Name ______________________________ Period __________ Date ______________

UNIT 6
STUDENT PACKET

MathLinks GRADE 6

EXPRESSIONS

Parent (or Guardian) signature ______________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

coefficient	constant term
distributive property	equation
exponential notation	expression equivalent expressions
terms like terms	variable

THE PROBLEM OF 4'S

Follow your teacher's directions.

(1)	(2)	(3)
(4)	(5)	(6)

(7)

value	work	value	work
1		*2*	
3		*4*	
5		*6*	
7		*8*	
9		*10*	

NUMERICAL EXPRESSIONS

We will apply the distributive property to rewrite numerical expressions. We will define and use exponential notation. We will evaluate expressions using conventions for the order of operations.
[6.NS.4, 6.EE.1, 6.EE.3; SMP1, 3, 6, 7, 8]

GETTING STARTED

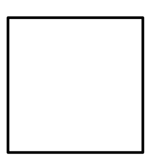
Find the greatest common factor (GCF) of each of the following pairs of natural numbers.

1. 8 and 10	2. 4 and 9	3. 24 and 36	4. 16 and 24

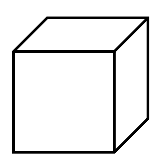
These are expressions			These are equations
$6 + 7$	$x + 3$	10	$10 = (6 - 4)5$
$\frac{1}{3}$	$(6 - 4)5$	$2(n + 5)$	$x + 3 = 6 + 7$

5. Describe what you think the difference is between an <u>expression</u> and an <u>equation</u>. Record the meaning of these words in **My Word Bank**.

6. Label the length and width of the square. Each side is 5 linear units.



7. Label the length, width, and height of the cube. Each edge is 4 linear units.



GCF AND THE DISTRIBUTIVE PROPERTY

Follow your teacher's directions for (1) – (5).

(1) – (2)

(3)

these are	these aren't

(4)	(5)

Use the distributive property to rewrite each expression and check.

6. 3(5 + 4)	7. (4 + 5)8
8. 7(6 – 2)	9. 16 + 56
10. 4(25-22)	11. 33 + 44 – 55

12. Record the meaning of the <u>distributive property</u> in **My Word Bank**. Include examples.

PRACTICE 1

1. Circle all of the equations below that correctly illustrate the distributive property.

a. 10 + 25 = (2 + 5)5	b. 9 + 24 = 3(3 + 24)
c. 5(6 + 2) = 5(6) + 5(2)	d. 40 – 16 = 8(5 – 2)

2. Circle all of the expressions below that are **not** equivalent to (9 – 2)4.

a. 7(4)	b. 9 – 2(4)	c. 28
d. 9(4) – 2(4)	e. 36 – 8	f. (2 – 9)4

Rewrite each expression using the distributive property. Check to show that the expressions are equivalent.

3. 4(5 + 7)	4. 5(8 – 3)	5. (6 + 1)

Rewrite each sum as a product by factoring out the GCF and applying the distributive property. Check that expressions are equivalent.

6. 14 + 21	7. 24 – 9
8. 5(3) + (5)5	9. 15 – 3

10. Ahmed thinks that 10 + 20 + 30 = 10(1 + 2 + 3). Prove that he is either correct or incorrect.

11. Rewrite 2 + 4 + 6 + 8 using the GCF and the distributive property.

12. A store bought 278 shirts for $7 each and sold them for $15 each. How much profit did the store make? How can you use the distributive property to make your computations easier?

EXPONENTIAL NOTATION

Follow your teacher's directions for (1) – (7).

(1)

(2)	(3)	(4)	(5)

Each small square on the grid below is one square unit of area.

(6)

(7)

8. Record the meaning of <u>exponential notation</u> with examples in **My Word Bank**.

Compute.

9. 2^5	10. six squared	11. ten cubed
12. 19^1	13. $3^2 + 3^4$	14. $2^3 \bullet 3^2$
15. $2^5 + 2^5$	16. $2(2^5)$	17. $3(3^1 + 3^2)$

ORDER OF OPERATIONS

Follow your teacher's directions.

(1)	(2)

(3)

Order of Operations

Step 1: Simplify expressions that are ____________________.

Step 2: Compute expressions with ____________________.

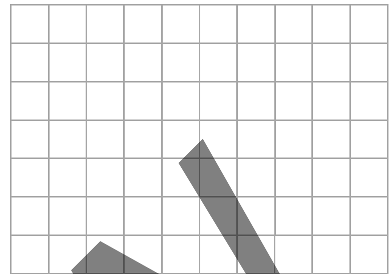
Step 3: Perform ____________________ and ____________________ from left to right.

