Name ______________________________ Period __________ Date ______________

MathLinks GRADE 6

UNIT 8
STUDENT PACKET

SOLVING EQUATIONS

Parent (or Guardian) signature ____________________________________

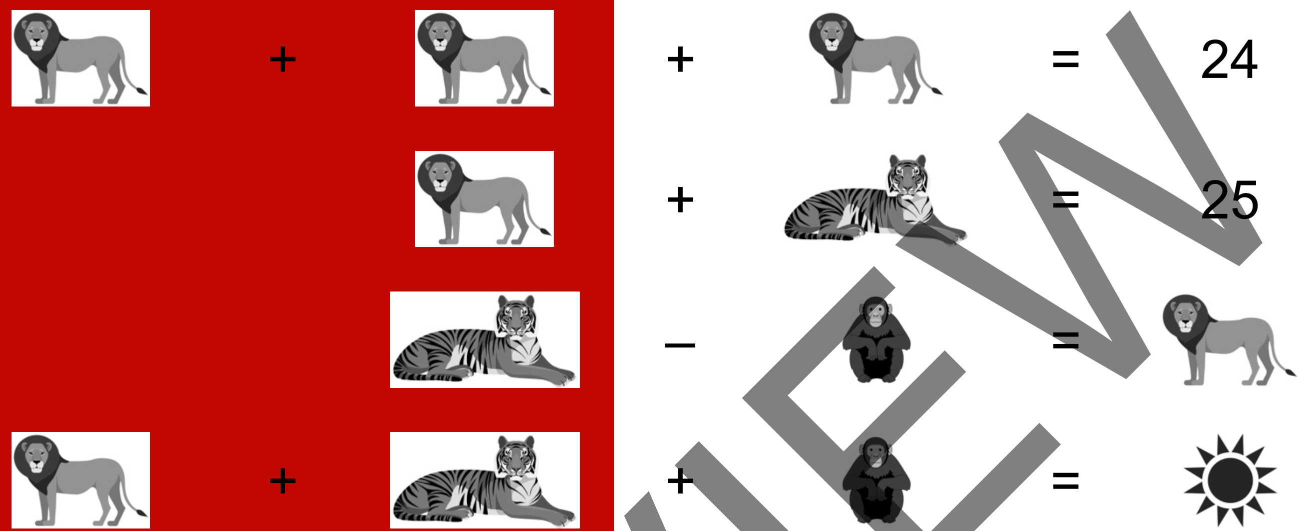
MY WORD BANK

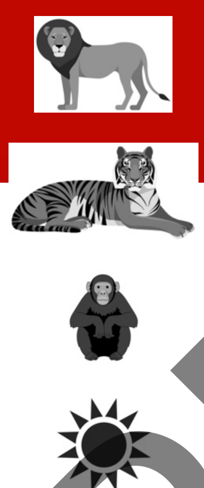
Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

distributive property	equation
evaluate	expression
simplify	solution to an equation
solve an equation	substitution

LIONS AND TIGERS AND BEARS

On this page, the same animals have the same value. Different animals have different values. Find the value of each animal and the value of . Explain your reasoning.





ALGEBRAIC EQUATIONS

We will explore mobiles and relate them to equations. We will revisit Nonna's Pizza Menu and find solutions to equations using substitution and a solution set.

[6.EE.5, 6.EE.6, 6.EE8; SMP1, 2, 6]

GETTING STARTED

Do you remember the difference between an expression and an equation? Use the statements a – h in the box below for problems 1 – 4.

a. $3 + 7 = 10$	b. $18 - 2 \bullet 5$	c. $4 = 20 - 16$	d. $5(7 + 4)$
e. $w + w + w = 3w$	f. $2x + 8 = 2(x + 4)$	g. $5y - 2y$	h. $10 = 5z$

1. List all of the expressions. ___________________

   Which of these are numerical expressions? __________

   Which of these are algebraic (variable) expressions? __________

2. List all of the equations. ___________________

   Which of these are numerical equations? __________

   Which of these are algebraic equations? __________

3. Choice "d" in the box above is an ___________________.

   Apply the distributive property to create a true equation: $5(7 + 4)$ = __________________

4. Choice "g" in the box above is an ___________________.

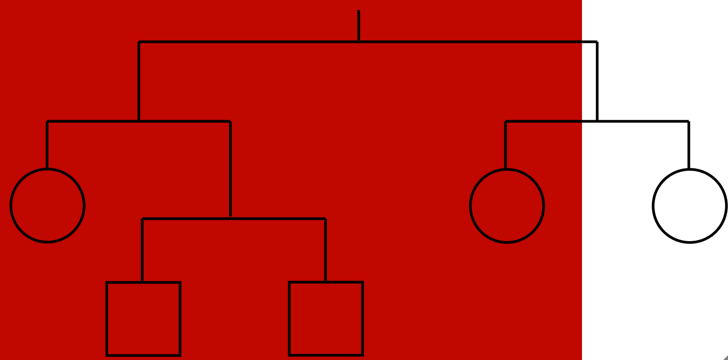
   Simplify it. ___________________ Then evaluate it for $y = 10$. ___________________

5. Record the meanings of <u>expression</u>, <u>equation</u>, <u>distributive property</u>, <u>simplify</u>, and <u>evaluate</u> in **My Word Bank**.

MOBILES AND BALANCE

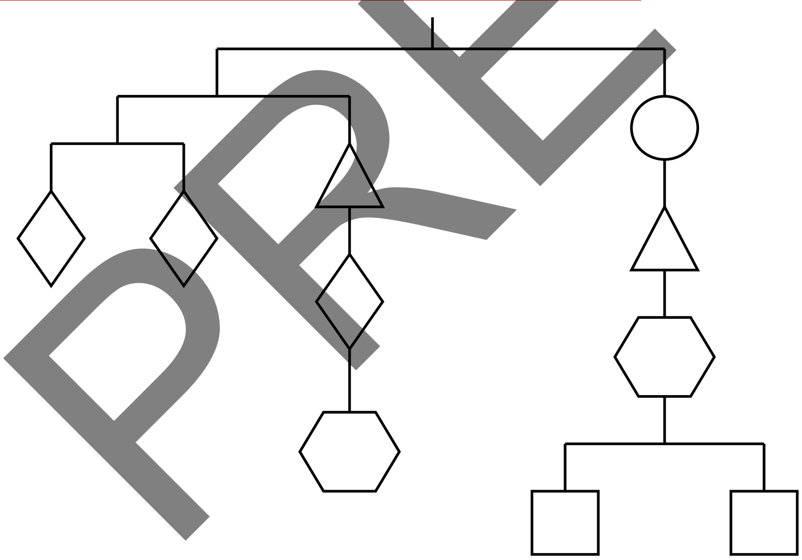
Follow your teacher's directions.

