Name ______________________________ Period __________ Date ______________

MathLinks GRADE 7

UNIT 4 STUDENT PACKET

RATIONAL NUMBER ADDITION AND SUBTRACTION

Materials | Grouping | Reproducibles | Slide Deck | Journal Idea

Parent (or Guardian) signature ______________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

integers	negative numbers
opposite of a number	positive numbers
rational numbers	zero pair

MR. MORTIMER'S MAGIC CUBES

As a child, Merrimack Mortimer loved chemistry, and he grew to become an inventor. He called one of his great inventions Magic Hot and Cold Cubes. Here we will learn about them in greater detail.

Follow your teacher's directions.

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)
(9)	(10)
(11)	

COUNTERS AND ADDING INTEGERS

We will use counters to develop concepts about integers and use this model to generalize rules for integer addition. We will add integers using these rules.

[7.NS.1ab; SMP1, 2, 3, 5, 6, 7, 8]

GETTING STARTED

1. Record the meanings of positive numbers, negative numbers, and integers in **My Word Bank**.

2. Look closely at the following list of numbers:

   -1   0   -5   23   1.5   -100 $\frac{1}{2}$ $-4\frac{3}{4}$

   a. Circle all of the numbers that are integers.

   b. Choose one number that is NOT an integer and explain why it is not an integer.

3. Record the meaning of rational numbers in **My Word Bank**.

4. Look closely at the following list of numbers:

   $\frac{3}{5}$ $\frac{9}{7}$ $\frac{6}{-11}$ -4   0.5 $\frac{0}{5}$ $\frac{-2}{17}$ $\frac{-4}{-3}$ $2\frac{1}{2}$ 0 $\frac{3}{0}$

   a. Circle all of the numbers that are rational.

   b. Choose one rational number above that is NOT in the form $\frac{m}{n}$ (where m and n are integers, and $n \neq 0$). Write it in the form $\frac{m}{n}$.

   c. Choose one number above that is NOT rational. Explain why it is not a rational number.

A COUNTER MODEL

Follow your teacher's directions for (1) – (5).

(1) A positive counter has a value of 1 and looks like _________.

Build a value of _____ and draw it here _________.

(2) A negative counter has a value of -1 and looks like _________.

Build a value of _____ and draw it here _________.

(3) A zero pair has a value of 0 and looks like _________.

Build two zero pairs and draw them here _________. This has a value of _____.

(4) Use counters to build _____ in three different ways and draw here.

a.	b.	c.

(5) Use counters to build _____ in three different ways and draw here.

a.	b.	c.

Build the given values using the given numbers of counters. Then record drawings.

6. Use 4 counters. Build and draw a value of 0.	7. Use 8 counters. Build and draw a value of 0.
8. Use 5 counters. Build and draw a value of 1.	9. Use 5 counters. Build and draw a value of -1.
10. Use 8 counters. Build and draw a value of -4.	11. Use 8 counters. Build and draw a value of 4.

12. Record the meanings of zero pair and opposite of a number in **My Word Bank**.

PRACTICE 1

1. The combination of one positive and one negative counter is called a ________ ________.

2. Describe a zero pair using Mortimer's magic cubes.

Build the given values using the given numbers of counters. Then record drawings.

	Value	# of counters	Drawing
3.	5	the least possible	
4.	-6	the least possible	
5.	0	2	
6.	0	10	
7.	5	7	
8.	5	more than 7, but less than 11	
9.	-2	8	
10.	-2	more than 2, but less than 6.	
11.	6	*at least* 7	
12.	-1	*more than* 7	

Build and draw the following situations.

13. Start with a value of 4. What can you place on your work space to change this into a value of zero? Draw the result.	14. Start with a value of -4. What can you place on your work space to change this into a value of zero? Draw the result.

15. Try to represent *any* odd value with an even number of counters. What do you notice?

ADDING INTEGERS WITH COUNTERS

Use these sentence frames to help think through integer addition. **Do not write in these.**

- Begin with a work space that has a value equal to 0.
- Build ___________ _____.
  positive/negative
- The plus (+) means to add.
- Add _____ ___________ counter(s).
  positive/negative
- The result is _____ ___________ counter(s).
  positive/negative

Follow your teacher's directions for (1) – (4).

(1)	(2)	(3)	(4)

Compute each sum. Draw using positive symbols (+) and negative symbols (–).

5. (7) + (-2)	6. (5) + (-5)	7. (2) + (6)
8. (-8) + (3)	9. (-2) + (-6)	10. (-4) + (6)

For problems 11 – 13, use **positive**, **negative**, and **zero** as choices to finish each sentence below. Use all that apply for each.

11. When adding two positive integers, the result will be _______________.

12. When adding two negative integers, the result will be _______________.

13. When adding a positive integer and a negative integer, the result will be ___________, ___________, or ___________.

INTEGER ADDITION RULES

Description of counters on your workspace	Drawing	Numerical example	Summarizing Shorthand (positive → pos) (negative → neg)
1. Positive Only			
Place some positives. Then place more positives.		____ + ____	pos + pos is
2. Negative Only			
Place some negatives. Then place more negatives.		____ + ____	pos + pos is
3. Positive and Negative			
a. Place some of each so that the result is positive.		____ + ____	pos + neg is pos when:
b. Place some of each so that the result is negative.		____ + ____	pos + neg is neg when:
c. Place some of each so that the result is zero.		____ + ____	pos + neg is 0 when:

PRACTICE 2

Without computing, determine whether each sum is positive (pos), negative (neg), or zero (0).

Positive Example 6 + (-4)	**Negative Example** -6 + (-4)	**Zero Example** -6 + 6
1. -2 + (-11)	2. 7 + (-3)	3. -2 + (-6)
4. 9 + 4	5. -6 + 4	6. 11 + (-4)
7. -6 + (-1)	8. -5 + 1	9. 1 + (-1)

Compute each sum. Use drawings if desired.

10. 7 + (-2)	11. -9 + 9	12. -1 + (-3)
13. 11 + 12	14. 3 + (-8)	15. -5 + 6
16. 2 + (-2)	17. -3 + (-6)	18. -13 + 3

Make each equation true using the given directions.

Directions	Equation
19. Both numbers are positive.	10 = _______ + _______
20. One positive number and one negative number.	10 = _______ + _______
21. Both numbers are negative.	-10 = _______ + _______
22. One positive number and one negative number.	-10 = _______ + _______

Write a number sentence and describe the change resulting from each action.

23. Jenelle earns $20, then loses $20.	24. Andres loses 5 yards, then gains 5 yards.
25. Minh's kite drops 10 ft, then climbs 10 feet.	26. Avani gets 15 new cards for her collection, then gives away 15 cards.

PRACTICE 3

1. Compute the following: 100 + 100 = _____ and -100 + (-100) = _____.

   How is adding two negative numbers the same as adding two positive numbers?

   How is it different?

2. Compute the following: 100 + (-10) = _____ -100 + 10 = _____

   How are these computations related to subtraction?