Step 4: Perform ____________________ and ____________________ from left to right

Copy each expression and evaluate	Notes
(4)	
(5)	
(6)	

PRACTICE 2

1. Draw a 2 by 2 square, a 3 by 3 square, and two 2 by 3 rectangles on the grid to the right. Then use exponents to write an expression for the total area, and find the total area.



2. Antonia bought five pencils for $1.15 each, three erasers for $0.50 each, and three pens for $2.45 each. Write an expression for the total cost and find the total cost.

Evaluate each expression.

3. $4 \div 2 \bullet 4$	4. $8 - 2 \bullet 3$	5. $16 \div 8 \bullet 2^3$
6. $(12 + 8) \div 4 - 2$	7. $\frac{12+8}{4-2}$	8. $12 + 8 \div 4 - 2$
9. $6^2 - 12 \div 6 \div 2$	10. $\frac{6^2 - 12 \div 6}{2}$	11. $\frac{6^2 - 12}{6 \div 2}$

12. Rewrite problem 7 above using a division symbol (÷) instead of the fraction bar. The new expression should include the same numbers and have the same value.

PRACTICE 3: EXTEND YOUR THINKING

1. Evaluate 2 • 8 + 12 ÷ 4.

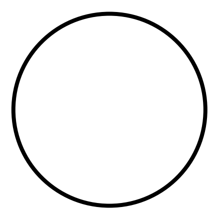
2. Copy the expression in problem 1. Then insert exactly one set of parentheses to make an expression whose value is 10. Show work to justify.

3. On October 26th, Mr. Stern challenged his class to create an expression with the value of his age, 36, using the two digits from the month number and the two digits in the day's date. The same rules applied as in the Problem of Fours. Try his challenge.

4. Fill in the table below. How does the square of a number greater than 1 differ from the square of a number between 0 and 1?

number	2	5	10	$\frac{1}{3}$	$\frac{2}{3}$	.7	0.06
square of the number							

5. Jose ate $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of one pizza.



   a. Draw a picture showing the amount of pizza Joe ate.

      Jose ate ______ of a pizza.

   b. Write two different expressions to represent this statement. Then evaluate each to confirm what your picture illustrates.

      Using multiplication symbols: _______________ Using exponents: __________

6. Recall you drew a 2×2×2 cube earlier in this lesson. Its volume was 8 cubic units. Sketch a cube here and label its length, width, and height as $\frac{1}{2}$ cm each. Then find the volume of this cube.

THE PROBLEM OF 4'S EXTENDED

1. Revisit your expressions for the numbers from 1-10 in **The Problem of 4's**. Revise them if needed on page 1 in another color so that the correct order of operations is clear.
2. Use four 4's to write expressions for as many of the numbers from 11-50 as you can. Use scratch paper if needed. Be sure to use the correct order of operations.

11	***12***	***13***	***14***
15	***16***	***17***	***18***
19	***20***	***21***	***22***
23	***24***	***25***	***26***
27	***28***	***29***	***30***
31	***32***	***33***	***34***
35	***36***	***37***	***38***
39	***40***	***41***	***42***
43	***44***	***45***	***46***
47	***48***	***49***	***50***

3. Describe an interesting strategy you used to find the numbers and circle one example above.

ALGEBRAIC EXPRESSIONS

We will write and evaluate algebraic expressions and use algebra vocabulary appropriately.
[6.NS.3, 6.NS.4, 6.EE.1, 6.EE.2,abc, 6.EE.3, 6.EE.4, 6.EE.6; SMP2, 3, 6]

GETTING STARTED

Complete the table below.

Equation	Factors	Product
Example 1: 3 • 8 = 24	3 and 8	24
Example 2: 3(2 + 6) = 24	3 and (2 + 6)	24
1. 60 = 12 • 5		
2. (19 – 11)6 = 48		
3. 49 = (2 + 5)(9 – 2)		

4. Rewrite the expression 3(2 + 6) using the distributive property. Check that the rewritten expression is still equal to 24.

5. Rewrite the expression (19 – 11)6 using the distributive property. Verify that the rewritten expression is still equal to 48.

6. One granola bar is $1.15. How much do 6 granola bars cost? Find your answer in more than one way.*

*For all problems in this lesson, we assume that tax is included.

VARIABLES AND EXPRESSIONS

Follow your teacher's directions for (1) – (5).

