, multiplying distances from the center by a common scale factor. The fixed center is referred to as a "center point." The transformation of the plane mapping $(x, y) \longrightarrow (2x, 2y)$ is a dilation with center at the origin and scale factor 2.
image	The <u>image</u> of a function or transformation is the collection of its output values. The input values are then referred to as the pre-image. See <u>transformation</u>.
rigid motion	A <u>rigid motion</u> is a transformation that preserves distances. Any rigid motion of the plane is a sequence of one or more translations, rotations, and reflections. Rigid motions also preserve lengths, angle measures, and parallel lines.
scale factor	A <u>scale factor</u> is a positive number which multiplies some quantity.
similar figures	Two figures in the plane are <u>similar figures</u> if one can be moved to exactly cover the other by a sequence of one or more translations, rotations, reflections, and dilations. In similar figures, corresponding angles are congruent, and lengths of corresponding sides are proportional.  similar not similar If $\triangle ABC$ is similar to $\triangle DEF$, we write $\triangle ABC \sim \triangle DEF$.
transformation	A <u>transformation</u> is a function that maps points in the plane (called the pre-image) to points in the plane (called the <u>image</u>). Translations, rotations, reflections, and dilations are transformations of the plane.

Dilations of the Plane

A <u>dilation</u> is a transformation of the plane that is used to resize an object.
Multiplying by a scale factor:

- greater than 1 results in an enlargement of the pre-image.
- between 0 and 1 results in a reduction of the pre-image.
- equal to 1 results in a figure congruent to the pre-image.



The transformation of the plane to the right

- has a center at the origin and a scale factor of 2, and
- can be represented by the rule $(x, y) \longrightarrow (2x, 2y)$.

Dilations share many but not all of the properties of translations, rotations, and reflections. Dilations

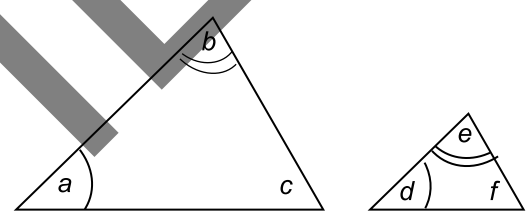
- map lines to lines (in fact to lines with the same slope),
- map parallel lines to parallel lines (that is, they preserve parallelism), and
- map angles to angles of the same measure.

However, dilations DO NOT, in general, preserve distances. The only dilation with the center at the origin that preserves distances is the identity transformation $(x, y) \longrightarrow (x, y)$, which has scale factor $s = 1$.

The Angle-Angle Criterion for Similarity of Triangles

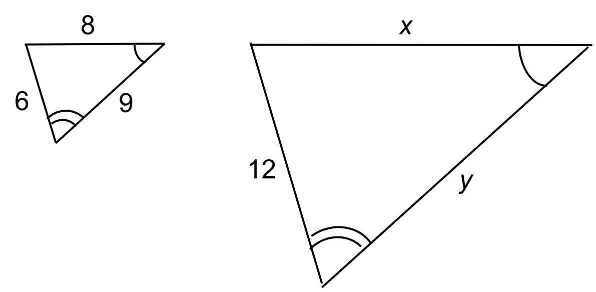
<u>A-A criterion</u>: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

In the figures to the right, if the sums of the angles in both triangles are to be 180º, then angles *c* and *f* must have the same measure. Therefore, the two triangles must be similar.



Finding Side Lengths of Similar Triangles

If the two triangles below are similar, there are two basic ways to set up proportions to find missing side lengths.



Method 1: Establish values of ratios of corresponding segments **between** the two figures.

$\frac{6}{12} = \frac{8}{x} \longrightarrow x = 16$ and $\frac{6}{12} = \frac{9}{y} \longrightarrow y = 18$

Method 2: Establish values of ratios of corresponding segments **within** the two figures.

$\frac{8}{6} = \frac{x}{12} \longrightarrow x = 16$ and $\frac{9}{6} = \frac{y}{12} \longrightarrow y = 18$

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
8.G.A	**Understand congruence and similarity using physical models, transparencies, or geometry software.**
8.G.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
8.G.5	Use informal arguments to establish facts ~~about the angle sum and exterior angle of triangles,~~ about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
8.G.B	**Understand and apply the Pythagorean Theorem.**
8.G.7	Apply the Pythagorean Theorem to determine the unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.EE.B	**Understand the connections between proportional relationships, lines, and linear equations.**
8.EE.2	~~Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where *p* is a positive rational number.~~ Evaluate square roots of small perfect squares ~~and cube roots of small perfect cubes. Know that √2 is irrational.~~
8.EE.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; ~~derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.~~
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; ~~give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*~~

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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