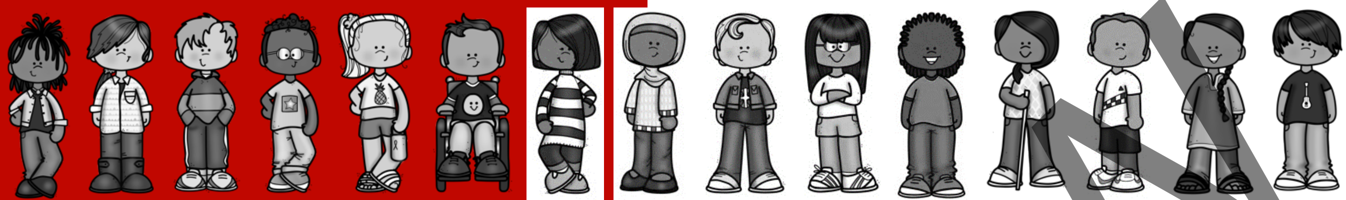
Name ______________________________ Period __________ Date ____________

Math Links GRADE 7

UNIT 7 STUDENT PACKET



EQUATIONS AND INEQUALITIES

Parent (or Guardian) signature ___________________________________

MY WORD BANK

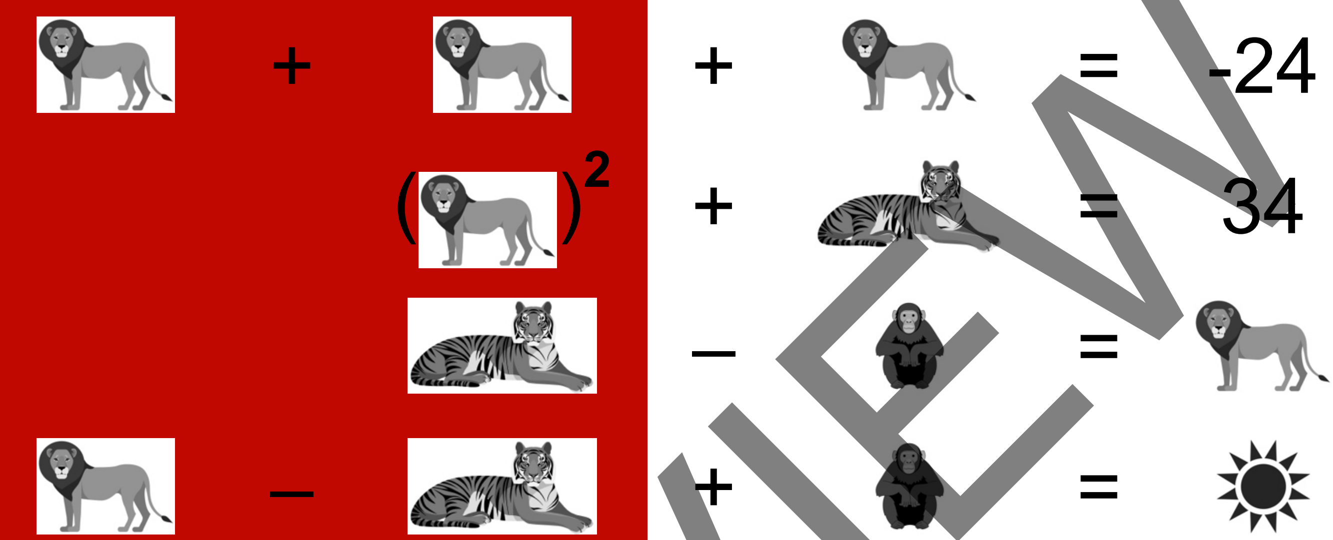
Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

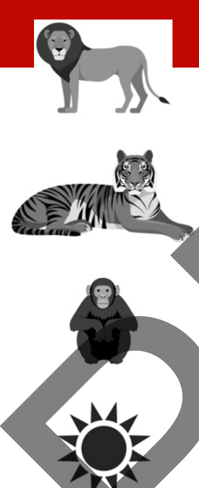
boundary point of a solution set	equation solution to an equation solve an equation
inequality	substitution

LIONS AND TIGERS AND BEARS

On this page, the same animals have the same value. Different animals have different values.

Find the value of each animal and the value of ☀. Explain your reasoning.





SOLVING EQUATIONS USING SUBSTITUTION

We will use substitution as a strategy to solve equations. Then we will use these equation-solving skills to solve number puzzle problems, given visual and written clues.

[7.NS.3, 7.EE.4a; SMP1, 7]

GETTING STARTED

Solve for the unknown. Write MM if you use mental math. Otherwise, show work. Check each solution by substituting it into the original equation.

1. $4 = v - 2$	2. $6u = 24$	3. $p + 55 = 95$
4. $\frac{1}{2} = 6$	5. $4.2 = n - 1.8$	6. $\frac{k}{4} = 5$
7. $1.25 = 5t$	8. $\frac{d}{3} = \frac{15}{9}$	9. $\frac{1}{3}x = \frac{4}{15}$

10. Record the meanings of equation and solution to an equation / solve an equation in **My Word Bank**.

THE COVER UP METHOD

Follow you teacher's directions for (1) – (7). Solve and check each equation using substitution.

(1)	(2)	(3)
(4)	(5)	(6)
(7)	8. $50 = 35 + (-5n)$ Check:	9. $-6(p - 5) = -54$ Check:
10. $98 = -9y - 1$ Check:	11. $\frac{-30 - w}{6} = -6$ Check:	12. $\frac{1}{3}(-2x + 4) = 2$ Check:

PRACTICE 1

1. Record the meaning of <u>substitution</u> in **My Word Bank**.

Solve and check each equation using substitution.

2. $4 + 12b = 100$	3. $15 = 5y + 20$	4. $6n - 5 = -65$
5. $0 = 23(0.5 + x)$	6. $\frac{m}{-4} + 6 = 7$	7. $-4(p - 10) = 100$
8. $-6 = \frac{42}{n + 1}$	9. $-\frac{1}{4}(x + 33) = -8$	10. $-(0.5y + 1.5) = -3$

11. The weight of a small bag of apples, *a*, is unknown. The weight of a small bag of oranges is 5 pounds.

a. Write an expression for the weight of a large grocery bag filled with a small bag of oranges and a small bag of apples.	b. Write an expression for the weight of 3 large grocery bags, each filled with a small bag of oranges and a small bag of apples.
c. Write an equation to show that the total weight of the 3 large grocery bags is 36 pounds. Then solve the equation.	d. What does the solution to the equation represent?

INTRODUCTION TO THE HUNDRED CHART PUZZLE

To the right is a hundred chart.

1. State two patterns you notice on the chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Suppose a hundred chart is cut into puzzle-type pieces. For problems 2 – 4, write in the missing numbers. For problems 5 – 7, write in the missing algebraic expressions.

2.

		45	

5.

n		

3.

68		

6.

	n

4.

		28	

7.

n		

THE HUNDRED CHART PUZZLE

Follow your teacher's directions.