NONNA'S PIZZA MENU

Pizza		**Drinks**	
Cheese slice	\$1.00	Small drink	\$0.95
Pepperoni slice	\$1.25	Medium drink	\$1.20
		Large drink	\$1.60

(1)

Let _____ = the cost of a ____________

Let _____ = the cost of a ____________

Let _____ = the cost of a ____________

Let _____ = the cost of a ____________

Let _____ = the cost of a ____________

(2)

(3)

(4)

(5)

Write the meaning of each order and find its cost.

6. $L + L + L + s$	7. $3(p + s)$

8. Record the meaning of <u>variable</u> in **My Word Bank**.

PRACTICE 4

NONNA'S PIZZA MENU (The variable represents the cost of the item.)			
Pizza		**Drinks**	
Cheese slice	\$1.00	Small drink	\$0.95
Pepperoni slice	\$1.25	Medium drink	\$1.20
		Large drink	\$1.60

A group of friends decide to go to Nonna's Pizza for lunch.

- Miguel orders a slice of cheese pizza, a slice of pepperoni pizza, and a medium drink.
- Barry orders two slices of pepperoni pizza and a large drink.
- Susie orders a slice of pepperoni pizza and a medium drink.
- Ronni orders two slices of cheese pizza and a large drink.

In the table below, record the variable expressions representing the costs of each order separately, and then the total order.

	Expression for the cost of the order	**Evaluate to find the cost**
1. Miguel		
2. Barry		
3. Susi		
4. Ronni		
5. **Total** (in simplest form)		

6. Explain why $3p + 2p$ is equivalent to $5p$, regardless of the cost of a slice of pepperoni.

7. Why can $3p + 2p$ be rewritten as $(3 + 2)p$?

8. The pizza shop owner decides to take \$0.10 off the cost of each slice of pizza. Write a numerical expression for the total cost of the order in problem 5, including this discount. Then find the cost.

ALGEBRA VOCABULARY

Follow your teacher's directions for (1) – (4).

(1) – (3)

(4) ______ + ______

For problems 5 – 8, explain why each statement below is **false**.

5. The expression $5x + 4$ is the same as the expression 9x.

6. The expression $y + 8$ has no coefficient for the variable y.

7. The expression $2x + 6 + x + 4$ has two terms.

8. After applying the distributive property, the expression $4(x + 3)$ has two factors.

9. Coach Patrick is going to get pizza and drinks from Nonna's for his team. There are 12 players. One-fourth of them want 2 slices of cheese pizza and the rest want 2 slices of pepperoni pizza. One-half of them want a medium drink, one-third want a small drink, and the rest want a large drink. Use the menu for the following.

   Write an algebraic expression for the cost of this order. Then evaluate the expression to find the total cost for the coach.

10. Record the meanings of terms, like terms, coefficient, constant term, and equivalent expressions in **My Word Bank**.

PRACTICE 5

Simplify each expression if possible. Then complete the rest of the table, referring to the simplified expression.

If possible, simplify each expression	Number of terms	Constant term(s)	Term(s) with variables	Coefficient of the variable(s)
1. $2m + 10n + 1$				
2. $11r$				
3. 12				
4. $a + 2b + c + 4$				
5. $a + 2b + a + 4b$				
6. $y + 2y + y + 6$				

Apply the distributive property. Then complete the rest of the table, referring to the expanded expression.

Apply the distributive property.	Number of terms	Constant term(s)	Term(s) with variables	Coefficient of the variable(s)
7. $3(x + 2)$				
8. $2(3x + 5)$				