(1)



(2)

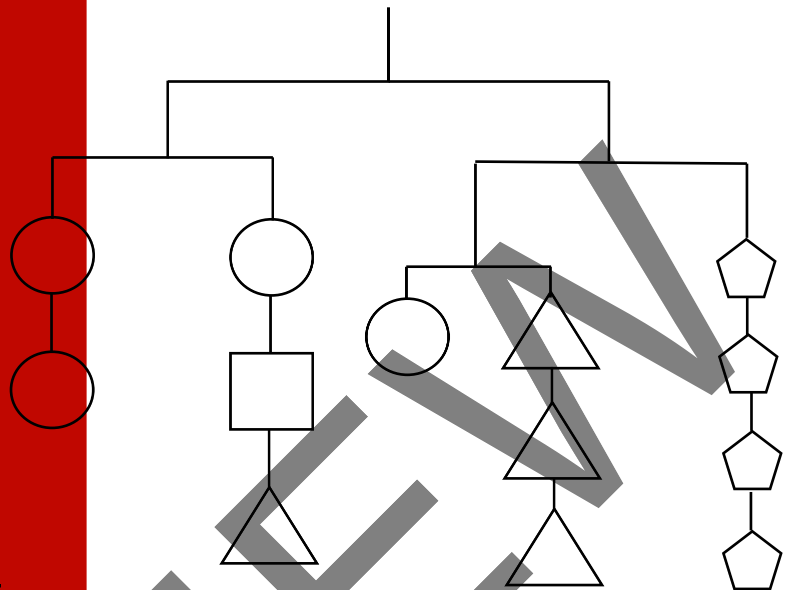
(3)



(4)

PRACTICE 1

The total weight of this mobile is 48.



1. Find the weight that each shape represents.

   Explain how you found the weight of the pentagon.

2. Write three statements or equations based on the weights on the mobile.

Fill in the blank shapes with numbers to make each equation true. The same shape must represent the same value in the same equation.

3. $24 = \square + \square + \square$

4. $\bigcirc^{3} + \bigcirc = 10$

5. ⬡ + 2 + ⬡ = 10 (the 3 is an exponent)

6. Julian looked at problem 5 and wrote ⬡8 ÷ 2 + ⬡5 = 9.

   Is this a true equation?

   Why can Julian NOT put 8 and 5 in the hexagons in problem 5?

PRACTICE 2: EXTEND YOUR THINKING

Fill in the blank shapes below with numbers to make each equation true. Squares and circles can represent different values in different equations, but the same shape must represent the same value in the same equation.

1. $20 = \square + \bigcirc$

2. $20 = \square - \bigcirc$

3. $20 = \square \bullet \bigcirc$

4. $20 = \square \div \bigcirc$

5. $20 = \square + \square + 4$

6. $20 = \square \bullet 4$

7. $20 = 2 \bullet \square - \square$

8. $20 = \dfrac{60}{\bigcirc}$

9. $8 \bullet 6 = 39 + \bigcirc$

10. $7 \bullet 6 = \square - \bigcirc$

11. Which problems must have exactly one (a unique) correct answer? Explain.

12. Choose two problems above where the values of the squares and circles are not unique. Copy them below, rewriting them with different solutions.

VARIABLES AND EQUATIONS

Follow your teacher's directions.

NONNA'S NEW AND IMPROVED PIZZA MENU
(The variable represents the cost of the item.)

Pizza		**Drinks**	
Cheese slice (c)	\$1.00	Small drink (s)	\$0.95
Pepperoni slice (p)	\$1.25	Medium drink (m)	\$1.20
Veggie slice (v)	\$1.75	Large drink (L)	\$1.60
		Extra-large drink (x)	\$1.90

(1)

(2)

(3)

(4)

PRACTICE 3

BOOM BURGERS MENU (The variable represents the cost of the item.)			
Burgers		**Drinks**	
Hamburger (*h*)	$4.00	Small drink (*s*)	$1.00
Cheeseburger (*c*)	$4.25	Medium drink (*m*)	$1.25
Veggie burger (*v*)	$4.75	Large drink (*L*)	$1.50
		Extra-large drink (*x*)	$1.75

For each equation below, find a menu item above with a cost that makes the equation true. Within the same problem below, the $\boxed{}$ refers to the same item. In different problems, the $\boxed{}$ need not represent the same menu item.

1. $2h + \boxed{} = 2v$ Menu item and its cost:	2. $2c = \boxed{} + 2m + 2s$ Menu item and its cost:
3. $h + s + m + L + x = 2 \bullet \boxed{}$ Menu item and its cost:	4. $2h = 2 \bullet (\boxed{} + m + s)$ Menu item and its cost:
5. $3(s + m + h) = 4 \bullet \boxed{} + x$ Menu item and its cost:	6. $2L + \boxed{} = 4 \bullet \boxed{}$ Menu item and its cost:

7. Record the meanings of <u>substitution</u> and <u>solution to an equation</u> in **My Word Bank**.

INEQUALITIES: EXTEND YOUR THINKING

1. Recall that "<" means "less than" and ">" means "greater than." Circle the true inequalities.

a. $13 > 2$	b. $24 > 73$	c. $10 > 10$
d. $0.12 < 0.12$	e. $0.059 > 0.1$	f. $\frac{7}{7} > 1.00$
g. $\frac{5}{12} < \frac{5}{9}$	h. $\frac{3}{5} < \frac{6}{13}$	i. $8 + 7 > 12$
j. $21 + 9 < 9 + 21$	k. $3(8 - 2) < 3(8) - 2$	l. $4(3) + 6(3) > (4 + 6)3$

2. Using the chart above, choose one false inequality involving a fraction or decimal. Explain why it is false.

3. Using the chart above, choose one false inequality involving mathematical operations and explain why it is false.

Under each inequality below are four potential solutions. Circle the solutions that make the inequality true. Then write a description of ALL of the numbers that could be solutions to the inequality.