3. Complete the puzzle below using the given expression. Then find total sums for rows and for columns (exclude the gray numbers). Make sure the sums are equal for the very bottom row and far right column.

		b			
	a* + *b		**40**	**-60**	**TOTAL SUMS (ROWS)**
a	**20**	0			
	-50				
				-90	
	TOTAL SUMS (COLUMNS)				

4. Devin is a running back on his high school football team. On first down (the first play), he loses 3 yards. On second down (the next play), he gains 17 yards. Where is Devin's team in relation to where they started before first down?

COUNTERS AND SUBTRACTING INTEGERS

We will use a counter model to generalize the rule for integer subtraction. We will subtract integers using the rule.

[7.NS.1abcd, 7.SP.7b, 7.SP.8a; SMP1, 2, 3, 4, 5, 6, 7, 8]

GETTING STARTED

1. Using at least 6, but no more than 12 counters, draw a value of -2 in two different ways.

2. How many ways are there to build any given integer with counters? ___________________

Compute each sum. Use positive symbols (+) and negative symbols (–) if desired.

3. -6 + (-6)	4. -5 + 3
5. 5 + (-3)	6. 6 + (-6)

Compute each sum without using counters or drawings. Show work if not done mentally.

7. 60 + (-30)	8. 27 + (-59)
9. -600 + (-300)	

10. Think about Mortimer's magic cubes. Regardless of the temperature of the liquid,
    a. what happens to it if we remove some cold cubes?
    b. what happens if instead we remove some hot cubes?

11. Abner thinks that -6 is greater than -3. What mistake is he making?

SUBTRACTING INTEGERS WITH COUNTERS 1

Use these sentence frames to help think through integer subtraction. **Do not write in these.**

- Begin with a work space that has a value equal to 0.
- Build ___________ _____.
  positive/negative
- The minus (–) means to subtract.
- Subtract _____ ___________ counter(s). Introduce zero pairs if needed.
  positive/negative
- The result is _____ ___________ counter(s).
  positive/negative

Follow your teacher's directions for (1) – (3).

(1)	(2)	(3)

Compute each difference. Draw using positive symbols (+) and negative symbols (–).

4. 6 – 3	5. -6 – (-3)	6. -7 – (-1)
7. -1 – (-1)	8. 4 – 4	9. -4 – (-4)

10. Mateo thinks that "when you subtract, the result is **less than** what you started with." Look at problems 1 – 9. Put stars by examples that illustrate Mateo is not correct.

SUBTRACTING INTEGERS WITH COUNTERS 2

Follow your teacher's directions for (1) – (4).

(1)	(2)
(3)	(4)

Compute each difference. Draw using positive symbols (+) and negative symbols (–).

5. 1 – 3	6. -4 – (-5)
7. 2 – (-4)	8. -6 – (3)
9. 8 – (-4)	10. 3 – (-3)
11. -5 – (5)	12. -2 – (-2)

13. Put a star next to all the problems above where the result (difference) is greater than number you started with (minuend). Then look at all the problems where you put stars in this lesson. What do you notice about the number that is being subtracted (subtrahend) EVERY time?

THE SUBTRACTION RULE

Compute. Show ACTIONS using positive symbols (+) and negative symbols (-).

1a. 3 – (1)	1b. 3 + (-1)
2a. -6 – (-4)	2b. -6 + (4)
3a. 3 – (-1)	3b. 3 + 1
4a. -5 – (-6)	4b. -5 + 6
5a. -5 – 2	5b. -5 + (-2)

Compare parts (a) and (b) for problems 1 – 5.

6. How are the actions for (a) different than the actions for (b)?

7. These examples show that subtracting a number gives the same result as ...

8. Generalize the **subtraction rule** for any numbers *m* and *n.*

Symbols:	$m - n = m +$ _____	$m - (-n) = m +$ _____
Words:	*m* minus *n* is equal to	*m* minus the opposite of *n* is equal to
Example:		

Complete each statement.

9. 3 – 2 = 3 + _____	10. -3 – 2 = -3 + _____	11. 3 – (-2) = 3 + _____
12. 5 – (-7) = 5 + _____	13. 5 + 7 = 5 – _____	14. 6 + 8 = 6 – _____

PRACTICE 4

1. Rewrite 7 – (-3) as an equivalent addition expression using the subtraction rule. Which is easier for you to compute, the addition or subtraction expression?

2. Circle all expressions that are equivalent to 5 – (-7).

   -5 – 7 5 – 7 5 + 7 -5 + 7

Rewrite each subtraction expression as an equivalent addition expression. Then compute.

3. 17 – 24	4. -56 – 18	5. 23 – (-9)
6. -19 – (-44)	7. -11 – 37	8. -41 – (-15)

9. On a cold winter afternoon in Minnesota, the temperature was 4° Fahrenheit. By evening the temperature had dropped 11°. What was the evening temperature? Write as a subtraction expression and its equivalent addition expression before answering the question.

10. Complete the puzzle below using the given expression. Then find total sums for rows and for columns (exclude the gray numbers). Make sure the sums are equal for the very bottom row and far right column.

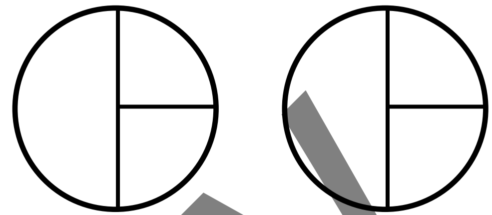
		b			
	a* – *b		**30**	**-60**	**TOTAL SUMS (ROWS)**
a	**-80**	-70			
	-100				
				85	
	TOTAL SUMS (COLUMNS)				

A ZERO-SUM GAME

1. Choose three numbers (two positive and one negative) whose sum is 0. Record them in Spinner 1 in any way you like.

2. Choose three numbers (two negative and one positive) whose sum is 0. Record them in Spinner 2 in any way you like.

Spinner 1 Spinner 2

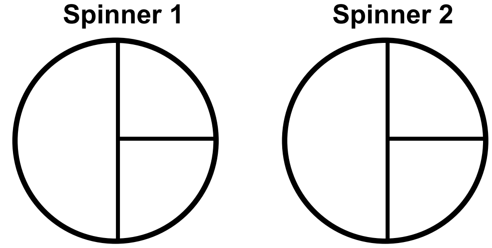


3. In this game, one turn is spinning both spinners once and finding the sum. If the sum is greater than 0, you win. If the sum is less than zero, you lose. For your chosen numbers, explain whether or not this is a fair game where $P(\text{winning}) = P(\text{losing}) = \frac{1}{2}$.

4. Using a paperclip as a spinner, find the sum for 20 trials and record. Did the results turn out as you expected? Explain.

5. Change the positions of the numbers you placed in Spinner 1 and/or Spinner 2. Is the probability of winning still the same? Do you have a better chance of winning?

Spinner 1 Spinner 2



ADDING AND SUBTRACTING RATIONAL NUMBERS

We will use number lines to extend the addition and subtraction rules for integers to the set of rational numbers.