(1) $\sum$ → This is the ______________________. It means __________.

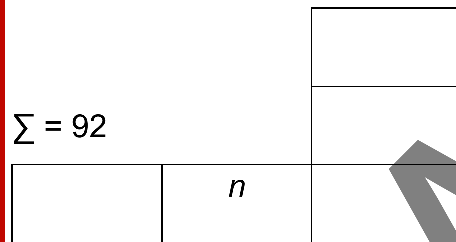
(2)	
(3)	
(4)	
(5)	

PRACTICE 2

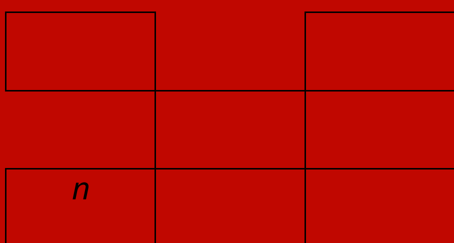
Here are some puzzle pieces from different hundred square puzzles. Write in variable expressions and use your equation-solving skills to find the numbers in the pieces.

1. $\sum = 138$

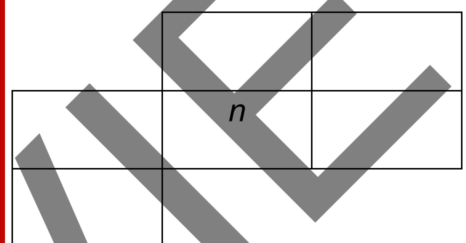
2. $\sum = 92$



3. $\sum = 110$



4. $\sum = 182$



Keith and Mick built square tile patterns and kept track of the steps in the tables below.

5. Fill in the number of tiles in step 5 for each, and then write input-output rules (equations).
6. Find the number of tiles in step 100 for each.
7. Find the step number for the given number of tiles in the last row for each.

Keith's tile pattern	
Step # (x)	**# of tiles (y)**
1	9
2	17
3	25
4	33
5	
100	
	401
Rule:	

Mick's tile pattern	
Step # (x)	**# of tiles (y)**
1	7
2	15
3	23
4	31
5	
100	
	159
Rule:	

8. How are these patterns the same?

9. How are they different?

SOLVING EQUATIONS USING ALGEBRA

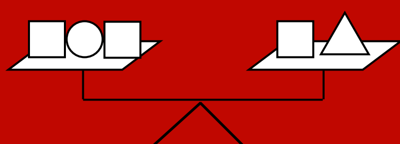
We will use the concept of balance with a cups and counters model to solve equations. Then we will use properties of arithmetic and equality to solve equations algebraically, and apply these skills to solving problems.

[7.NS.3, 7.EE.1, 7.EE.4a, 7.RP.2ac; SMP1, 2, 3, 4, 5, 6, 7, 8]

GETTING STARTED

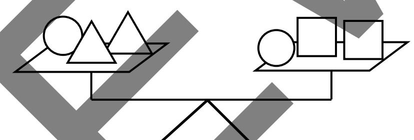
In each problem below, all the shapes have some weight, the same shapes have the same weight, and different shapes have different weights. All problems are independent of one another. Use what you know about balance to answer each question.

1.



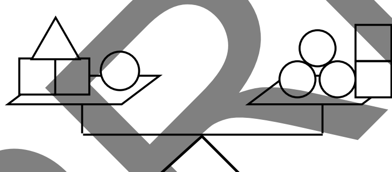
What weighs the same as △ ?

2.



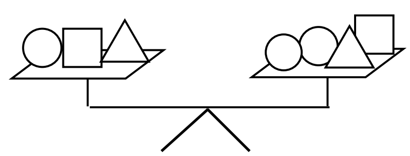
What weighs the same as □ ?

3.



What weighs the same as △ ?

4.



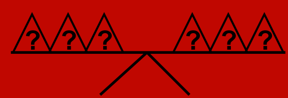
This scale shows balance. Why is this incorrect?

BALANCED AND UNBALANCED SCALES

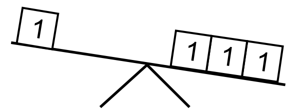
We can picture equalities with balanced scales and inequalities with unbalanced scales. Imagine that each [1] represents one unit of weight and each [?] represents an unknown weight (not equal to zero). To represent unknowns, a popular variable is x.

This is a balanced scale (equation). →

Equation
$3x = 3x$

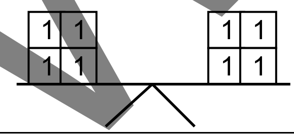


Inequality
$1 < 3$



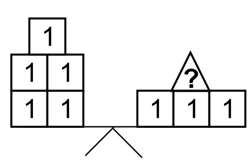
← This is an unbalanced scale (inequality)

For each problem, **start with this original balanced scale**, 4 = 4.
Draw a sketch to illustrate the action described.
Write the resulting equation or inequality.



1. Three units (1's) are removed from both sides of the original balanced scale. equation or inequality:	2. One unknown (x) is added to both sides of the original balanced scale. equation or inequality:
3. Two units are removed from the right side of the original scale. equation or inequality:	4. Two x's are added to the right side, and one x to the left side of the original scale. equation or inequality:
5. The number of units on both sides of the original scale is doubled. equation or inequality:	6. One-half the units are removed from each side of the original scale. equation or inequality:

7. Iggy built the balanced scale to the right.



a. Write the equation it represents.

b. Why can Iggy remove 3 units from both sides?

c. Draw the new balanced scale and write the equation it represents.

d. Does the equation in part c represent the solution to the equation in part a?

SOLVING EQUATIONS WITH BALANCE

Follow your teacher's directions.

(1)

Positive Counter	Negative Counter	Cup	Upside-down Cup

(2)

(3)

(4)

(5)

PRACTICE 3

For each equation, first build it with cups and counters. Continue the building process until it is solved. Record the process with drawings. Write the solution and check it using substitution.

1. $-6 + x + 3x + 5 = -7 - 2$

   Build and solve:

   Check:

2. $8 = 5 + 2(x - 4) - x$

   Build and solve:

   Check:

3. $-x + 1 + 5 - 4x = 1 - 5$

   Build and solve:

   Check:

4. $-7 = x - 4(x - 1) - 2$

   Build and solve:

   Check:

Explain each student's mistakes below.

5. Hoagy sees 2 positive counters and 2 negative counters and says, "I'm going to remove them because they are zero pairs."

   V + + | – –

6. Tito says, "I see 3 cups on one side and 6 positive counters on the other, so $x = 2$."

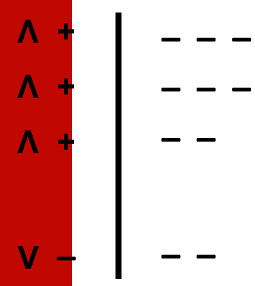
   Λ Λ Λ | + + + + + +

SOLVING EQUATIONS ALGEBRAICALLY

Follow your teacher's directions.

(1)	
(2)	
(3)	
(4)	
(5)	(6)

PRACTICE 4

1. Write the equation pictured to the right so that there are parentheses on the left side, and the distributive property must be applied. Then solve the equation, showing all algebraic steps. Draw as desired.	