9. Evaluate each expression below for $v = 10$.

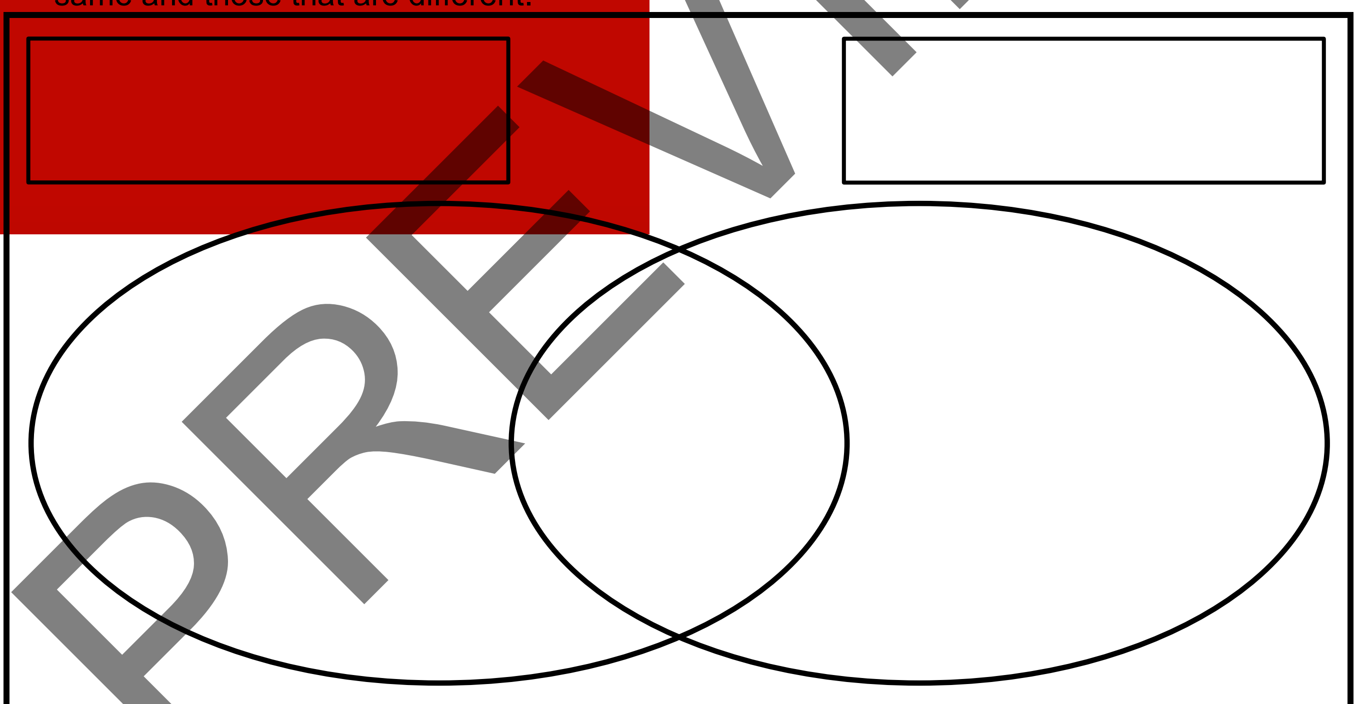
a. $\frac{v}{2}$	b. $\frac{1}{2}v$	c. $\frac{v}{5}$	d. $\frac{1}{5}v$

MATCH AND COMPARE SORT: EXPRESSIONS

1. Individually, match words with descriptions. Record results.

Card set △			Card set ○		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I			**I**		
II			**II**		
III			**III**		
IV			**IV**		

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.



3. Partners, choose another pair of numbered matched cards and discuss the attributes that are the same and those that are different.

PRACTICE 6

1. Explain why $\frac{n}{4}$ and $\frac{1}{4}n$ are equivalent.

2. Evaluate each expression below for $m = 9$.

a.	$5m + 8 - 2m + 7 + 3m + 10 - 6m - 13$
b.	$4(m + 2) + m + 7 + 3(m - 1) - 8m$
c.	$3(m + 5) + 4m + 6 + 3(m + 1) - 10m - 12$

3. What do you notice about problems 2a, 2b, and 2c above?

WORDS, NUMBERS, AND SYMBOLS

We will translate between word statements, numerical expressions, and algebraic expressions. We will evaluate algebraic expressions for different values of the variables.

[6.NS.3, 6.NS.4, 6.EE.1, 6.EE.2abc, 6.EE3, 6.EE.4, 6.EE6; SMP2, 3, 6, 7, 8]

GETTING STARTED

Consider the variable expression $x + y + x + y + x + y$.

1. Write this expression as the sum of two terms.

2. Use the distributive property to rewrite this expression as a product of a whole number and a variable expression.

Consider the variable expression $4a + b \div c$.

3. Evaluate this expression for $a = 6$, $b = 8$, $c = 2$

4. Rewrite the expression using a fraction bar instead of the division (÷) symbol. Evaluate the new expression to verify that it is equivalent to the original.

5. Rewrite the expression $6x + 4x$ in in at least two different ways.

6. Use the distributive property to rewrite the expression $12m + 36n$ in at least two different ways so that it is the product of a whole number and a variable expression.

7. Manoj says the expressions $2m$ and $2 + m$ are equivalent. He believes he is correct because if $m = 2$, then both expressions are equal to 4. Explain why Manoj is NOT correct.

PERIMETER OF A RECTANGLE

Follow your teacher's directions for (1) – (3).