4. $v + 5 > 12$	5. $w - 4 < 5$	6. $8x > 40$	7. $\frac{y}{3} < 6$
$v = 9$ $v = 7$ $v = 7.1$ $v = 6.9$	$w = 4$ $w = 9$ $w = 7$ $w = 8.5$	$x = 10$ $x = 5$ $x = 5.5$ $x = 4.5$	$y = 21$ $y = 24$ $y = 15$ $y = 12$
Description:	Description:	Description:	Description:

8. Why must the inequality $n > n$ always be false?

9. Why must the inequality $n + 1 > n$ always be true?

SOLVING EQUATIONS 1: ADDITION AND MULTIPLICATION

We will solve addition and multiplication equations in various ways. We will use "mental math" with substitution as a strategy to solve equations. We will solve balance scale problems as a way to examine equality. Then we will use tape diagrams and a balance strategy to solve equations.

[6.EE.5, 6.EE.6, 6.EE.7; SMP1, 6, 7]

GETTING STARTED

1. The ratio of males to females in the Mayberry High School band is 5:7. There are 84 band members.

   Draw a tape diagram to represent the quantities of band members and the way they are related in this situation. Clearly show how many males and females are in the band.

2. This tape diagram below illustrates the number of chocolate and vanilla cupcakes at Lara's party. Each rectangle labeled *c* or *v* represents an equal quantity of each cupcake flavor.

<table>
<tr><td colspan="5" align="center">60</td></tr>
<tr><td>c</td><td>c</td><td>c</td><td>v</td><td>v</td></tr>
</table>

   a. What is the ratio of chocolate to vanilla cupcakes at Lara's party?

      How many of each flavor were there?

   b. Catarina had twice as many of each cupcake at her party. Draw a tape diagram to depict this. Is the ratio of chocolate to vanilla cupcakes still the same as for Lara's party?

   c. Flavia had the same number of chocolate cupcakes as Lara, but 12 more of the vanilla cupcakes. Draw a tape diagram to depict this. Is the ratio of chocolate to vanilla cupcakes still the same as Lara's party?

SOLVING EQUATIONS WITH MENTAL MATH AND SUBSTITUTION

Follow your teacher's directions for (1) – (4).

(1)	(2)	(3)	(4)

Solve each equation using substitution and mental math. Check.

5. $56 = 8x$	6. $37 = y + 17$	7. $7m = 42$
8. $16 = n + 2$	9. $120 + v = 150$	10. $\frac{1}{4}f = 3$

11. The weight of one bag of apples, x, is unknown. There are 5 bags of apples that are all of this weight. The total weight is 40 pounds. For this situation:

a. Draw a tape diagram.	b. Write an equation and its solution.

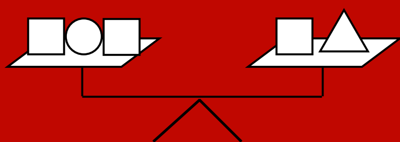
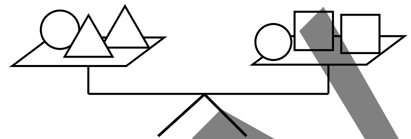
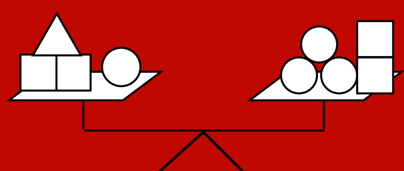
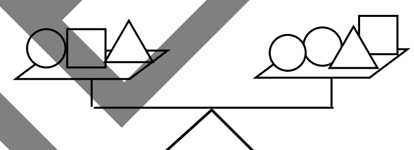
c. Use the value for x from part b above. Write an equation to express that the sum of the weights of the bag of apples plus the weight of a bag of oranges, y, is 12 pounds.

Solve for y.

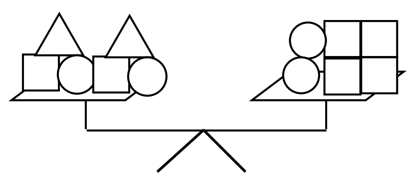
12. Record the meaning of solve an equation in **My Word Bank**.

BALANCE SCALE PUZZLES

In each problem below, all shapes have some weight, the same shapes have the same weight, and different shapes have different weights. All problems are independent of one another. Use what you know about balance to answer each question.

1.  What weighs the same as △ ?	2.  What weighs the same as □ ?
3.  What weighs the same as △ ?	4.  Why is this picture impossible?

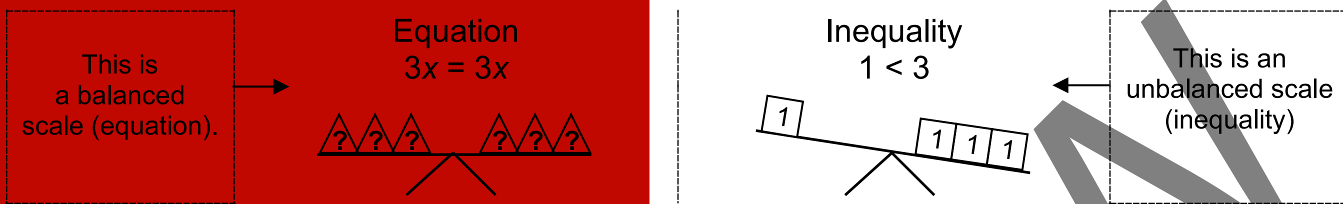
Use the scale to the right for problems 5 – 7.



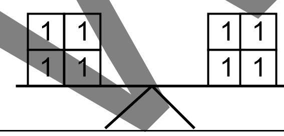
	Action on the left side	Action on the right side	Still balanced ?	Reasoning
5.	add ○△	add ○△		
6.	remove ○○ □□	remove ○○ □□		
7.	remove △ □○	remove ○□□		

BALANCED AND UNBALANCED SCALES

We can picture equalities with balanced scales and inequalities with unbalanced scales. Imagine that each $\boxed{1}$ represents one unit of weight and each ? represents an unknown weight. To represent unknowns, a popular variable is *x*.