Solve by showing all algebraic steps.

2. $-2x - 1 - 6x = 23$	3. $12 = 4(x + 2) + 4 + x$	4. $4 - 4(x - 1) + x = -7$

Circle the part of each equation-solving process that contains a mistake. Correct it and continue the solution process underneath the problem.

5. $-12 = -3x + 15-6x$ $-12 = 3x + 15$ $-27 = 3x$ $-9 = x$	6. $0 = 10 - 2(x + 5) + 30 + 12x$ $0 = 10 - 2x + 5 - 30 + 12x$ $0 = 45 + 10x$ $-45 = 10x$ $-4.5 = x$	7. $7x - 3(2x - 1) - 2x = -2$ $7x - 6x + 3 - 2x = -2$ $-x + 3 = -2$ $-x = -5$
Correction(s):	Correction(s):	Correction(s):

8. Circle all of the following equations that are equivalent to $4 + 6x = 22$.

$6x = 18$ $4 + x = 16$ $22 = 4 + 6x$ $\frac{4 + 6x}{2} = \frac{22}{2}$ $3(4 + 6x) = 66$

PRACTICE 5

Joey and Tommy discussed how they might solve the equation $20d + 78 = 12$.

1. Joey said, "First I'm going to divide the expressions on both sides of the equation by 20." Even though Joey's strategy is permissible, why might it be difficult to execute?

2. Tommy said, "First I'm going to subtract 78." Even though Tommy has the right idea, explain why this language is not precise.

3. Solve the equation above. Show your work.

4. Dee Dee has \$240 in his savings account. He deposits \$20 per month for several months.

   a. Write a numerical expression for the amount of money that is in Dee Dee's account after 6 months.

   b. Write a variable expression for the amount of money that is in Dee Dee's account after n months.

   c. Write an equation to represent that Dee Dee has \$580 in his account after n months. Then solve the equation for n.

   d. Dee Dee decides to be more ambitious about saving. With the same initial amount of money in his account and with deposits of \$25 per month, how long will it take for him to save \$580? Show all work by writing and solving an equation.

5. Circle all the equations below that are equivalent to $0 = 3(x – 2) + 5$.

$-5 = 3(x – 2)$ $3(x – 2) + 5 = 0$ $3x – 2 + 5 = 0$

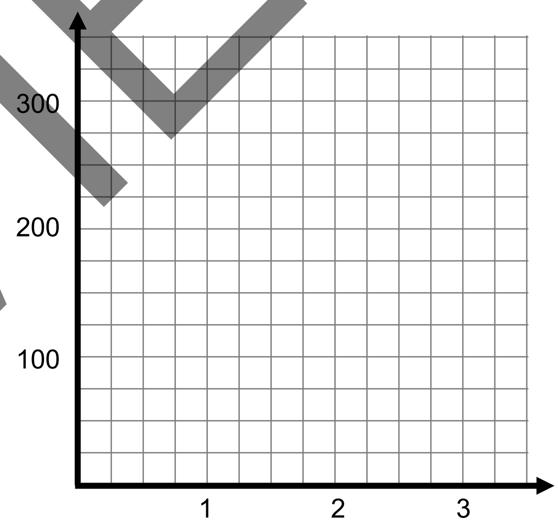
JOAN'S PHONES

Joan's Phones sells cheap calling plans. There is a set-up cost of $20, and then there is an annual charge of $144. If you wish, you may be billed semi-annually or quarterly.

1. Before ever using the phone (0 months, 0 days), you pay $____.
2. If billed annually, after 1 year you will pay ____ + ____ = $____.
3. If billed semi-annually, after half a year, you will pay ____ + ____ = $____.
4. Complete the table below showing various costs for Joan's phone plan.

Time in years (*t*)	0	1	2	3	0.5	0.25	1.5	3.25
Cost in dollars (*c*)								

5. Draw and label the graph to the right for Joan's phone plan.
6. Write an equation that relates *t* and *c*.
7. Is this a proportional relationship? Why?



Determine the number of years of phone plan use that result in the phone costs indicated below. For non-whole numbers of years, also write the solution in years and months.

8. $c = 272$	9. $c = 632$	10. $c = 128$

11. Ric thinks that the point (1, 144) should lie on the graph since the cost is $144 per every 1 year of service. Explain to Ric why this isn't true.

INEQUALITIES

We will explore the conditions under which the sign of an inequality is preserved or reversed, and solve and graph inequalities.

[7.NS.3, 7.EE.1, 7.EE.4ab; SMP1, 3, 6, 7, 8]

GETTING STARTED

1. Circle all of the true inequalities below. Then use these inequalities for problems 2 – 3.

a. $(-1)(7) > (-1)(8)\frac{1}{1}$ b. $\frac{5}{12} < \frac{5}{9}$ c. $3(4 - 8) < 3(4) - 8$

d. $21 + 9 < 9 + 21\frac{1}{1}$ e. $\frac{3}{5} < \frac{6}{13}$ f. $-4(3) + 6(3) < -(4 + 6)$

2. Choose one false inequality above with a fraction or decimal and explain why it is false.

3. Choose one false inequality above with mathematical operations and explain why it's false.

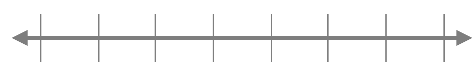
Under each inequality below are four potential solutions. Circle all the two solutions that make each inequality true. Then write a description of ALL of the numbers that could be solutions to the inequality.

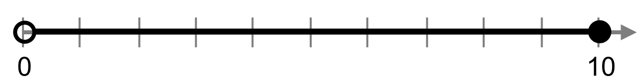
4. $v + 5 > -2$	5. $w - 4 < -4$	6. $2x > -20$	7. $\frac{y}{3} < -6$
$v = -8$	$w = 0$	$x = 10$	$y = 18$
$v = -7$	$w = -1$	$x = -10$	$y = -18$
$v = -6$	$w = 0.5$	$x = -9$	$y = -24$
$v = -6.9$	$w = -0.5$	$x = -10.5$	$y = -21$
Description:	Description:	Description:	Description:

8. Record the meaning of <u>inequality</u> in **My Word Bank**.

GRAPHING INEQUALITIES

Follow your teacher's directions.

	Equation or inequality	Written description of all solutions	Graph of all solutions
(1)			
(2)			
(3)			
(4)			
(5)			
(6)			
(7)			

(8)	
(9)	

PRACTICE 6

1. Record the meaning of <u>boundary point of a solution set</u> in **My Word Bank**.

Fill in the table for each inequality below.