(1)	(2)

(3)

Ayla's formula:	Maya's formula:	Jordan's formula:
P = ______________	*P* = ______________	*P* = ______________

4. Ayla:	Maya:	Jordan:

5. Suppose the dimensions of Emmett's fence are $20\frac{1}{2}$ feet and $16\frac{3}{4}$ feet. Find the perimeter in two different ways.

6. Use Jordan's formula to find the perimeter of a rectangular piece of paper with dimensions 8.5 inches and 11 inches. (Notation note: 8.5 inches = 8.5 in = 8.5")

7. One rectangle has dimensions of 8.5" by 11". Another has dimensions of 7.5" by 12". Are the perimeters the same or different? How do you know?

8. Write the dimensions of three more rectangles that have the same perimeter as the sheet of paper.

9. In general, what is true about the dimensions of all rectangles that have a perimeter of 39 inches?

TRANSLATING WORDS INTO NUMBERS AND SYMBOLS

Follow your teacher's directions for (1) – (6).

	Drew's description	**Aisha's description**		**Drew's description**	**Aisha's description**
(1) a.			(2) a.		
b.			b.		
(3) a.			(4) a.		
b.			b.		
(5) a.			(6) a.		
b.			b.		

Translate each statement below into an algebraic expression. Then evaluate for $v = 4$, $w = 8$.

7. The sum of two numbers, v and w.

8. Twice the sum of two numbers, v and w.

9. One-half of the sum of two numbers, v and w.

10. The sum of two numbers, v and w, then divided by 2.

11. Add a number v to another number, which is w divided by 2.

12. The sum of two numbers v divided by 2, and w divided by 2.

PRACTICE 7

Write a numerical and algebraic expression for each quantity described below.

1. The total number of puppies and kittens.
   a. The number of puppies is 6 and the number of kittens is 8.
   b. The number of puppies is p and the number of kittens is k.

2. The number of trading cards KC has after giving some away.
   a. KC had 12 trading cards and gave away 8 of them.
   b. KC had x trading cards and gave away y of them.

3. The number of Simon's ribbons.
   a. Sara has 4 ribbons. Simon has 6 times as many ribbons as Sara.
   b. Sara has n ribbons. Simon has 6 times as many ribbons as Sara.

4. The number of crackers in each group.
   a. Salim has 20 crackers. He puts them into 5 equal groups.
   b. Salim has m crackers. He puts them into 5 equal groups.

Translate each algebraic expression into words. Be sure that your word statements are clear and unmistakable.

5. $d - f + 2$	6. $2(d - f)$
7. $\frac{d - f}{2}$	8. $\frac{1}{2}d - f$

9. Write two possible different algebraic expressions that could be translated from this unclear statement: three times a number n plus two.

PRACTICE 8

Copy the three rectangle perimeter formulas from **Perimeter of a Rectangle** into the chart below. Then use them to find the perimeters with the given lengths and widths in linear units.

***Perimeter* formulas →**			
1. $L = 5$ $W = 10$			
2. $L = 5.6$ $W = 10.7$			
3. $L = 5\frac{1}{3}$ $W = 10\frac{3}{4}$			

Complete the table.

	Words	**Symbols**	**Evaluate if $x = 2$, $y = 5$**
4.	The difference when x is taken from y		
5.	The quotient when y is divided by x		
6.		$x + y$	
7.		xy	
8.	4 times x, plus y		
9.	4 times the sum of x and y		
10.		$4x - y$	
11.		$4(y - x)$	

PRACTICE 9: EXTEND YOUR THINKING

A square with side length x has area $A = x^2$, measured in square units.
A cube with edge length x has volume $V = x^3$, measured in cubic units.

1. Find the area and volume measures with appropriate units, given the values for x.

	x	A	V
a.	1 ft		
b.	4 cm		
c.	0.3 m		
d.	$\frac{3}{4}$ in		

2. Sondra thinks that the measures above for A and V are the same when x = 1 ft. What is correct about her statement and what is incorrect about her statement?

3. Find two numbers for which $x^2 < x^3$, two numbers for which $x^2 = x^3$, and two numbers for which $x^2 > x^3$. Justify your answers by showing work.

$x^2 < x^3$		
$x^2 = x^3$		
$x^2 > x^3$		

4. Translate each verbal instruction into an algebraic expression. Then evaluate for $y = 3$.

a. Square a number *y*, then multiply it by 4.	b. Multiply a number *y* by 4, then square the product.