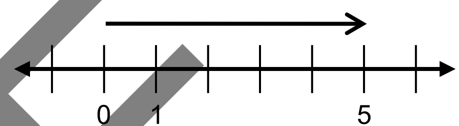
[7.NS.1abcd; SMP1, 2, 5, 6, 7, 8]

GETTING STARTED

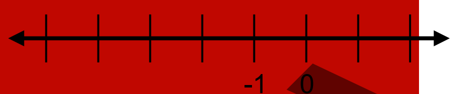
Use appropriate words to make the following sentences true.

1. As we move from left to right on the number line, numbers have ______________ value.
2. As we move from right to left on the number line, numbers have ______________ value.

This arrow begins at 0 and ends at 5. It represents the number 5. Draw arrows to represent each of the following numbers on the number lines below.

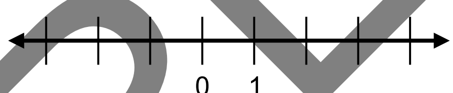


3. -5



4. 4.5

5. $-1\frac{1}{2}$



6. A number is 2 units from 3 on the number line. Fill in the blanks in the addition equations below to represent the two possible equations for this number. Use a number line above if helpful.

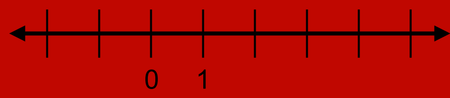
   3 + ☐ = ☐ 3 + ☐ = ☐

   How do the two numbers added to 3 compare?

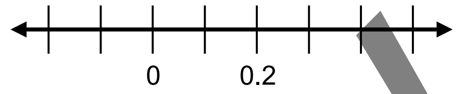
NUMBER LINE ADDITION

Follow your teacher's directions for (1) – (6).

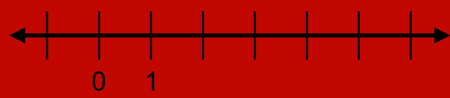
(1)



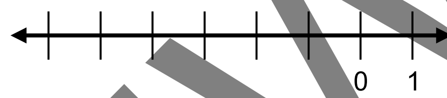
(2)



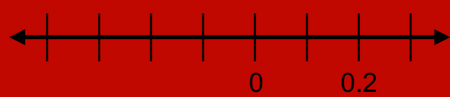
(3)



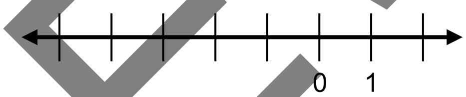
(4)



(5)

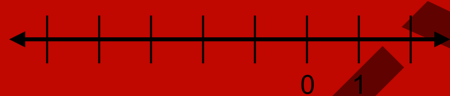


(6)

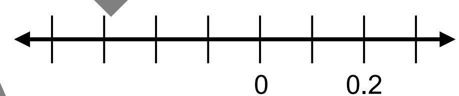


Predict each sum. Then compute using arrows.

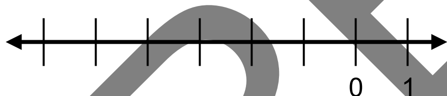
7. $-3 + 2$



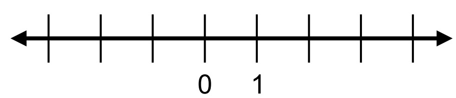
8. $-0.3 + 0.2$



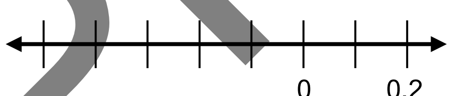
9. $-3\frac{1}{2} + \left(-2\frac{1}{2}\right)$



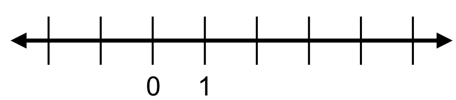
10. $3 + (-2)$



11. $-0.3 + (-0.2)$



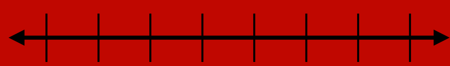
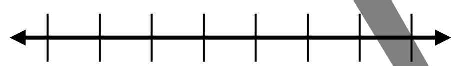
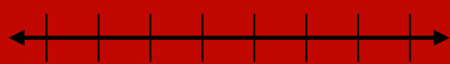
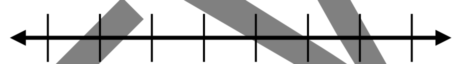
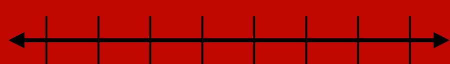
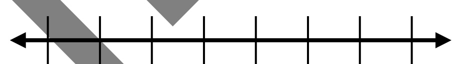
12. $3\frac{1}{2} + \left(-2\frac{1}{2}\right)$



13. Look at problems 1 – 12 above. Do the addition rules we learned in a previous lesson hold for these problems? Do you think that these rules hold for all rational number addition?

PRACTICE 5

Predict each sum. Then compute using arrows. Label tick marks appropriately.

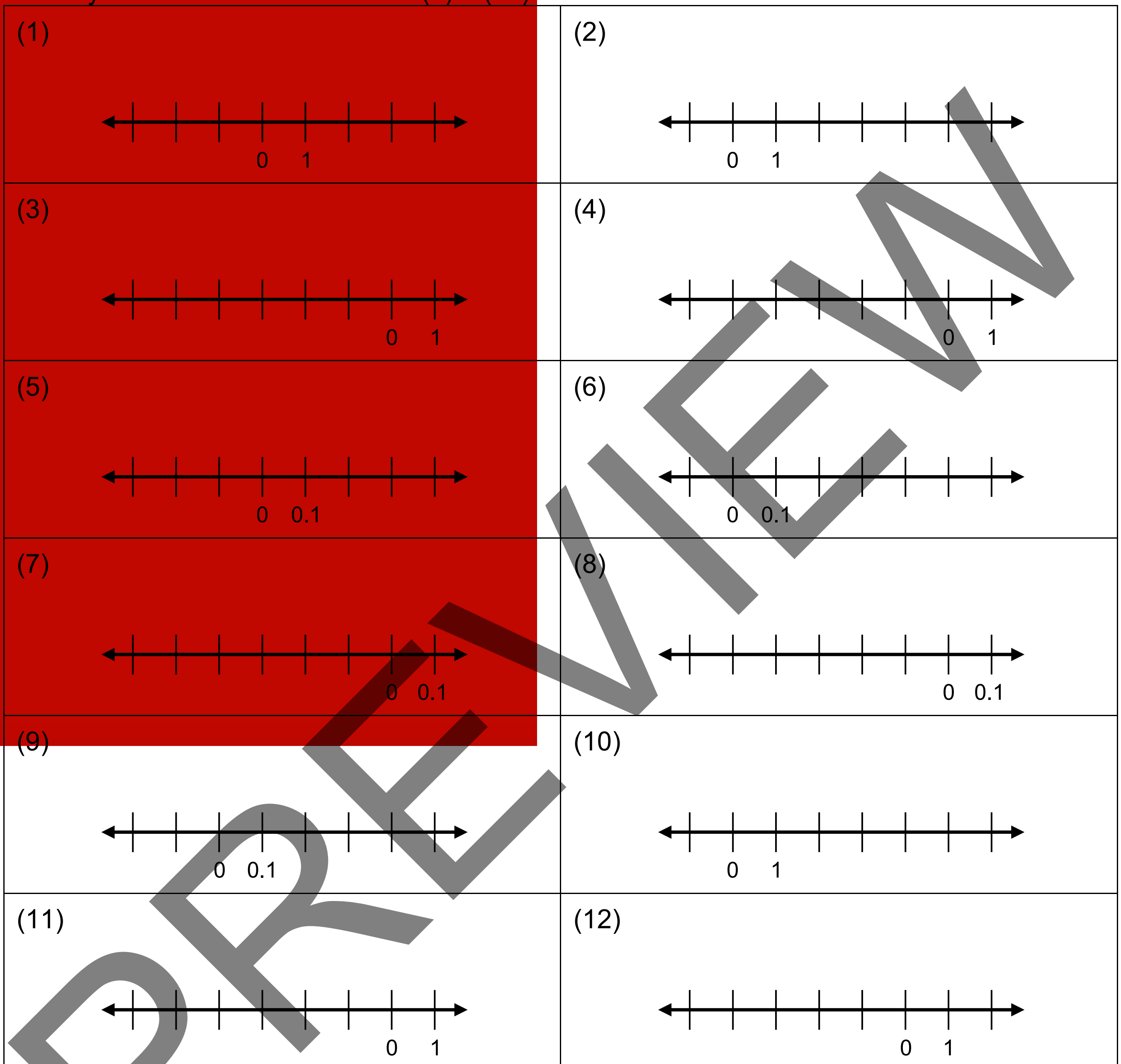
1. $6 + (-1)$ 	2. $-5 + 4$ 
3. $-0.5 + 0.3$ 	4. $-0.2 + (-0.2)$ 
5. $\left(-2\frac{1}{4}\right) + -1\frac{1}{4}$ 	6. $2\frac{3}{4} + \left(-1\frac{1}{4}\right)$ 