Inequality		Written description of all solutions	Graph of all solutions
2.	$v > -3$		
3.	$w \leq 1$		

4. Circle all numbers below that are solutions to the inequality $x + 1 \geq -5$.

-7 -6 -5.9999 -6.0000001 $-\frac{30}{5}$ 0 $\frac{3}{17}$

5. List four integers that are less than or equal to -2. ______ ______ ______ ______

6. List four non-integers that are less than or equal to -2. ______ ______ ______ ______

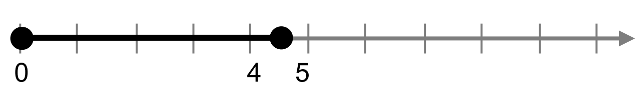
7. How many numbers exist that are less than or equal to -2? ____________________________

8. Why is -1 not a solution to the inequality $x \leq -2$? ________________________________

Write an inequality for each situation below. Label and scale each graph appropriately.

Situation		Inequality	Graph
9.	You must be at least 48 inches tall to ride the rollercoaster.	(let h = height)	
10.	To ride the rollercoaster, wait time is more than 16 minutes.	(let t = time)	

Write situations of your choice for the graphs below.

11.		

EXPLORING INEQUALITIES

Complete the tables below. For the last column, write a new inequality that reflects the change in values after operating on the original inequality each time. Be sure that your new inequality is in fact a true statement.

1. Begin each operation with this inequality:	…then do this to both sides…	Steps: Left	Steps: Right	True inequality with the resulting values
4 < 10	Add 5	4 + ____	10 + ____	9 < 15
	Add -5			
	Subtract 3			
	Subtract -3	4 – (____)		
	Multiply by 8			
	Multiply by -8	4(____)		
	Divide by 2			
	Divide by -2	4 ÷ (____)		

2. Begin each operation with this inequality:	…then do this to both sides…	Steps: Left	Steps: Right	True inequality with the resulting values
-3 > -6	Add 5	-3 + ____	-6 + ____	
	Add -5			
	Subtract 7			
	Subtract -7			
	Multiply by 2			
	Multiply by -2			
	Divide by 3			
	Divide by -3			

3. In the table above, look closely at the last column and circle every result where the inequality changed direction compared to the original inequality.

4. Under what circumstances did the direction of the inequality symbol change?

SOLVING INEQUALITIES

Follow your teacher's directions.

(1) Equation:	Check solution:
(2) Related Inequality	Check solution:
(3) Equation:	Check solution:
(4) Related Inequality	Check solution:
(5) $1 < 13 - x - 2x$	

PRACTICE 7

Rewrite each inequality below so that the variable is on the left side and graph it.

1. $-2 \geq x$	2. $-4 < y$	3. $3 \leq z$

4. Circle all the inequalities below that are equivalent to $7 > -2x - 1$.

$-4 > x$	$-4 < x$	$x > -4$	$x < -4$
$-3 > x$	$-3 < x$	$x > -3$	$x < -3$

For each inequality below, solve, check the boundary point and one other point, and graph.

5. $20 - 5x + 8x > -1$	6. $-2 \geq 4x - 3(x - 1)$
7. $8 - 6x + 4(x - 6) \leq 6 - 12$	8. $-12 - 4 < 2(5 - x)$

EQUATIONS AND INEQUALITIES WITH RATIONAL NUMBERS

We will solve equations and inequalities that include rational numbers. We will use equations and inequalities to solve problems.

[7.NS.3, 7.EE.1, 7.EE.4a, 7.EE.4b; SMP1, 3, 4, 6, 8]

GETTING STARTED

For each verbal statement below, choose an appropriate symbolic representation from the given choices. Some choices may not be used at all, and some may be used more than once.

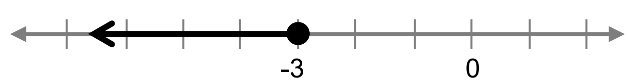
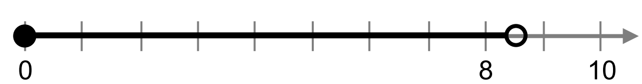
Words		Choices
1. Matt has less than \$2.50.		A. $x = 2.5$
2. Derek has exactly \$2.50.		B. $x \approx 2.5$
3. Shannon has \$2.50 or more.		C. $x > 2.5$
4. Ron has at least \$2.50.		D. $x \geq 2.5$
5. Steve has \$2.50 at the most.		E. $x < 2.5$
6. Lamar has approximately \$2.50.		F. None of the above

7. Graph $2.5 < x$.

8. Why must the inequality $n > n$ always be false?

9. Why must the inequality $n + 1 > n$ always be true?

Describe a situation that could be represented by each graph.

10.	
11.	

EQUATIONS WITH RATIONAL NUMBERS

Follow your teacher's directions.

(1)	(2)
(3)	(4)
(5)	(6)

PRACTICE 8

Solve each equation below and check.

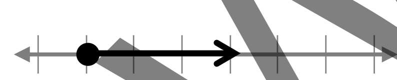
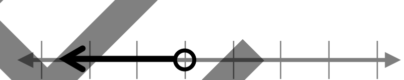
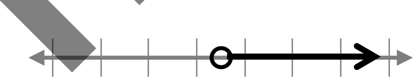
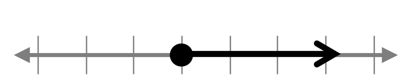
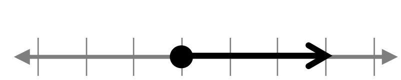
1. $-1.4 = 2.2 - 4(y - 1.1)$	2. $-\frac{1}{6} = \frac{2}{3}(h - 1)$
3. $4p + 1\frac{1}{2} - 6p = -\frac{3}{4}$	4. $\frac{-k - 2 - 3k}{-10} = -1$

Identify your variable(s), write an equation, solve the equation, and answer the question.

5. Hamish is making a large dinner for his family. He buys 6 pounds of lamb chops, and then spends another \$11.70 on potatoes and vegetables. The total comes to \$61.20. What is the price per pound of lamb chops?	6. A rectangle has a perimeter of 60 cm. Its width is one-third its length. What are its dimensions?

GRAPH MATCH

The solutions to problems 1 - 4 on **Practice 8** are the boundary points for problems 1 – 8 below. Use what you know about inequalities to write the solutions and match the them to their correct graphs.

____	1. $-\frac{1}{6} < \frac{2}{3}(h-1)$	a.	
____	2. $4p + 1\frac{1}{2} - 6p > -\frac{3}{4}$	b.	
____	3. $\frac{-k-2-3k}{-10} \geq -1$	c.	
____	4. $-\frac{1}{6} \leq \frac{2}{3}(h-1)$	d.	
____	5. $-1.4 \geq 2.2 - 4(y-1.1)$	e.	
____	6. $\frac{-k-2-3k}{-10} = -1$	f.	
____	7. $-1.4 < 2.2 - 4(y-1.1)$	g.	
____	8. $4p + 1\frac{1}{2} - 6p \leq -\frac{3}{4}$	h.	

PRACTICE 9

Solve each equation or inequality and graph it.

1. $\frac{2}{3}n - \frac{1}{2} > -\frac{5}{6}$

2. $7.5(2 - y) > 0$

3. $\frac{1}{2}\left(m + \frac{1}{4}\right) > 1\frac{3}{8}$

4. $\frac{-(6d - 8)}{2.5} = 4$

Solve each problem using algebra (an equation or inequality). Define variables, answer the question and check the solution(s).