5. Explain why $10n^2$ and $(10n)^2$ are not equivalent.

REVIEW

MATCH 'EM UP

1. Your teacher will give you some cards to cut up.

2. Match verbal descriptions to their equivalent numerical expressions and numerical values.

3. Each matched trio of cards should spell the name of an animal. List the animals here.

4. Make up another set of equivalence cards. Label them with the letters of a different animal.

BIG SQUARE PUZZLE: EXPRESSIONS

1. Your teacher will give you a puzzle to assemble.

2. List any four equivalent expressions from the puzzle of each kind below:

Numerical (all four equivalent)	Algebraic (all four equivalent)

POSTER PROBLEM: EXPRESSIONS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)
■ One-half of the sum of *x* and *y* ■ The sum of *x* and *y*, divided by 2	■ 2 times *x* plus *y* ■ 2 times the sum of *x* and *y*
Poster 3 (or 7)	**Poster 4 (or 8)**
■ The product of *x* and *y*, minus 1 ■ 1 subtracted from *x* times *y*	■ *x* times the quotient of *y* divided by 4 ■ the product of *x* and *y*, divided by 4

A. Copy the first word statement. Translate it into a variable expression.

B. Copy the second word statement. Translate it into a variable expression.

C. Evaluate both expressions for $x = 4$, $y = 6$.

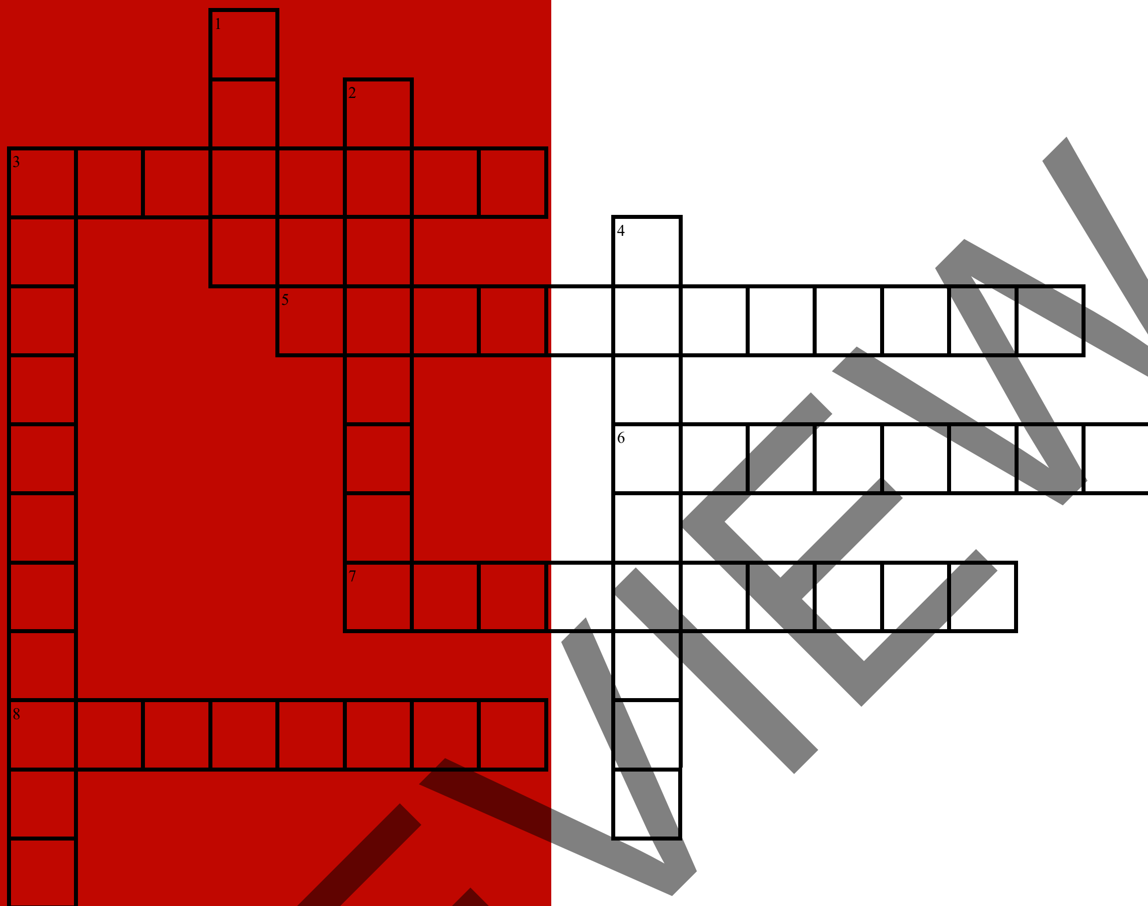
D. Evaluate both expressions for $x = 2$, $y = 8$.