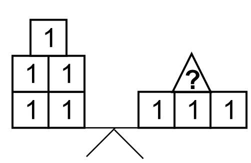


For each problem, **start with this original balanced scale**, $4 = 4$.
Draw a sketch to illustrate the action described.
Write the resulting equation or inequality.



1. Three units (1's) are removed from both sides of the original balanced scale. Equation or inequality:	2. One unknown (*x*) is added to both sides of the original balanced scale. Equation or inequality:
3. Two units are removed from the right side of the original scale. Equation or inequality:	4. Two *x*'s are added to the right side, and one *x* to the left side of the original scale. Equation or inequality:
5. The number of units on both sides of the original scale is doubled. Equation or inequality:	6. One-half the units are removed from each side of the original scale. Equation or inequality:

7. Harry built the balanced scale to the right.



   a. Write the equation it represents.

   b. Why can Harry remove 3 units from both sides?

   c. Draw the new balanced scale and write the equation it represents.

   d. Does the equation in part c represent the solution to the equation in part a?

SOLVING EQUATIONS WITH TAPES AND BALANCE

Follow your teacher's directions for (1) – (9). Tape diagrams need not be to scale.

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)

10. Write a story problem to represent this tape diagram.

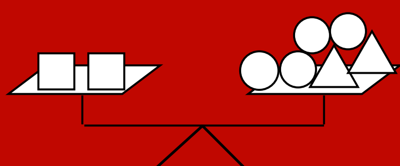
x	135
450	

PRACTICE 4

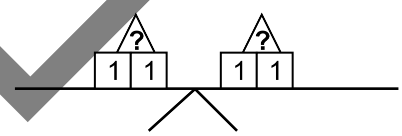
Solve each equation using substitution and mental math. Check.

1. $12m = 36$	2. $450 = w + 200$	3. $\frac{1}{4} + k = 1$	4. $\frac{1}{3}p = 10$

5. What group of shapes weighs the same as $\square$?



6. Let $\boxed{1}$ represent one unit of weight and ? represent an unknown weight, x. Add $2x + 1$ to both sides of this balanced scale, and write the new equation that it represents.



Write an equation for each tape diagram and solve using an algebraic process. Check.

7.

m	13.5
82.4	

8.

231		
n	n	n

9.

v	v	v	v
$3\frac{1}{5}$			

10.

$5\frac{1}{6}$	h
$12\frac{2}{3}$	

SOLVING EQUATIONS 2: ALL FOUR OPERATIONS

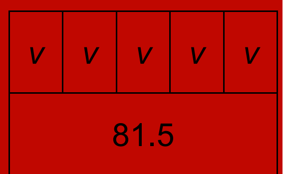
We will solve equations using all four operations. We will continue to solve equations using mental math with substitution, plus tape diagrams and balance. We will solve problems using algebra, and apply new strategies to solve these equations.

[6.EE.5, 6.EE.6, 6.EE.7, 6.EE.9; SMP1, 2, 4, 8]

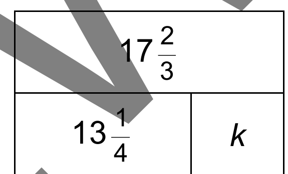
GETTING STARTED

Write an equation for each tape diagram and solve using an algebraic process. Check.

1.

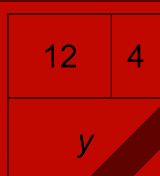


2.



Follow the directions for each tape diagram below.

3. For this tape:



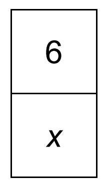
a. Write an equation the diagram represents.

b. Increase the amount of tape so that 8 is added to the top and bottom parts.

c. Write the new equation the diagram represents.

d. Did this change to the tape diagram change the value of *y*?

4. For this tape:



a. Write an equation the diagram represents.

b. Increase the amount of tape so that the top and bottom parts are multiplied by 3 (show a total of 3 copies).

c. Write the new equation the diagram represents.

d. Did this change to the tape diagram change the value of *x*?

REVISITING MENTAL MATH AND SUBSTITUTION

Follow your teacher's directions for (1) – (4).

(1)	(2)	(3)	(4)

Solve each equation below using substitution and mental math. Check.

5. $18 - y = 10$	6. $x - 4 = 24$	7. $9m = 54$
8. $\frac{n}{4} = 6$	9. $\frac{27}{v} = 3$	10. $\frac{1}{5}f = 6$
11. $100 = w + 2$	12. $2h = 1$	13. $\frac{r}{16} = \frac{1}{2}$

14. The number of oranges in a crate, x, is unknown. There are 5 people who will share them equally. Everyone expects to get 12 oranges. Write an algebraic equation for this situation, then solve for x. Be sure to state what the solution for x means.

15. Use the value for x from problem 14. Write an equation for which y people share x oranges so each person gets 20 of them. Solve for y and state what the solution for y means.

REVISITING TAPES AND BALANCE

Follow your teacher's directions for (1) – (9).

	Tape Diagram	Equation
Drew (1) – (3)		
Aisha (4) – (6)		
Maya (7) – (9)		

Solve each equation below.

10. $x - 53 = 137$	11. $\frac{x}{3} = 49$	12. $\frac{2x}{5} = 48$

PRACTICE 5

Solve each equation below.

1. $x - 296 = 712$	2. $10 - 2.6 = x - 4.5$	3. $638 = x + 527$
4. $\frac{2x}{9} = 12$	5. $\frac{x}{1.8} = 0.04$	6. $\frac{x}{6} = 2\frac{1}{3}$
7. $273 = 10x + 3x$	8. $11.4 = x + 1.1 + 1.6$	9. $4x - 0.5x = 14.7$
10. $x - 1\frac{1}{2} = \frac{1}{4}$	11. $\frac{3}{4} = x + \frac{2}{3}$	12. $\frac{1}{6} = 1\frac{2}{3}x$

TRANSLATING PROBLEMS INTO EQUATIONS

Write and solve the equations below as directed.

1. The total number of puppies and kittens is 150.

   a. Write a numerical equation if there are 80 puppies and 70 kittens.

   b. Write a variable equation if there are p puppies and k kittens.

   c. Write an equation and solve for p if there are 57 kittens.

2. The number of trading cards KC has after giving some away is 49.

   a. Write a numerical equation if KC started with 77 cards and gave away 28.

   b. Write a variable equation if KC had x trading cards and gave away y of them.

   c. Write an equation and solve for x if KC gave away 33 cards.