5. Gerardo is a salesperson. He is paid $300 per week plus $15 per sale. This week he wants his pay to be more than $900. How many sales does he have to make this week?

6. There are three numbers: an original number, half of the original number, and twice the original number. The sum of these three numbers is -49. What are the three numbers?

IESHA'S SUMMER

For each problem below, write an inequality, solve it, and graph the solutions. Then explain each answer in the context of the problem.

1. Iesha has $460 in a checking account at the beginning of summer. She wants to leave at least $200 in her account by the end of summer. She withdraws $25 each week for her expenses. How many weeks can Iesha withdraw this amount of money from this account?

2. A taxi service charges a $2.25 flat rate in addition to $0.64 per mile. Iesha wants to spend no more than $10 on a ride. How many miles can Iesha travel without exceeding her limit?

3. Iesha goes to the Fun Golf Arcade with her friends. They play golf, have lunch, and then play some video games. A round of golf is $6.20. Lunch is $5.60. Video games are $0.50 each. If Iesha wants to spend no more than $20.00, how many video games can she play?

REVIEW

BIG SQUARE PUZZLE: EQUATIONS AND INEQUALITIES

Your teacher will give you a puzzle to assemble. Below is one of the equations in the puzzle. Explain or show two different ways to solve it.

$\frac{1}{4}(26 - x) = 5$	$\frac{1}{4}(26 - x) = 5$

WHY DOESN'T IT BELONG?: EQUATIONS AND INEQUALITIES

Solve and graph each of the algebraic sentences below. Choose one and explain why its solutions don't belong with the others. Then choose at least one more and explain why its solutions don't belong.

A $-4x + 2 \leq -10$	**B** $2 - 4x \geq -10$
C $-8 = -6x + 10$	**D** $-28 > -10 - 6x$

OPEN MIDDLE PROBLEMS: EQUATIONS AND INEQUALITIES

Use exactly three of the digits 1 through 9 one time each.

1. Structure:

$$\square x + \square = \square$$

   a. Write an equation and find its solution.

   b. Write an equation so that it has the greatest possible solution.

   c. Write an equation so that it has the least possible solution.

2. Structure:

$$\square x - \square = \square$$

   a. Write an equation and find its solution.

   b. Write an equation so that it has the greatest possible solution.

   c. Write an equation so that it has the least possible solution.

POSTER PROBLEMS: EQUATIONS AND INEQUALITIES

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is __________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
$\frac{1}{2}\left(v+\frac{1}{3}\right)-\frac{1}{4}v \geq -1$	$1\frac{1}{2}-1\frac{1}{2}(w-1) > 2\frac{1}{4}$	$\frac{5}{3} < \frac{1}{5}x-2\left(x-\frac{1}{3}\right)$	$-\frac{3}{4} \leq -\frac{4}{5}-\left(y-\frac{1}{2}\right)$

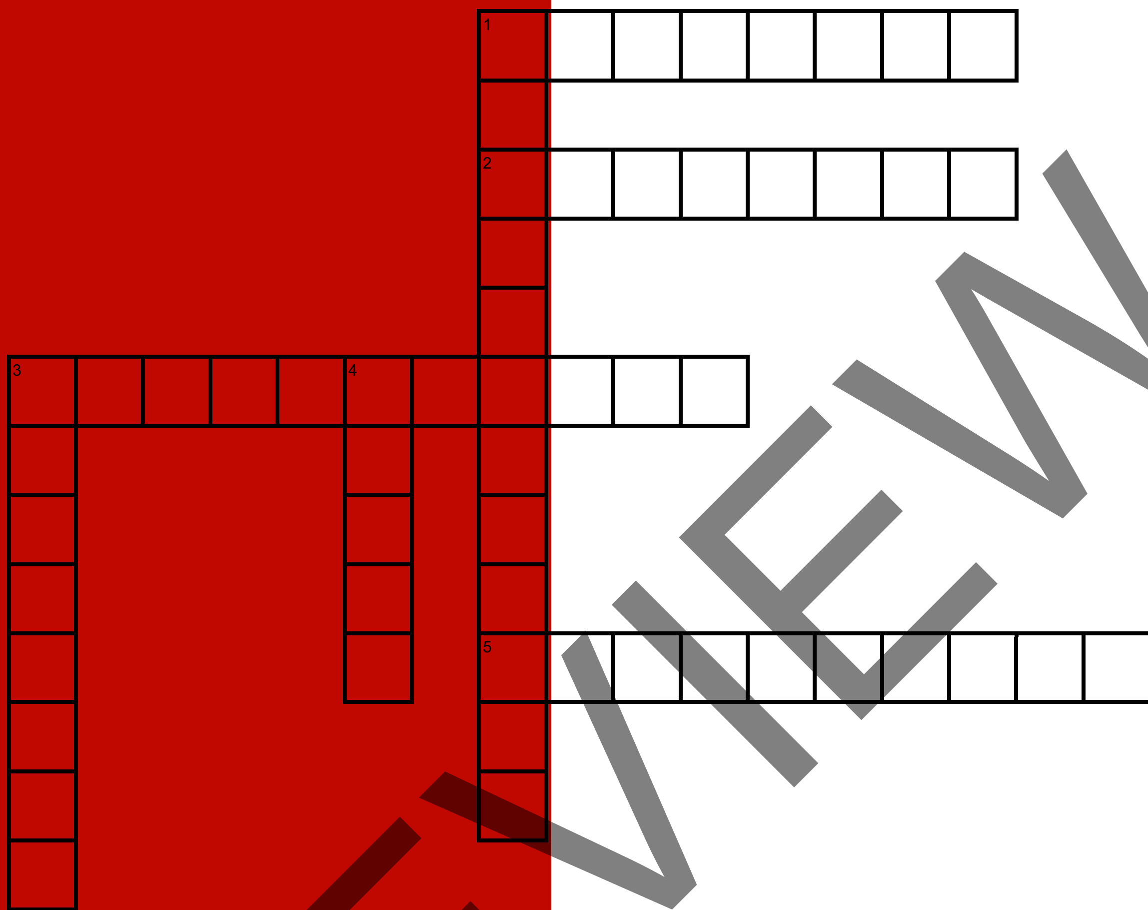
A. Copy the inequality and apply the distributive property (one step).
B. Collect like terms (one more step).
C. Solve for the values of the variable (two more steps).
D. Check the boundary point and at least one other point. Graph the solutions.