Part 3: Go to your start poster and check the work done by other groups. Copy the two expressions on your paper below. Return to your seats. Work with your group and show all work.

First expression: ________________ Second expression: ________________

Are these two expressions equivalent? Explain.

VOCABULARY REVIEW



Across

3 For the expression $10n + 2$, 2 is the ___ term.

5 The equation $3(x - 4) = 3x - 3(4)$ illustrates the ___ property.

6 In the expression 4^5, 5 is the ___.

7 a combination of numbers, variables, and operation symbols is a(n) ___.

8 a statement asserting the equality of two expressions

Down

1 In the expression 4^5, 4 is the ___.

2 For the expression $10n + 2$, n is a(n) ___.

3 For the expression $10n + 2$, 10 is the ___ of n.

4 For the expression $3x + 2y$, these two terms cannot be combined because they are not ___ ___ (two words).

SPIRAL REVIEW

1. **Computational Fluency Challenge**: This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

   a. Start with 4.5. Multiply by 4. Multiply the result by 0.7. Multiply the result by 8. Multiply the result by 10. Now you have a “big number”. My big number is _____.

   b. Start with your big number. Divide it by 14. Divide the result by 0.2. Divide the result by 1.8. Divide the result by 4. What is the final result? _____

2. Chase is packing his backpack for school and wants to make sure it does not weigh too much. Researchers say that backpacks should weigh no more than 10% of what the student weighs, and Chase weighs 80 pounds.

   a. What is the maximum amount Chase’s backpack should weigh?

   b. If Chase’s laptop computer weighs 60 ounces and his water bottle weighs 56 ounces, write a numerical expression for the weight (in pounds) in his backpack.

   c. Chase says he can carry both the laptop and the water bottle in his backpack. Is Chase in compliance with recommendations? Explain.

SPIRAL REVIEW

Continued

3. Olivia is training for a marathon (26.2 miles). She typically runs 24 days and rests for 6 days. She tries to run at a constant rate. She keeps a journal of how far she ran and how much time it took. Here is her journal.

Time (min)	8	16		40		160
Distance (mi)	1	2	3		13	

   a. Write the ratio of days Olivia runs to the total days.

   b. Some of the entries in Olivia's journal got erased. Complete her times and distances.

   c. Based on Olivia's constant rate, how long will it take her to run the marathon?

   d. Carlos is also training for a marathon as well. In 30 minutes, he runs 4 miles. Who runs faster, Olivia or Carlos? Explain.

4. Complete the chart below.

Fraction	**Decimal**	**Percent**	**Percent of $10**
$\frac{4}{5}$			
	8		
		8%	

5. Joel bought a shirt for $40 and then paid 8% tax on the purchase. How much was the total purchase?

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- ○ Investigate concepts and solve problems involving length, area, and volume.
- Extend the number system to include negatives. ○
- ○ Use statistical measures and displays to describe center and spread.
- Explore relationships between inputs and outputs. ○
- ○ Gain computational fluency with positive rational numbers.
- Rewrite and evaluate expressions and solve equations. ○
- ○ Explore and apply ratio and rate reasoning and representations.

Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to Monitor Your Progress and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Give an example of how you used variables to represent quantities in problems (SMP2).

4. **More Connections.** Describe an important connection between algebra and geometry that you made in these lessons.