3. Simon has 144 keychains.

   a. Write a numerical equation if Simon has 6 times as many keychains as Sarah, and Sarah has 24 keychains.

   b. Write a variable equation if Simon has m times as many keychains as Sarah, and Sarah has n keychains.

   c. Write an equation and solve for n if Simon has 12 times as many keychains as Sarah.

4. The number of playing cards in each stack is 35.

   a. Write a numerical equation if Salim has 280 cards, and puts them into stacks that each have 8 cards.

   b. Write a variable equation if Salim has f cards, and puts them into stacks that each have h cards.

   c. Write an equation and solve for f if each stack has 10 cards.

PRACTICE 6: EXTEND YOUR THINKING

Translate each of the following word problems into an algebraic equation. Clearly state what quantity your variable stands for. Solve the equation. Answer the question. For percent problems it may be helpful to change percent values to equivalent decimal values.

1. Preston spent $56.99 on a pair of jeans and then some more on a shirt. The tax amount was $7.74, and the total amount he spent altogether was $93.72. How much did the shirt cost?	2. A rectangle has length equal to $1\frac{1}{4}$ inches and area equal to $4\frac{1}{2}$ square inches. What is its width?
3. A jacket was on sale for 25% off, making for a saving of $12.15. How much was the jacket originally?	4. $18.60 was taken off the price of a different jacket, which was originally priced at $62. What percent was saved?
5. Peyton paid $4.32 tax on a new hair dryer. Out of curiosity she found out that where she lives, she is charged a 9% sales tax. How much was the hair dryer she purchased?	6. Parker is a server in a restaurant. He received an $21.60 tip on a bill that totaled $120. What percent tip did he receive?

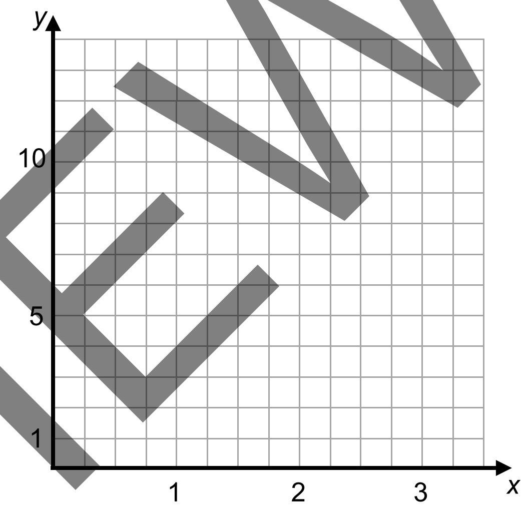
LETICIA'S TRAINING

1. Leticia is training for a marathon. Each run is at a constant rate of speed.

   a. Complete the table below that shows various times and distances she has run recently.

Time in hours (*t*)	1	2			0.5	0.25	1.5	3.25
Distance in miles (*d*)	5		15	20				

   b. Draw and label a graph for Leticia's training data.



   c. Write an equation that relates *t* and *d*.

   d. What is the number of miles per hour Leticia runs? Circle where you see this in the table, the graph, and the equation.

2. How many minutes are in 0.1 hour?

3. Write equations for each of the following distances Leticia ran on other days. Solve each equation for t. Then write the time in hours and minutes.

a. *d* = 13 miles	b. *d* = 2 miles	c. *d* = 26 miles

4. Adelina went on several runs of distance *a*, and decided that each one would be 3.2 miles less than some of Leticia's runs, *d*. Write an equation that relates *a* and *d*.

5. Write and solve an equation to find *d* for each of these distances run by Adelina. Then write the distance in miles.

a.	b.	c.

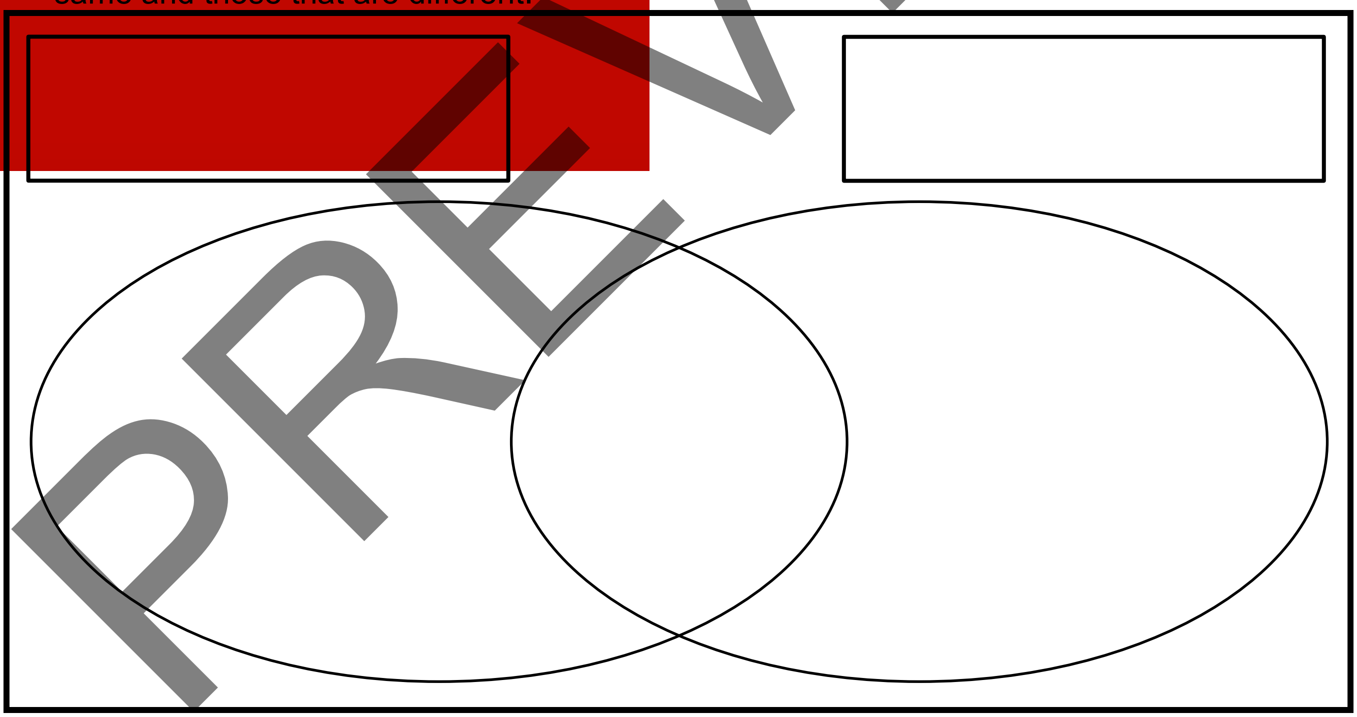
REVIEW

MATCH AND COMPARE SORT: SOLVING EQUATIONS

1. Individually, match words with descriptions. Record results.

Card set △			Card set ○		
Card number	word	Card letter	Card number	word	Card letter
I			I		
II			II		
III			III		
IV			IV		

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.