Compute each sum using rules. Show work as needed.

7. $-16 + (-31)$	8. $-57 + 44$	9. $-39 + 93$
10. $-5.5 + 8.6$	11. $-0.21 + (-0.245)$	12. $6.81 + (-0.44)$
13. $-6\frac{1}{3} + \left(-2\frac{1}{4}\right)$	14. $10\frac{2}{3} + \left(-8\frac{1}{5}\right)$	15. $7\frac{1}{2} + \left(-12\frac{9}{16}\right)$

NUMBER LINE SUBTRACTION

Follow your teacher's directions for (1) – (12).



13. Look at problems 1 – 12 above. Does the subtraction rule we learned in a previous lesson hold for these problems? Do you think that this rule holds for all rational number subtraction?

PRACTICE 6

Predict each difference. Then compute using arrows. Label tick marks appropriately.

1. $6 - (-1)$	2. $-5 - 4$
3. $-0.5 - 0.3$	4. $-0.2 - (-0.2)$
5. $-1\frac{1}{4} - \left(-2\frac{1}{4}\right)$	6. $2\frac{3}{4} - \left(-1\frac{1}{4}\right)$

Compute each difference using rules. Show work as needed.

7. $-1\frac{1}{4} - \left(-2\frac{1}{4}\right)$	8. $5\frac{3}{4} - \left(-3\frac{1}{4}\right)$	9. $-2\frac{1}{4} - \left(-6\frac{3}{4}\right)$
10. $-16 - (-31)$	11. $-57 - 44$	12. $39 - (-93)$

PRACTICE 7

Compute using any method. If mental math is used, write MM. Otherwise show all work.

1. -25.1 + 4.8	2. -25.1 – 4.8	3. 25.1 – (-4.8)
4. $\frac{1}{3} - \left(-\frac{1}{6}\right)$	5. $-\frac{3}{4} - \frac{5}{6}$	6. $-\frac{3}{8} - \left(-\frac{5}{12}\right)$
7. $\frac{1}{7} + \left(-\frac{5}{7}\right) + \frac{4}{7}$	8. $-\frac{3}{5} + 1\frac{1}{2} + \left(-\frac{2}{5}\right)$	9. $-\frac{1}{4} + 1\frac{1}{4}$
10. -5.5 – 8.6	11. 6.81 – (-0.44)	12. -0.21 – (-0.245)

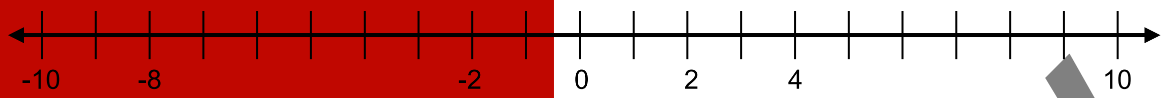
PRACTICE 8

Compute using any method. If mental math is used, write MM. Otherwise show all work.

1. $\frac{1}{5} - 1$	2. $\frac{1}{5} - 2$	3. $\frac{3}{5} - 2$
4. $-3\frac{1}{2} + 4\frac{3}{10}$	5. $-2\frac{3}{4} + 1\frac{2}{5}$	6. $-5\frac{1}{6} + \left(-1\frac{2}{9}\right)$
7. $-3\frac{1}{2} - 4\frac{3}{10}$	8. $-2\frac{3}{4} - \left(-1\frac{2}{5}\right)$	9. $5\frac{1}{6} - \left(-1\frac{2}{9}\right)$
10. $-6\frac{1}{3} - \left(-2\frac{1}{4}\right)$	11. $10\frac{2}{3} - \left(-8\frac{1}{5}\right)$	12. $7\frac{1}{2} - 12\frac{9}{16}$

EXPLORING DIFFERENCE AND DISTANCE ON THE NUMBER LINE

Use the number line below as needed for problems 1 – 6 to count the distance between the given points. Recall that distances are always represented by nonnegative numbers.



	Points on a line	Distance counted between points	Difference between points	Absolute value of the differences
1.	5 and 8		8 – 5 = _____ 5 – 8 = _____	
2.	0 and 4			
3.	-7 and -5			
4.	-4 and 0			
5.	2 and -9			
6.	3 and 3			

7. The distance between two points on a number line is the __________ __________ of their difference.

For the given pairs of points on a line below, find the distance between them without counting.

8. 25 and 105	9. -30 and -70	10. 50 and -50

11. A bird is flying 50 meters above sea level. A dolphin is swimming 35 meters below sea level. What is the vertical distance between the bird and the dolphin?

PRACTICE 9: EXTEND YOUR THINKING

1. A rancher is digging a well. Ground level has an elevation of zero. First write an expression to describe his actions. Then solve the problem.

   From ground level he digs down 13 feet, and then stops for the day. Overnight wind blew 2 feet of dirt back into the hole. The second day he digs another 9 feet. The third day he decides the hole is now too deep, and fills in 6 feet of dirt. What is the elevation at the bottom of the well after his work is complete?

Recall that properties like the commutative and associative properties of addition allow us to add numbers in different orders. Use these properties to make the following calculations easier. Describe your process.