Part 3:

1. Copy the problem from your start poster.

2. Solve your start poster problem in a different way.

VOCABULARY REVIEW



Across

1 a value that makes an equation or inequality true

2 The point where two expressions in an inequality are equal to each other.

3 examples include $3x – 5$, 4, x^2

5 an examples is $2x > 5 + 1$

Down

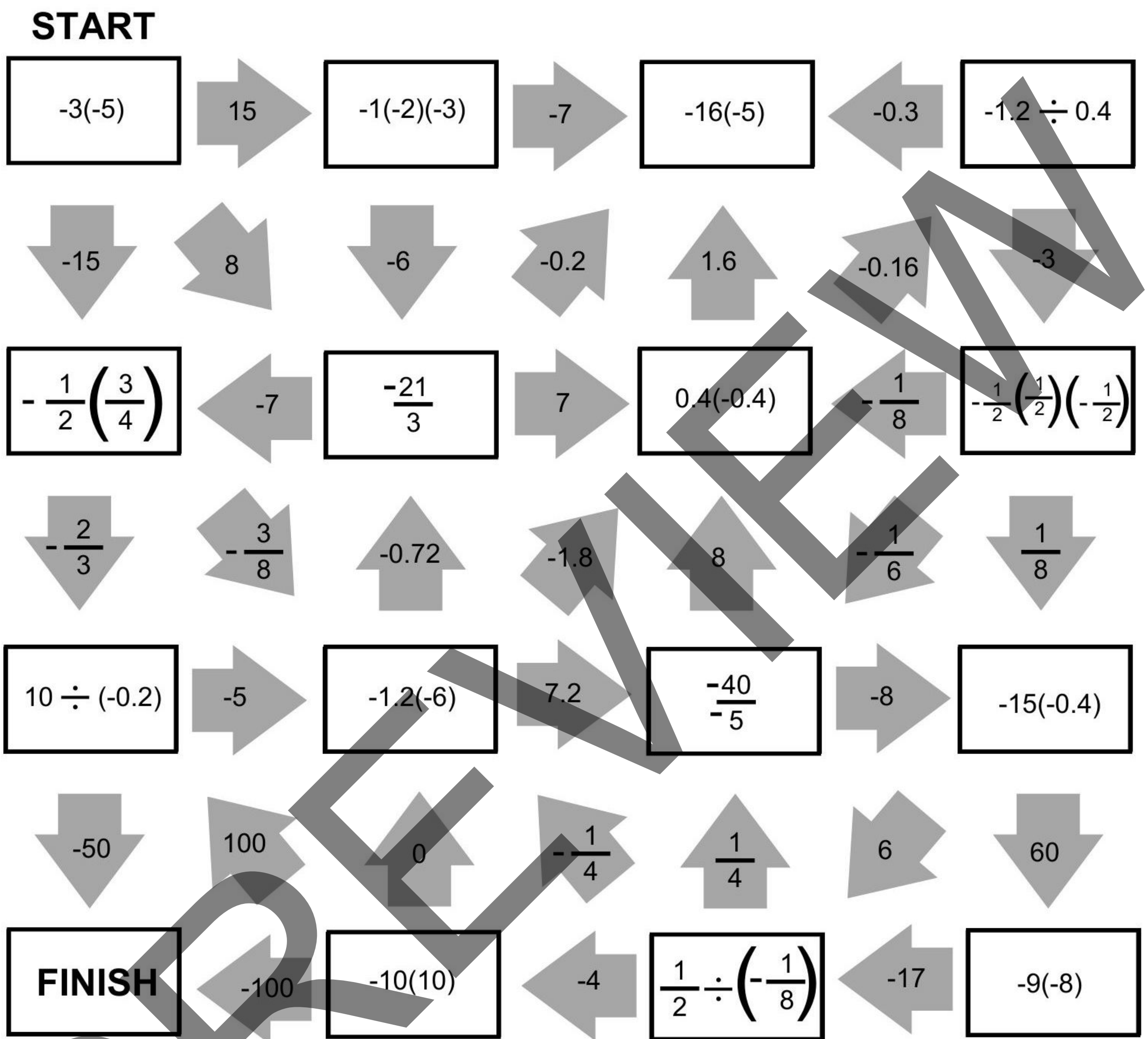
1 replacement of a quantity with one that is equal to it

3 a statement asserting that two expressions are equal

4 find all the values of the variable that make it true

SPIRAL REVIEW

1. Follow the math path to computational fluency.



2. Complete the table.

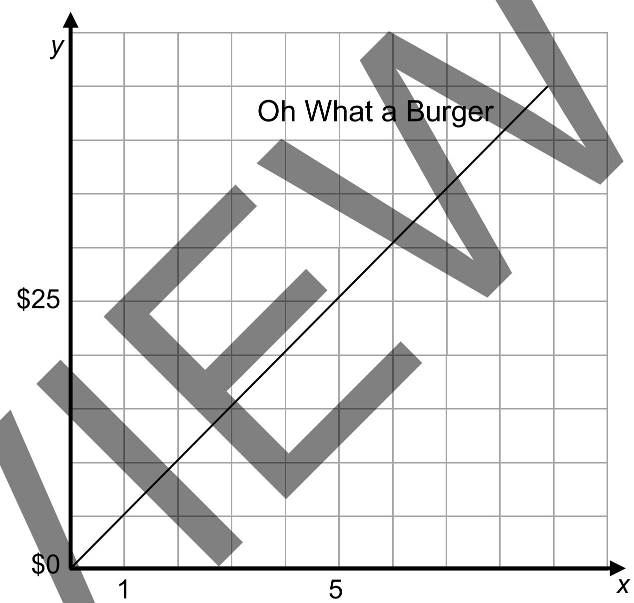
Fraction	$\frac{3}{40}$		$\frac{38}{25}$			
Decimal				0.92		0.015
Percent		1%			112.5%	

SPIRAL REVIEW
Continued

3. Linus is choosing from two different restaurants, Patty On Burgers and Oh What a Burger. Linus created tables and graphs to show how much he would have to pay for different amounts of burgers. Unfortunately, water spilled on parts of his tables and the graph for Patty On Burger and some information was erased.

PATTY ON BURGERS	
# of burgers (x)	**cost (y)**
1	4.50
2	
3	
4	18
5	
x	

OH WHAT A BURGER	
# of burgers (x)	**cost (y)**
0	
1	
2	
3	
4	
x	



a. Complete Linus' tables.

b. Complete the graph with labels, title, and graph Patty On Burgers.

c. Do both graphs represent proportional relationships? Explain.

d. What is the unit rate for Patty On Burgers?

e. What is the unit rate for Oh What a Burger?

f. Which burger place is the better buy? Explain.

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- Find the likelihood of events with probability.
- Sample to understand populations with statistics.
- Apply proportional reasoning to ratios, rates, percent and scale.
- Solve problems involving measurements of geometric figures.
- Operate with rational numbers and solve problems.
- Develop spatial reasoning in two- and three-dimensions.
- Use algebra as a problem-solving tool.

Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.

3. **Mathematical Practices.** Which properties of equality hold for inequalities? Which do not? Give a counter example to prove a property that does not hold [SMP3, 7]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **Making Connections.** What strategies that you used for solving simple equations helped you when the equations became more complex?