3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

MATCH 'EM UP

1. Your teacher will give you some cards. Match the equations with the solutions.

2. Create equations with solutions for your own Match 'Em Up game. Be creative and challenge yourself with the equations you write.

Equation cards:			
Solution cards:			

3. Exchange papers with a classmate. Check each-others' work. Make corrections as needed.

4. Bonus Fun: Your teacher will give you blank cards. Write your equations and solutions on the cards. Combine your cards with other students in your class and create a new game. It might be Match 'Em Up, or something else. Rummy? Memory? Play your game.

BIG SQUARE PUZZLE: SOLVING EQUATIONS

Your teacher will give you a puzzle to assemble. Divide up the cards among your partners. Before putting the puzzle together, copy four of your equations from the puzzle below, and show the work to solve them.

1. A multiplication equation with a fraction:	2. A multiplication equation with a decimal:
3. A division equation with a fraction:	4. A subtraction equation with a decimal:

POSTER PROBLEMS: SOLVING EQUATIONS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is ________.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

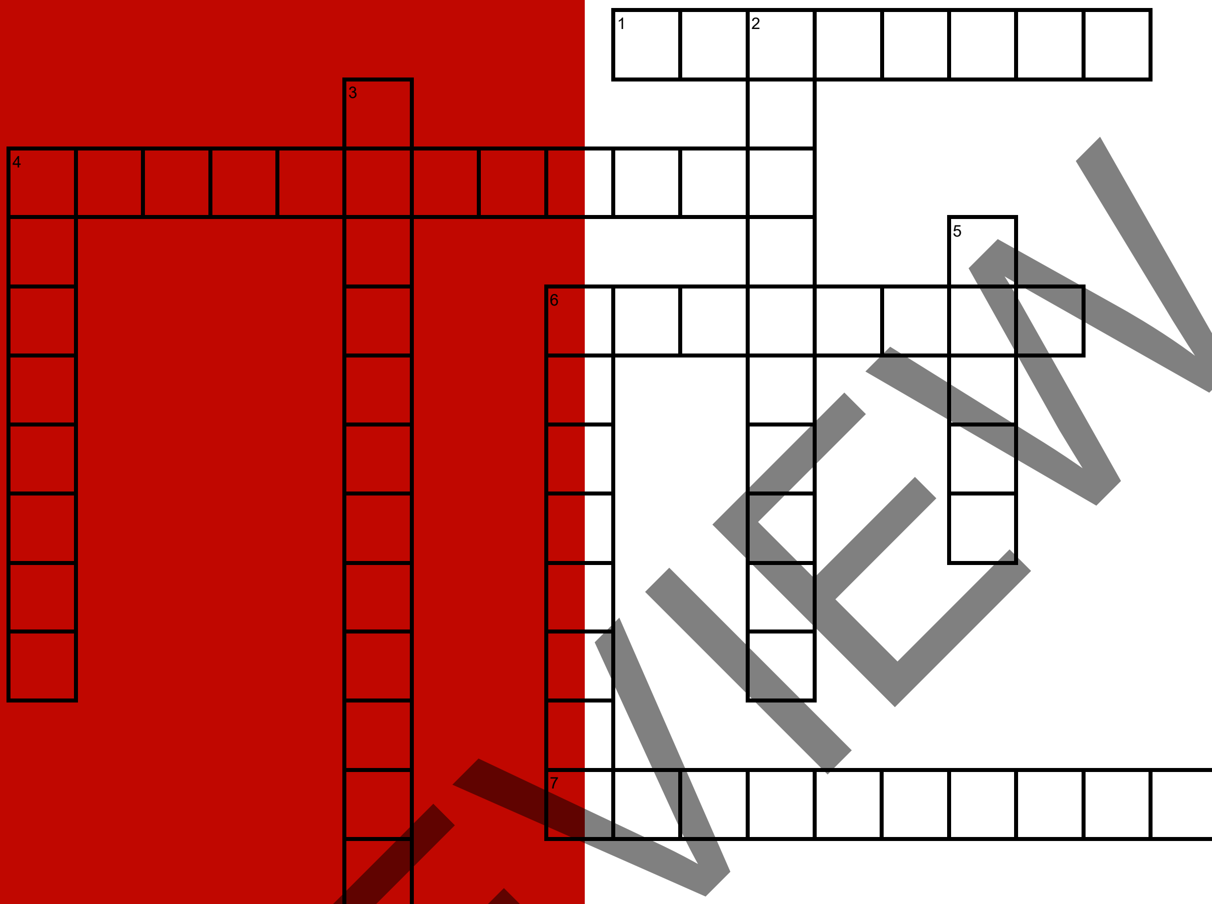
Poster 1 (or 5)			
1. $\frac{2}{3}v = \frac{5}{6}$	2. $9.1 = w + 2.8$	3. $2.1 = \frac{x}{0.7}$	4. $y - 1\frac{3}{4} = 2\frac{1}{8}$
Poster 2 (or 6)			
1. $\frac{2v}{3} = \frac{4}{15}$	2. $1\frac{5}{9} + w = 2\frac{1}{6}$	3. $\frac{x}{5} = 0.08$	4. $y - 0.94 = 1.02$
Poster 3 (or 7)			
1. $\frac{5}{6} = 15v$	2. $0.15 + w = 4.07$	3. $\frac{1}{2} = \frac{x}{\frac{1}{4}}$	4. $y - 2.5 = 8\frac{1}{4}$
Poster 4 (or 8)			
1. $\frac{1}{10} = \frac{3v}{5}$	2. $6\frac{2}{3} = w + 1\frac{3}{4}$	3. $\frac{x}{8.1} = 0.04$	4. $8.2 = y - 1\frac{1}{5}$

A. Copy and solve equation 1.
B. Check equation 1. Copy and solve equation 2.
C. Check equation 2. Copy and solve equation 3.
D. Check equation 3. Copy and solve equation 4.