2. $37 + (-21) + (-37)$	3. $-8.6 + 2.7 + 8.6 - 2.7$
4. $-\frac{5}{7} + \left(-\frac{2}{3}\right) + \frac{2}{3} - \frac{1}{7}$	5. $-\frac{1}{2} + \left(-\frac{1}{4}\right) + \left(-1\frac{1}{2}\right)$

Insert plus (+) and minus (–) signs to make the equations true.

6. $-3.8 \square (-4.2) \square 6.4 = -6$	7. $0.14 \square 0.86 \square (-0.05) = -0.77$
8. $-\frac{1}{2} \square \left(-\frac{1}{3}\right) \square \frac{5}{6} = -\frac{5}{3}$	9. $-2\frac{1}{4} \square 4\frac{1}{6} \square \left(-3\frac{1}{2}\right) = 5\frac{5}{12}$

REVIEW

COMPARING ADDITION AND SUBTRACTION

Complete the tables below using patterns.

1.

Expression			Sum
5	+	(__)	
5	+	(__)	
5	+	(1)	
5	+	(0)	5
5	+	(-1)	4
5	+	(-2)	
5	+	(__)	
5	+	(__)	
5	+	(__)	
5	+	(__)	

2.

Expression			Difference
5	–	(__)	
5	–	(__)	
5	–	(-1)	
5	–	(0)	5
5	–	(1)	4
5	–	(2)	
5	–	(3)	
5	–	(__)	
5	–	(__)	
5	–	(__)	

Complete the problems below based on the results (sums or differences) in the tables above.

3. Under what circumstances are the results **less than 5**?

   Adding a ________________ number or subtracting a _______________ number.

4. Under what circumstances are the results **greater than 5**?

   Adding a ________________ number or subtracting a ________________ number.

5. What two expressions have a result of 4? _______________ and _______________

6. What two expressions have a result of 8? _______________ and _______________

7. Subtracting 6 from a number gives the same result as adding _____ to it.

8. Subtracting -2 from a number gives the same result as adding _____ to it.

9. Write the related addition expression for each subtraction expression below.

a. -5 – 1	b. -5 – (-1)	c. 0 – (-1)

INTEGER BATTLE

You will need:

- 2 or more players
- R4-2ab Integer Cards (or playing cards with picture cards removed – for black and red cards, define one color as positive and the other as negative)

Integer Battle is like the classic card game War. It may be played one-on-one or two-on-two.

Addition version

- Shuffle all the cards and deal them out equally to each player/team.
- Both players/teams place two cards from the top of their stack in front of them.
- Each team adds the values on both pairs of cards. The player/team with the greater sum wins, and that player/team collects all four cards in a pile in front of them.
- When a player/team runs out of cards, and there are still collected cards in their pile, they shuffle and reuse those cards like before.
- When a player/team completely runs out of cards, and have none left at all, the other team is declared the winner.

Subtraction version

- The game is played exactly like the addition version, with one exception. When two cards are placed down, order matters. The second card placed down is subtracted from the first card placed down. Therefore, this version requires that players are careful to note which card is placed first, and which is placed second.

1. Play the **addition** version of Integer Battle. Record two winning hands:

   1st _____ + _____ = _____ is greater than _____ + _____ = _____

   2nd _____ + _____ = _____ is greater than _____ + _____ = _____

2. Play the **subtraction** version of Integer Battle. Record two winning hands:

   1st _____ – _____ = _____ is greater than _____ – _____ = _____

   2nd _____ – _____ = _____ is greater than _____ – _____ = _____

BIG SQUARE PUZZLES: RATIONAL NUMBER ADDITION AND SUBTRACTION

1. Complete the Big Square Puzzle(s) provided by your teacher.

2. Describe a strategy you use to complete the puzzle(s).

POSTER PROBLEMS: RATIONAL NUMBER ADDITION AND SUBTRACTION

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

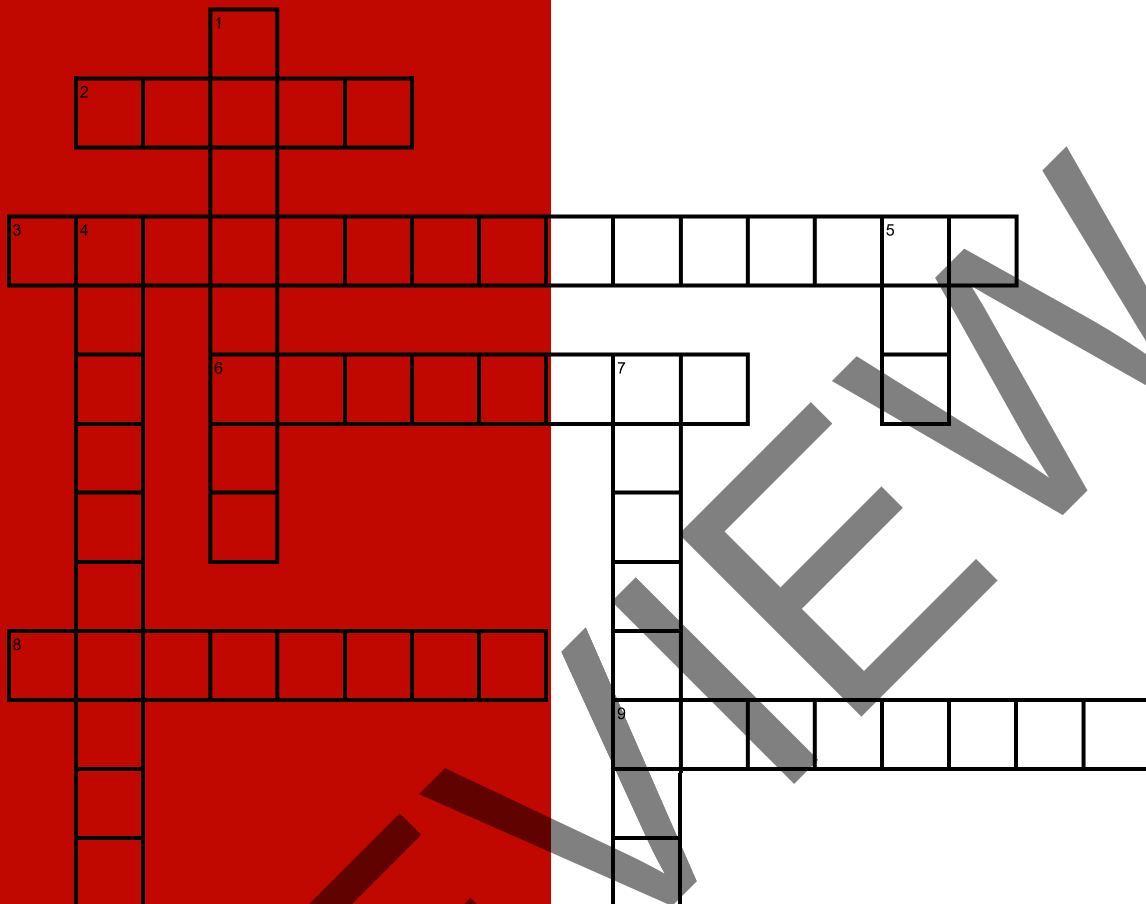
Part 2: Do the problems on the posters by following your teacher's directions. Show all computations neatly on the posters.