STUDENT RESOURCES

Word or Phrase	Definition
boundary point of a solution set	A <u>boundary point of a solution set</u> is a point for which any interval surrounding the point on the number line contains both solutions and non-solutions. If the solution set is an interval, the boundary points of the solution set are the endpoints of the interval. The boundary point for BOTH $x < -1$ AND $x \leq -1$ is $x = -1$. In the first case the boundary point IS NOT part of the solution set (open circle). In the second case it IS (closed circle).  
equation	An <u>equation</u> is a mathematical statement that asserts the equality of two expressions.
expression	A mathematical <u>expression</u> is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are $7x$, $a + b$, $4v - w$, $\frac{8+x}{10}$, and 19.
inequality	An <u>inequality</u> is a mathematical statement that asserts the relative size or order of two objects. When the expressions involve variables, a <u>solution to the inequality</u> consists of values for the variables which, when substituted, make the inequality true. $5 > 3$ is an inequality. $x + 3 > 4$ is an inequality. All values for x that are greater than 1 are solutions to this inequality.
solution to an equation	A <u>solution to an equation</u> involving variables consists of values for the variables which, when substituted, make the equation true. The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.
solve an equation	To <u>solve an equation</u> refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. To solve the equation $2x = 6$, one might think “two times what number is equal to 6?” Since $2(3) = 6$, the only value for x that satisfies this condition is 3. Therefore 3 is the solution.
substitution	<u>Substitution</u> refers to replacing a value or quantity with an equivalent value or quantity. If $x + y = 10$, and $y = 8$, then we may substitute this value for y in the equation to get $x + 8 = 10$.

Properties of Arithmetic

Properties of arithmetic govern the manipulation of expressions (mathematical phrases).
For any three numbers *a*, *b*, and *c*:

- ✓ Associative property of addition
  $a + (b + c) = (a + b) + c$
- ✓ Commutative property of addition
  $a + b = b + a$
- ✓ Additive identity property
  (addition property of 0)
  $a + 0 = 0 + a = a$
- ✓ Additive inverse property
  $a + (-a) = -a + a = 0$
- ✓ Associative property of multiplication
  $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
- ✓ Commutative property of multiplication
  $a \bullet b = b \bullet a$
- ✓ Multiplicative identity property
  (multiplication property of 1)
  $a \bullet 1 = 1 \bullet a = a$
- ✓ Multiplicative inverse property
  $a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1$
- ✓ Distributive property relating addition and multiplication
  $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers *a*, *b*, and *c*.

Properties of Equality

Properties of equality govern the manipulation of equations (mathematical sentences).

For any three numbers *a*, *b*, and *c*:

- ✓ Addition property of equality
  (Subtraction property of equality)
  If $a = b$ and $c = d$, then $a + c = b + d$.
- ✓ Multiplication property of equality
  (Division property of equality)
  If $a = b$ and $c = d$, then $ac = bd$
- ✓ Reflexive property of equality: $a = a$
- ✓ Symmetric property of equality:
  If $a = b$, then $b = a$
- ✓ Transitive property of equality:
  If $a = b$, and $b = c$, then $a = c$

Solving Equations Using a Substitution Strategy

Method 1: To solve an equation using substitution, apply your knowledge of arithmetic facts to find values that make the equation true.

Example 1: Solve $-3x = 15$.

- Think: ***What number times -3 is 15?***
- Since $-3(-5) = 15$, $x = -5$.

Example 2: Solve $12 = 20 - k$.

- Think: ***20 minus what equals 12?***
- Since $20 - 8 = 12$, $k = 8$.

Method 2: Use the "cover-up" method and proceed as above.

Example 3: Solve $\frac{n + 20}{3} = 8$

- Cover up $n + 20$ → $\frac{\square}{3} = 8$
- Think: ***What divided by 3 equals 8?***
- Since $\frac{24}{3} = 8$, you are covering up 24
- Think: ***What plus 20 equals 24?***
- Since $4 + 20 = 24$, $n = 4$

Example 4: Solve $5(m - 2) = -20$.

Cover up $m - 2$ → $5(\square) = -20$

- Think: ***5 times what equals -20?***
- Since $5(-4) = -20$, you are covering up -4
- Think: ***What minus 2 equals -4?***
- Since $-2 - 2 = -4$, $m = -2$

Similar Phrases with Different Meanings

Sometimes it is useful to "translate" a string of words into symbols.

String of Words	Example	Symbols	Classification
is less than	4 is less than 10	$4 < 10$	inequality
less than	4 less than 10	$10 - 4$	expression
is greater than	7 is greater than 2 + 3	$7 > 2 + 3$	inequality
greater than	7 greater than 2 + 3	$(2 + 3) + 7$	expression

"Is greater than" includes the word "is." Therefore, it behaves like a mathematical verb. This string of words is used to make a mathematical sentence (an inequality in this case).

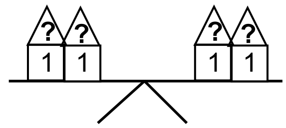
"Greater than" is a string of words without a verb. It translates into an expression. In English, we connect phrases with verbs to make sentences. The same is true in mathematics.

Balance Scales and Laws of Equality

Balance scales are physical representations of equations because both sides of a balanced scale must have the same weight, and both sides of an equation must have the same value.

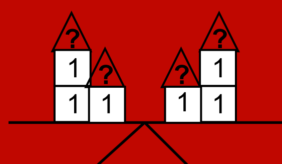
We imagine that each 1 represents one unit of weight and each ? represents an unknown weight (not equal to zero). To represent unknowns, a popular variable is *x*.

Start with a balanced scale like the one to the right, which represents the equation $2x + 2 = 2x + 2$.



Example 1: Add the same thing to both sides, like 1.

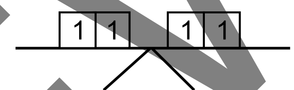
New scale: (still balanced)



New equation: $2x + 2 + 1 = 2x + 2 + 1$
$2x + 3 = 2x + 3$

Example 2: Subtract the same thing from both sides, like $2x$.

New scale: (still balanced)

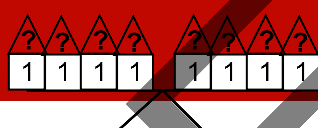


New equation: $2x + 2 – 2x = 2x + 2 – 2x$
$2 = 2$

In examples 1 and 2 the addition property of equality is applied (see Properties of Equality). Note that this property extends to subtraction as well. (See example 2 above).

Example 3: Multiply both sides by the same thing, like 2.

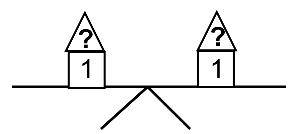
New scale: (still balanced)



New equation: $2(2x + 2) = 2(2x + 2)$
$4x + 4 = 4x + 4$

Example 4: Divide both sides by the same thing, like 2.

New scale: (still balanced)



New equation: $\frac{2x+2}{2} = \frac{2x+2}{2}$
$x + 1 = x + 1$

In examples 3 and 4 the multiplication property of equality is applied (see Properties of Equality). Note that this property extends to division as well.