Part 3: Go to your start poster and check equation 4. Then return to your seats. Work with your group and do the following.

Create a multiplication equation with at least one fraction that has the solution $x = \frac{1}{2}$.	Create a subtraction equation with at least one decimal that has the solution $n = 2.5$.

VOCABULARY REVIEW



Across

1 To write an expression in a simpler form
Example: $4n + 2n$ is $6n$

4 replacing one quantity with another that is equal to it

6 a statement asserting that two expressions are equal

7 a combination of numbers, variables, and operation symbols

Down

2 a strategy for solving an equation that involves substitution and no work shown (2 words)

3 a property that rewrites a product as a sum or vie versa

4 a value for a variable that makes an equation true

5 to find a value for a variable that makes an equation true

6 To find the value of an expression

SPIRAL REVIEW

1. **Computational Fluency Challenge**: This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

   a. Start with 7.8. Multiply by 10. Multiply the result by 0.5. Multiply the result by 3. Multiply the result by 2. Now you have a "big number". My big number is _____.

   b. Start with your big number. Divide it by 9. Divide the result by 13. What is the final result? _____

2. Evaluate each expression below if $a = 5$.

a. $3a^2$	b. $a^2 - 3$	c. $\frac{(3a)^2}{a}$

3. Complete each equivalence statement. Then write the name of an object that might represent that measurement.

a. 16 oz = ____ lb	b. 16 oz = _____ qt	c. 50 g = ____ kg
d. 25 liter ≈ ____ qt	e. 3000 lb = ____ ton	f. 1 cup = ____ oz

SPIRAL REVIEW
Continued

4. Paxton Middle School collected 1,000 cans for their food drive. The leadership class will put all the collected cans into bags that can hold up to 21 pounds. Each can weighs between 6 oz and 18 oz.

Description	Numerical expression	Simplified expression
a. Lightest can in pounds		
b. Heaviest can in pounds		
c. Least number of bags used		
d. Greatest number of bags used		
e. What is the average weight per can? f. For a sturdy bag that can hold up to 40 pounds, using the average weight, about how many bags would be needed?		

5. Circle the equivalent expressions. What property is demonstrated by their equivalence?

$6(2) + 6(7)^2$ $\qquad$ $[6(2) + 7]^2$ $\qquad$ $6(2 + 7^2)$

6. Complete the table below.

Fraction	Decimal	Percent	Percent of $1000
			$750
		125%	
	1.5		
		32.5%	

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this packet. Draw lines to show connections that you noticed.

- Investigate concepts and solve problems involving length, area, and volume.
- Extend the number system to include negatives.
- Use statistical measures and displays to describe center and spread.
- Explore relationships between inputs and outputs.
- Gain computational fluency with positive rational numbers.
- Rewrite and evaluate expressions and solve equations.
- Explore and apply ratio and rate reasoning and representations.

Give an example from this packet of one of the connections above.

2. **Packet Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** What mathematical representations did you use to explore Leticia's training problem [SMP4]? Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **More Connections.** What are some different strategies you learned for solving equations? Which do you prefer? Why?

STUDENT RESOURCES

Word or Phrase	Definition
distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. $3(4 + 5) = 3(4) + 3(5)$ and $(4 + 5)8 = 4(8) + 5(8)$
equation	An equation is a mathematical statement that asserts the equality of two expressions. $18 = 8 + 10$ is an equation that involves only numbers. This is a numerical equation. $18 = x + 10$ is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.
evaluate	Evaluate refers to finding a numerical value. To evaluate an expression, replace each variable in the expression with a value and then calculate the value of the expression. To evaluate the numerical expression $3 + 4(5)$, we calculate $3 + 4(5) = 3 + 20 = 23$. To evaluate the variable expression $2x + 5$ when $x = 10$, we calculate $2x + 5 = 2(10) + 5 = 20 + 5 = 25$.
equivalent expressions	Two mathematical expressions are equivalent if, for any possible substitution of values for the variables, the two resulting numbers are equal. In particular, two numerical expressions are equivalent if they represent the same number. See expression. The numerical expressions $3 + 2$ and $9 – 4$ are equivalent, since both are equal to 5. The algebraic expressions $3(x + 4)$ and $3x + 12$ are equivalent. For any value of the variable x, the expressions represent the same number.
expression	A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8 + x}{10}$, and $4v – w$.
inequality	An inequality is a mathematical statement that asserts the relative size or order of two objects. When the expressions involve variables, a solution to the inequality consists of values for the variables which, when substituted, make the inequality true. $5 > 3$ is an inequality. $x + 3 > 4$ is an inequality. All values for x that are greater than 1 are solutions to this inequality.

Solving Equations

Word or Phrase	Definition
simplify	Simplify refers to converting a numerical or variable expression to a simpler form. A variable expression might be simplified by combining like terms. A fraction might be simplified by dividing numerator and denominator by a common divisor. $\frac{8}{12}$ may be simplified to the equivalent numerical expression $\frac{2}{3}$. $2x + 6 + 5x + 3$ may be simplified to the equivalent variable expression $7x + 9$.
solution to an equation	A solution to an equation involving variables consists of values for the variables which, when substituted, make the equation true. The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.
solve an equation	To solve an equation refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. To solve the equation $2x = 6$, one might think "two times what number is equal to 6?" Since $2(3) = 6$, the only value for x that satisfies this condition is 3. Therefore 3 is the solution.
substitution	Substitution refers to replacing a value or quantity with an equivalent value or quantity. If $x + y = 10$, and $y = 8$, then we may substitute this value for y in the equation to get $x + 8 = 10$.
tape diagram	A tape diagram is a graphical representation that uses length to represent relationships between quantities. We draw rectangles with a common width to represent quantities, and rectangles with the same length to represent equal quantities. Tape diagrams are typically used to represent quantities expressed in the same unit. This tape diagram represents a drink mixture with 3 parts grape juice for every 2 parts water. | G | G | G | W | W |