ROW	Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
I	$-3\frac{1}{5} + 4\frac{3}{10}$	$-\frac{2}{3} + \frac{5}{6}$	-2.8 + 4.35	-0.064 + 0.54
II	$-3\frac{1}{2}$	$-\frac{3}{4}$	-5.6	-0.51
III	$1\frac{9}{10}$	$\frac{1}{2}$	6.1	0.056
IV	$-4\frac{2}{5}$	$-\frac{1}{3}$	-10.1	-0.29

A. Copy and compute row I.
B. Add the number in row II to the result of row I.
C. Subtract the number in row III from the result of row II.
D. Subtract the number in row IV from the result of row III.

Part 3: Return to your original poster. Verify computations and correct errors if needed.

VOCABULARY REVIEW



Across

2 set of numbers that includes natural numbers and zero

3 -(-4) is the ___ ___ of -4 (2 words)

6 … -3, -2, -1, 0, 1, 2 …

8 one positive counter and one negative counter (two words)

9 a number less than zero

Down

1 a number greater than zero

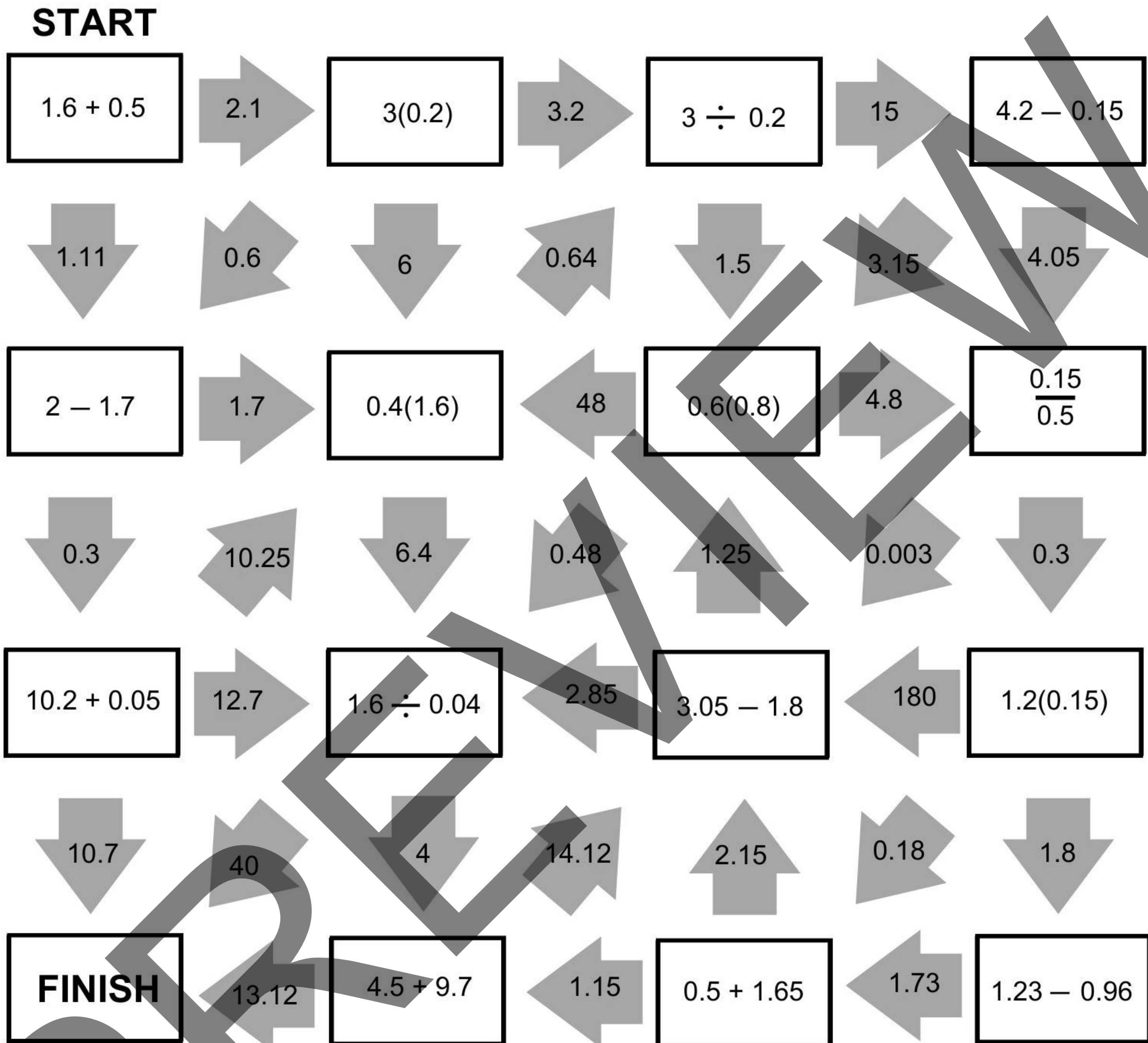
4 the result of subtraction

5 the result of addition

7 a number that can be written as $\frac{a}{b}$ (a and b are integers, $b \neq 0$)

SPIRAL REVIEW

1. **Math Path Fluency Challenge**: Use what you know about decimal operations to find the correct path from Start to Finish.



2. Complete table.

	Shirt 20% discount	Pants 15% discount	Shoes 35% discount	Socks 10% discount	Jacket 50% discount
Original Price	$17		$25		
Amount of discount		$6		$1	
New price					$25

SPIRAL REVIEW

Continued

3. Compute. (Remember: first simplify expressions in grouping symbols, then calculate exponents, then multiply and divide left to right, and finally add and subtract left to right).

a. $6^2 + 2\left(7 - \frac{15}{5}\right)$	b. $25 - 5\left(\frac{10 \bullet 2}{4}\right)$
c. $(2 + 3)^2 - 24$	d. $48 - 3^2(5) + 10$

4. Simplify.

a. $3x + 5y + 2y$	b. $3(f + 3) - f$
c. $4(w + 3) + 2(4 - w)$	d. $a + a - a + b + b$

5. Evaluate for $x = 3$ and $y = 5$.

a. $2x + 2y$	b. $x + x + x + y$
c. $20x - 2y$	d. $\frac{6}{x} + 5y$

6. JM buys $58.20 worth of schools supplies to donate to the local after school program. JM receives a 35% discount on the purchase.

   a. What is the discounted price of the school supplies?

   b. JM pays 9.25% tax on the discounted price. What is the total that JM spent?

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- ○ Find the likelihood of events with probability.
- ○ Sample to understand populations with statistics.
- ○ Apply proportional reasoning to ratios, rates, percent and scale.
- ○ Solve problems involving measurements of geometric figures.
- ○ Operate with rational numbers and solve problems.
- ○ Develop spatial reasoning in two- and three-dimensions.
- ○ Use algebra as a problem-solving tool.

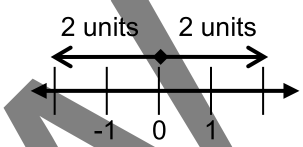
Give an example from this unit of one of the connections above.