Solving Equations Using a Model

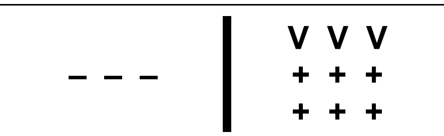
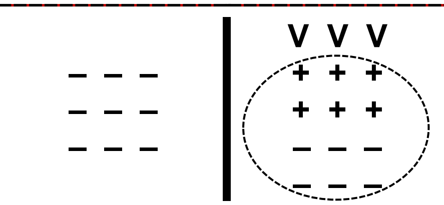
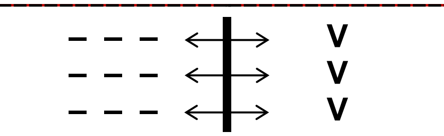
Let **+** represent 1 Let **V** represent the unknown (like *x*)
Let **–** represent -1 Let **Λ** represent the opposite of the unknown (like -*x*)

As you solve equations, think:

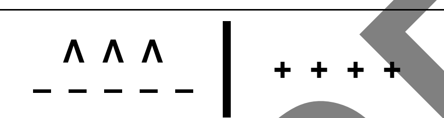
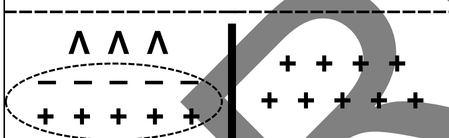
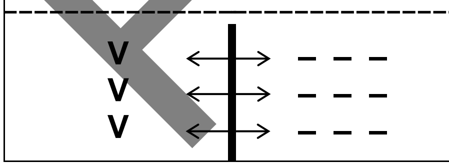
- Can I simplify one or both sides? That is, focus on what can be done to each expression alone.
- What can I do to both sides? That is, focus on what can be done to the equation.

The following examples illustrate one solution path. Others paths are possible to arrive at the same solutions.

- Solve -3 = 3(*x* + 2). Check (after solving): 3(-3) + 6 → -9 + 6 → -3 = -3

Picture	Equation	Comments
	$-3 = 3x + 6$	build the equation (think: 3 groups of *x* + 2) and rewrite
	$-3 = 3x + 6$ $+(-6) = +(-6)$ $-9 = 3x$	add -6 to both sides and remove zero pairs
	$\frac{-9}{3} = \frac{3x}{3}$ $-3 = x$	divide both sides by 3 to put counters equally into cups

- Solve: -2*x* – 5 – *x* = 4 Check (after solving): -2(-3) – 5 – (-3) → 6 – 5 + 3 → 1 + 3 = -3 + 7 → 4 = 4

Picture	Equation	Comments
	$-3x - 5 = 4$	build the equation and rewrite (collect like terms)
	$-3x - 5 = 4$ $+5 = +5$ $-3x = 9$	add 5 to both sides and remove zero pairs
	$(-1)(-3x) = (-1)(9) \rightarrow 3x = -9$ $\frac{3x}{3} = \frac{-9}{3}$ $x = -3$	take the opposite of both sides (accomplished by multiplying both sides by -1), and then divide both sides by 3
	$\frac{-3x}{-3} = \frac{9}{-3}$ $x = -3$	The above actions have the same effect as diving by -3 in a single step

Using Algebraic Techniques to Solve Equations

To solve equations using algebra:

- Use the properties of arithmetic to simplify each side of the equation (e.g., associative properties, commutative properties, inverse properties, distributive property).
- Use the properties of equality to isolate the variable (e.g., addition property of equality, multiplication property of equality).

- Solve: -2 – 3 = 5*x* – 2*x* + 7 for *x*

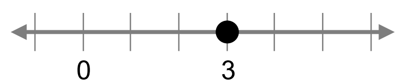
Equation	**Comments**
$-5 = 3x + 7$	• Collect like terms (-2 – 3 = -5; 5*x* – 2*x* = 3*x*). Note that this is an application of the distributive property because (5 – 2)*x* = 3(*x*).
$-5 = 3x + 7$ $\underline{-7} = \underline{-7}$ $-12 = 3x$	• Addition property of equality (subtract 7 from both sides) • Additive inverse property (7 + (-7) = 0)
$\frac{-12}{3} = \frac{3x}{3}$ $-4 = x$	• Multiplication property of equality (divide both sides by 3 or multiply both sides by $\frac{1}{3}$) • Multiplicative identity property (1*x* = *x*)

Graphing Inequalities

When graphing solutions to inequalities on the number line, we will use arrows to represent sets of solutions that extend indefinitely in one direction or the other. These arrows should not be confused with the arrows used to denote distance and direction above number ines in Units 3 and 4.

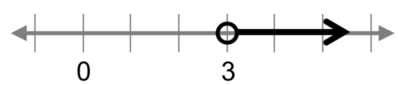
- Integers are graphed as closed circles.

  Example: *x* = 3



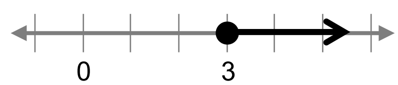
- Solutions to inequalities that involve the statements “is less than” (<) and “is greater than” (>) are graphed with an open circle, indicating that the boundary number *is not* included in the solution set.

  Example: *x* > 3



- Solutions to inequalities that involve the statements “is less than or equal to” (≤) and “is greater than or equal to” (≥) are graphed with a closed circle, indicating that the boundary number *is* included in the solution set.

  Example: *x* ≥ 3



Reversing the Direction of an Inequality

When multiplying or dividing both sides of an inequality by a negative number, the direction of the inequality reverses.

Original inequality	Do to both sides	Resulting inequality	Direction reverses?
$10 > -4$	Add 2	$12 > -2$	No
	Subtract 2	$8 > -6$	No
	Multiply by 2	$20 > -8$	No
	Divide by 2	$5 > -2$	No
$-10 < 4$	Add -2	$-12 < 2$	No
	Subtract -2	$-8 < 6$	No
	Multiply by -2	$20 > -8$	**Yes**
	Divide by -2	$5 > -2$	**Yes**

The direction of an inequality reverses **ONLY** when multiplying or dividing both sides of an inequality by a negative number.

Note that it does not matter if there are negative numbers in the original inequality or not.

Solving Inequalities in One Variable

When solving a linear inequality, treat the inequality as if it were an equation. When multiplying or dividing both sides of the inequality by a negative number, reverse the direction of the inequality.

Example 1	Comments
$-4x + 1 \le 13$ $-1 \quad -1$	(Subtraction) Do not reverse the inequality.
$-4x \le 12$ $\frac{-4x}{-4} \ge \frac{12}{-4}$	(Division by a negative number) **Reverse the inequality symbol.**
$x \ge -3$	Solutions

Example 2	Comments
$2x - 9 \le -13$ $+9 \quad +9$	(Addition) Do not reverse the inequality.
$2x \le -4$ $\frac{2x}{2} \le \frac{-4}{2}$	(Division by a positive number) **Do not reverse the inequality symbol.**
$x \le -2$	Solutions

Error alert. The inequality does not always reverse when solving an inequality that includes negatives. Example 2 illustrates this.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
7.RP.A	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
7.RP.2	Recognize and represent proportional relationships between quantities:
a.	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
c.	Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.
7.NS.A	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
7.EE.A	**Use properties of operations to generate equivalent expressions.**
7.EE.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.EE.B	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers.
a.	Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
b.	Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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