Variables in Algebra	
Loosely speaking, variables are quantities that can vary. Variables are represented by letters or symbols. Variables have many different uses in mathematics. The use of variables, together with the rules of arithmetic, makes algebra a powerful tool. Three important ways that variables appear in algebra are the following.	
Usage	**Examples**
Variables can represent an *unknown quantity* in an equation or inequality. In this case, the equation or inequality is valid only for specific value(s) of the variable.	$x + 4 = 9$ $5n = 20$ $y < 6$
Variables can represent *quantities that vary* in a relationship. In this case, there is always more than one variable in the equation.	Formula: $P = 2\ell + 2w$, $A = s^2$ Function (input-output rule): $y = 5x$, $y = x + 3$
Variables can represent *quantities in statements that generalize* rules of arithmetic. In this case, there may be one or more variables.	Commutative property of addition: $x + y = y + x$ Distributive property: $x(y + z) = xy + xz$

Using Shapes to Represent Variables		
If the same shape (variable) is used more than once in an equation, it must represent the same value each place it appears. Two different shapes (variables) in an equation may represent the same value or different values.		
This is allowed	This is allowed	This is NOT allowed
□ + □ = ○	□ + ⬯ = ◇	○ + ○ = △
7 + 7 = 14	6 + 6 = 12	6 + 4 = 10

Evaluate or Simplify?
We use the word "evaluate" when we want to calculate the value of an expression. To evaluate 16 – 4(2), follow the rules for order of operations and compute: 16 – 4(2) = 16 – 8 = 8. To evaluate $6 + 3x$ when $x = 2$, substitute 2 for x and calculate: 6 + 3(2) = 6 + 6 = 12. We use the word "simplify" when rewriting a number or an expression in a form more easily readable or understandable. To simplify $2x + 3 + 5x$, combine like terms: $2x + 3 + 5x = 7x + 3$. Sometimes it may not be clear what is the simplest form of an expression. For instance, by the distributive property, $4(x + 2) = 4x + 8$. For some applications, $4(x + 2)$ may be considered simpler than $4x + 8$, but for other applications, $4x + 8$ may be considered simpler than $4(x + 2)$.

How to Determine if an Equation is True

THE PIZZA PLACE MENU
(The variable represents the cost of an item.)

Pizza		**Drinks**	
Cheese slice (*c*)	\$1.00	Small drink (*s*)	\$0.95
Pepperoni slice (*p*)	\$1.25	Large drink (*L*)	\$1.75

.

What value from the menu above makes this equation true?

$p + \square = 3c$

Substitute: $1.25 + \square = 3(1.00)$

$1.25 + 1.00 = 3.00$? NO

$1.25 + 1.25 = 3.00$? NO

$1.25 + 0.95 = 3.00$? NO

$1.25 + 1.75 = 3.00$? YES

The equation is true when $\square$ represents the cost of a large drink ($L = 1.75$).

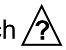
Solving Equations Using a Mental Math and Substitution Strategy

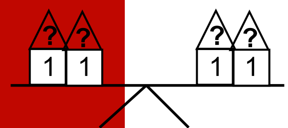
To solve an equation using mental math and substitution, apply your knowledge of arithmetic facts to find a value for the unknown that makes the equation true.

Example 1: $8x = 40$ Think: *8 times what number is 40?* Since $8(5) = 40$, $x = 5$ Check: $8(5) = 40$ $40 = 40$	Example 2: $8 + h = 20$ Think: *8 plus what number equals 20?* Since $8 + 12 = 20$, $h = 12$ Check: $8 + 12 = 20$ $20 = 20$
Example 3: $4 = 12 - k$ Think: *4 is equal to 12 minus what number?* Since $4 = 12 - 8$, $k = 8$ Check: $4 = 12 - 8$ $4 = 4$	Example 4: $\frac{n}{3} = 8$ Think: *What number divided by 3 is 8?* Since $\frac{24}{3} = 8$, $n = 24$ Check: $\frac{24}{3} = 8$ $8 = 8$

Balance Scales and Laws of Equality

Balance scales are physical representations of equations because both sides of a balanced scale must have the same weight, and both sides of an equation must have the same value.

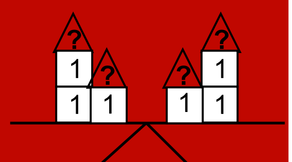
Imagine that each $\boxed{1}$ represents one unit of weight and each  represents an unknown weight (not equal to zero). To represent unknowns, a popular variable is *x*.



The balanced scale above represents the equation $2x + 2 = 2x + 2$.

Example 1: Start with the balance scale above. Add the same thing to both sides, like 1.

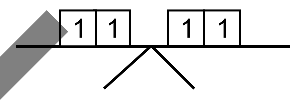
New scale:
(still balanced)



New equation: $2x + 2 + 1 = 2x + 2 + 1$
$2x + 3 = 2x + 3$

Example 2: Start with the balance scale above. Subtract the same thing from both sides, like $2x$.

New scale:
(still balanced)



New equation: $2x + 2 - 2x = 2x + 2 - 2x$
$2 = 2$

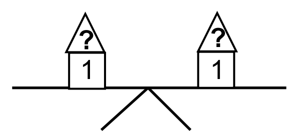
Example 3: Multiply both sides by the same thing, like 2.

New scale:
(still balanced)

New equation: $2(2x + 2) = 2(2x + 2)$
$4x + 4 = 4x + 4$

Example 4: Divide both sides by the same thing, like 2. Here we are halving the weight on each side.

New scale:
(still balanced)



New equation: $\dfrac{2x+2}{2} = \dfrac{2x+2}{2}$
$x + 1 = x + 1$

The <u>addition property of equality</u> states that if $a = b$ and $c = d$, then $a + c = b + d$. In other words, equals added to equals are equal. (See example 1 above.)

Note that this property extends to subtraction as well. (See example 2 above.)

The <u>multiplication property of equality</u> states that if $a = b$ and $c = d$, then $ac = bd$. In other words, equals multiplied by equals are equal. (See example 3 above.)

Note that this property extends to division as well. (See example 4 above.)

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.EE.B	**Reason about and solve one-variable equations and inequalities.**
6.EE.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
6.EE.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; ~~represent solutions of such inequalities on number line diagrams.~~
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

9 781614 454281