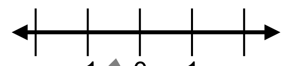
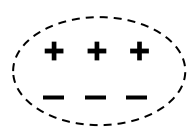
2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.

3. **Mathematical Practices.** Suppose you were asked to explain how to add integers to a younger student. What model or strategy would you use, and why? Give an example and explain in words [SMP3, 5]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **Making Connections.** Why do you think that some students may have a misconception that subtraction makes things smaller? Give an example that might correct this misconception.

STUDENT RESOURCES

Word or Phrase	Definition
absolute value	The absolute value $\|x\|$ of a number x is the distance from x to 0 on the number line. $\|2\| = 2$ and $\|-2\| = 2$, because both 2 and -2 are 2 units from 0 on the number line. 
addend	See sum.
additive identity property	The additive identity property states that $a + 0 = 0 + a = a$ for any number a. In other words, the sum of a number and 0 is the number. We say that 0 is an additive identity. The additive identity property is sometimes called the addition property of zero. $3 + 0 = 3$, $0 + 7 = 7$, $-5 + 0 = -5 = 0 + (-5)$
additive inverse	The additive inverse of a is the number b such that $a + b = b + a = 0$. The additive inverse of a is denoted by $-a$. -4 is the additive inverse of 4.
additive inverse property	The additive inverse property states that $a + (-a) = 0$ for any number a. In other words, the sum of a number and its opposite is 0. The number $-a$ is the additive inverse of a. $3 + (-3) = 0$, $-5 + 5 = 0$
difference	In a subtraction problem, the difference is the result of subtraction. The minuend is the number from which another number is being subtracted, and the subtrahend is the number that is being subtracted. 12 – 4 = 8 minuend subtrahend difference
integers	The integers are the whole numbers and their opposites. They are the numbers 0, 1, 2, 3, … and -1, -2, -3, … .
minuend	See difference.
negative numbers	Negative numbers are numbers that are less than zero, written $a < 0$. The negative numbers are the numbers to the left of 0 on a horizontal number line, or below zero on a vertical number line. The numbers -2, -4.76, and $-\frac{1}{4}$ are negative. The numbers 2 and 5.3, and 0 are NOT negative.

Word or Phrase	Definition
opposite of a number	The <u>opposite of a number</u> *n*, written $-n$, is its additive inverse. Algebraically, the sum of a number and its opposite is zero. Geometrically, the opposite of a number is the number on the other side of zero at the same distance from zero. The opposite of 1 is -1, because $1 + (-1) = -1 + 1 = 0$. The opposite of -1 is $-(-1) = 1$. Thus, the opposite of a number does not have to be negative. 
positive numbers	<u>Positive numbers</u> are numbers that are greater than zero, written $a > 0$. The positive numbers are the numbers to the right of 0 on a number line, or above zero on a vertical number line. The numbers 3, 2.6, and $\frac{3}{7}$ are positive. The numbers -3, -2.6, $-\frac{3}{7}$, and 0 are NOT positive.
rational numbers	<u>Rational number</u> are numbers expressible in the form $\frac{m}{n}$, where *m* and *n* are integers, and $n \neq 0$. $\frac{3}{5}$ is rational because it is a quotient of integers. $2\frac{1}{3}$ and 0.7 are rational numbers because they **can be** expressed as quotients of integers, namely $\frac{7}{3}$ and $\frac{7}{10}$, respectively. $\sqrt{2}$ and π are NOT rational numbers. They cannot be expressed as a quotient of integers.
subtrahend	See <u>difference</u>.
sum	A <u>sum</u> is the result of addition. In an addition problem, the numbers to be added are <u>addends</u>. 7 + 5 = 12 addend addend sum
whole numbers	The <u>whole numbers</u> are the natural numbers together with 0. They are the numbers 0, 1, 2, 3, … .
zero pair	In the counter model, a positive and a negative counter together form a <u>zero pair</u>. Let + represent a positive counter and let − represent a negative counter.  Then the figure to the right is an example of a collection of (three) zero pairs.

Mr. Mortimer's Magic Cubes

Mr. Mortimer discovered an amazing way to control the temperature of liquid. He invented magic hot and cold cubes to change the liquid's temperature. These magic cubes never melt or change in any way. For example, ice cubes melt, but magic cold cubes do not.

Hot Cubes (the basics):

- If you add 1 hot cube to a liquid, the liquid heats up by 1 degree.
- If you remove 1 hot cube from the liquid, the liquid cools down by 1 degree.

Cold Cubes (the basics):

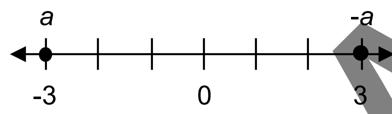
- If you add 1 cold cube to the liquid, the liquid cools down by 1 degree.
- If you remove 1 cold cube from the liquid, the liquid heats up by 1 degree.

How this temperature change model works				**For 1 cube**
Hot Cubes Positive (+)	Put in Heat	→	Hotter	add (+1) → +(+1) = +1
	Remove Heat	→	Colder	subtract (+1) → -(+1) = -1
Cold Cubes Negative (–)	Put in Cold	→	Colder	add (-1) → +(-1) = -1
	Remove Cold	→	Hotter	subtract (-1) → -(-1) = +1

Here are a few examples to show temperature change using magic hot and cold cubes.

	Simplest ways:		**Other Ways:**	
+4 degrees	Put in 4 hot cubes +(+4) = 4	Remove 4 cold cubes -(-4) = 4	Put in 6 hot cubes and put in 2 cold cubes +(+6) + (-2) = 4	Remove 6 cold cubes and remove 2 hot cubes -(-6) – (+2) = 4
-2 degrees	Remove 2 hot cubes -(+2) = -2	Put in 2 cold cubes +(-2) = -2	Remove 3 hot cubes and remove 1 cold cube -(+3) – (-1) = -2	Put in 3 cold cubes and put in 1 hot cube +(-3) + (+1) = -2
0 degrees	Do nothing 0		Put in 4 hot cubes and put in 4 cold cubes +(+4) + (-4) = 0	Remove 3 hot cubes and remove 3 cold cubes -(+3) – (-3) = 0

Representing the Additive Inverse

The minus sign may be used to show additive inverses. The identity $a + (-a) = 0$ means that $-a$ is the additive inverse of a. It is what we add to a to get 0.	Example: If $a = -3$, then $-a = 3$ The statement, “If a is equal to minus 3, then minus a is equal to 3” can be read: • If a is equal to the opposite of 3, then the opposite of a is equal to 3. When we add -3 and 3, the result is 0. 

A Counter Model

This counter model is used to model integers.

Let + represent a positive counter with a value of positive 1
Let – represent a negative counter with a value of negative 1.

A <u>zero pair</u> is a pair with one positive counter and one negative counter. Both representations below have a value of zero.

one zero pair: + / –

three zero pairs: + + + / – – –

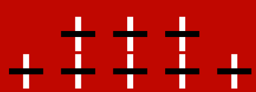
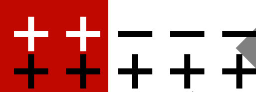
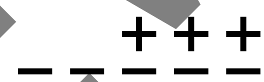
Below are some counter diagrams that represent the given integers:

	+4	-2	0
Simplest representation:	+ + + +	– –	(no counters)
Other representations:	+ + + + + –	– – + + – –	+ –
	+ + + + + + + – – –	– – – – – – + + + +	+ + + + – – – –

Counter Addition Sentence Frames

- Begin with a work space that has a value equal to 0.
- Build __________ _____.
  (positive/negative)
- The plus (+) means to add.
- Add _____ __________ counter(s).
  (positive/negative)
- The result is _____ __________ counter(s).
  (positive/negative)

Integer Addition Using Counters

-3 + (-5) = -8	-3 + 5 = 2	3 + (-5) = -2
		
• Start with a work space equal to zero.	• Start with a work space equal to zero.	• Start with a work space equal to zero.
• Build negative 3.	• Build negative 3.	• Build positive 3.
• The (+) means to add.	• The (+) means to add.	• The (+) means to add.
• Add 5 negative counters.	• Add 5 positive counters.	• Add 5 negative counters.
• The result is 8 negative counters.	• The result is 2 positive counters.	• The result is 2 negative counters.

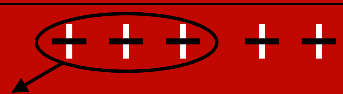
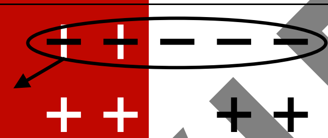
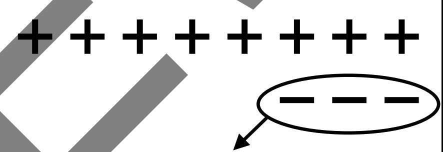
Rules for Addition of Integers

Rule 1: When the addends have the same sign, add the absolute values. Use the original sign in the answer.

Rule 2: When the addends have different signs, subtract the absolute values. Use the sign of the addend with the greatest absolute value in the answer.

Counter Subtraction Sentence Frames
• Begin with a work space that has a value equal to 0. • Build ___________ _____. positive/negative • The minus (–) means to subtract. • Subtract _____ ___________ counter(s). Introduce zero pairs if needed. positive/negative • The result is _____ ___________ counter(s). positive/negative

Integer Subtraction Using Counters

-5 – (-3) = -2	-3 – (-5) = 2	5 – (-3) = 8
		
• Start with a work space equal to zero. • Build negative 5. • The (–) means to subtract. • Subtract 3 negative counters. I do not need zero pairs. • The result is 2 negative counters.	• Start with a work space equal to zero. • Build negative 3. • The (–) means to subtract. • Subtract 5 negative counters. I need zero pairs to do this. • The result is 2 positive counters.	• Start with a work space equal to zero. • Build positive 5. • The (–) means to subtract. • Subtract 3 negative counters. I need zero pairs to do this. • The result is 8 positive counters.

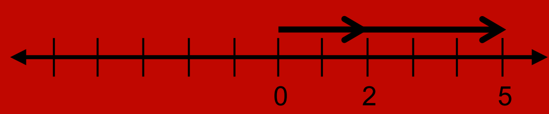
Rule for Subtraction of Integers
Rule: In symbols, $a - b = a + (-b)$ and $a - (-b) = a + b$. In words, the result is the same whether subtracting a quantity or adding its opposite. Examples: 6 – 4 = 6 + (-4) = 2 -3 – (-2) = -2 + 2 = -1

Addition and Subtraction on a Number Line

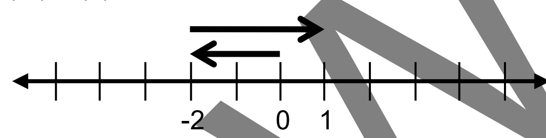
We can use arrows to represent addition and subtraction on a number line. For **adding** any two numbers:

- The absolute value of a number is represented by the arrow length.
- The first arrow begins at zero. If it's representing a positive number, the arrow points to the right. If it's representing a negative number, the arrow points to the left.
- If the second number is positive, the arrow points right. If the second number is negative, the arrow points left.
- The sum is represented by the end (tip) position of the second arrow.

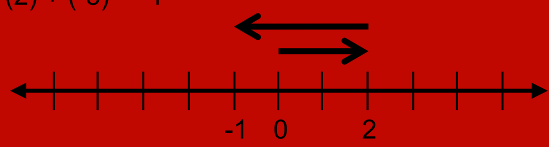
(2) + (3) = 5



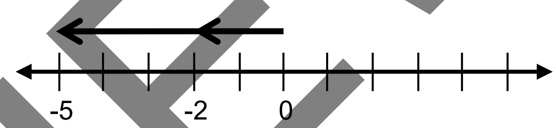
(-2) + (3) = 1



(2) + (-3) = -1



(-2) + (-3) = -5

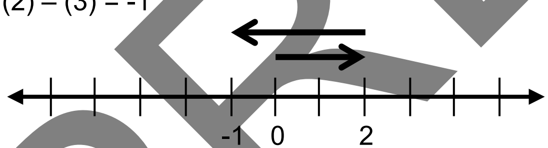


For **subtracting** any two numbers, remember that any minus sign signals doing the "opposite:"

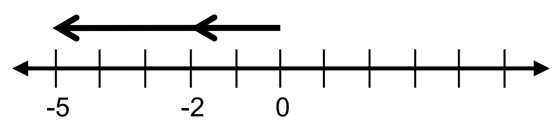
- The absolute value of a number is represented by the arrow length.
- The first arrow begins at zero. If it represents a positive number, the arrow points to the right. If it represents a negative number, the arrow points to the left.
- If the second number is positive, move the opposite of right (LEFT). If the second number is negative, move the opposite of left (RIGHT).
- The difference is represented by the end (tip) position of the second arrow.

Compare the subtraction problems below to the addition problems above. Notice that the first numbers and arrows are all identical to those above. Notice that the numbers subtracted are identical as well, and so the second arrows all point in the opposite direction.

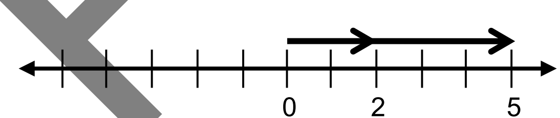
(2) – (3) = -1



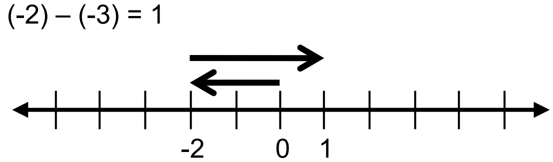
(-2) – (3) = -5



(2) – (-3) = 5



(-2) – (-3) = 1



COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
7.NS.A	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
7.NS.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
a	Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
b	Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending upon whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
c	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
d	Apply properties of operations as strategies to add and subtract rational numbers.
7.SP.C	**Investigate chance processes and develop, use, and evaluate probability models.**
7.SP.7	Develop a probability model and use it to find probabilities of events.
b	Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation:
a	Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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