STUDENT RESOURCES

Word or Phrase	Definition
coefficient	A coefficient is a number or constant factor in a term of an algebraic expression. In the expression $3x + 5$, 3 is the coefficient of the linear term $3x$, and 5 is the constant coefficient.
constant term	A constant term in an algebraic expression is a term that has a fixed numerical value. In the expression $5 + 2x + 3$, the terms 5 and 3 are constant terms. If this expression is rewritten as $2x + 8$, the term 8 is the constant term of the new expression.
distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. $3(4 + 5) = 3(4) + 3(5)$; $(4 + 5)8 = 4(8) + 5(8)$; $6(8 – 1) = 6(8) – 6(1)$
equation	An equation is a mathematical statement that asserts the equality of two expressions. $18 = 8 + 10$ is an equation that involves only numbers. This is a numerical equation. $18 = x + 10$ is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.
equivalent expressions	Two mathematical expressions are equivalent if, for any possible substitution of values for the variables, the two resulting numbers are equal. In particular, two numerical expressions are equivalent if they represent the same number. See expression. The numerical expressions $3 + 2$ and $6 – 1$ are equivalent, since both are equal to 5. The algebraic expressions $3(x + 2)$ and $3x + 6$ are equivalent. For any value of the variable x, the expressions represent the same number.
evaluate	Evaluate refers to finding a numerical value. To evaluate an expression, replace each variable in the expression with a value and then calculate the value of the expression. To evaluate the numerical expression $3 + 4(5)$, we calculate $3 + 4(5) = 3 + 20 = 23$. To evaluate the variable expression $2x + 5$ when $x = 10$, we calculate $2x + 5 = 2(10) + 5 = 20 + 5 = 25$.
exponential notation	The exponential notation b^n (read as "b to the power n") is used to express n factors of b. The number b is the base, and the number n is the exponent. $2^3 = 2 \bullet 2 \bullet 2 = 8$; The base is 2 and the exponent is 3. $3^2 \bullet 5^3 = 3 \bullet 3 \bullet 5 \bullet 5 \bullet 5 = 1{,}125$; The bases are 3 and 5. The exponents are 2 and 3.

Word or Phrase	Definition
expression	A mathematical <u>expression</u> is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8+x}{10}$, and $4v - w$.
greatest common factor	The <u>greatest common factor</u> (GCF) of two numbers is the greatest factor that divides the two numbers. The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 18 are 1, 2, 3, 6, 9, and 18. Therefore the GCF of 12 and 18 is 6.
like terms	Terms of a mathematical expression that have the same variable part are referred to as <u>like terms</u>. See <u>term</u>. In the mathematical expression $2x + 6 + 3x + 5$, the terms $2x$ and $3x$ are like terms, and the terms 6 and 5 are like terms.
simplify	<u>Simplify</u> refers to converting a numerical or variable expression to a simpler form. A variable expression might be simplified by combining like terms. A fraction might be simplified by dividing numerator and denominator by a common divisor. $2x + 6 + 5x + 3 = 7x + 9$ $\frac{8}{12} = \frac{2}{3}$
square number	A <u>square number</u>, or <u>perfect square</u>, is a number that is a square of a natural number. The area of a square with side-lengths that are natural numbers is a square number. The square numbers are $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$, … .
terms	The <u>terms</u> in a mathematical expression involving addition (or subtraction) are the quantities being added (or subtracted). Terms that have the same variable part are referred to as <u>like terms</u>. The expression $2x + 6 + 3x + 5$ has four terms: $2x$, 6, $3x$, and 5. The terms $2x$ and $3x$ are <u>like terms</u>, since each is a constant multiple of x. The terms 6 and 5 are <u>like terms</u>, since each is a constant.
variable	A <u>variable</u> is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to quantities that vary in a relationship (as in a formula or an input-output rule). They may refer to unknown quantities in expressions, equations or inequalities. Finally, they may be used to generalize rules of arithmetic. In the equation $d = rt$, the quantities d, r, and t are variables. In the equation $2x = 10$, the variable x may be referred to as the unknown. The equation $a + b = b + a$ generalizes the commutative property of addition for all numbers a and b.

The Distributive Property

The distributive property relates the operations of multiplication and addition. The term “distributive” arises because the property is used to distribute the factor outside the parentheses over the terms inside the parentheses.

Suppose you earn $9.00 per hour. If you work 3 hours on Saturday and 4 hours on Sunday, one way to compute your earnings is to compute your wages for each day and then add them. Another way is to multiply the hourly wage by the total number of hours. This example illustrates the distribute property.

$$(9 \times 3) + (9 \times 4) = 9(3 + 4)$$
$$27 + 36 = 9(7)$$

Order of Operations

There are many mathematical conventions that enable us to interpret mathematical notation and to communicate efficiently. The agreed-upon rules for interpreting mathematical notation, important for simplifying arithmetic and algebraic expressions, are called the standard order of operations.

Step 1: Do the operations in grouping symbols first (e.g., use rules 2-4 inside parentheses).
Step 2: Calculate all the expressions with exponents.
Step 3: Multiply and divide in order from left to right.
Step 4: Add and subtract in order from left to right.

Example: $$\frac{3^2 + (6 \bullet 2 - 1)}{5} = \frac{3^2 + (12 - 1)}{5} = \frac{3^2 + (11)}{5} = \frac{9 + (11)}{5} = \frac{20}{5} = 4$$

There are many times when these rules make complete sense and are quite natural. Take this case, for example:

You purchase 2 bottles of water for $1.50 each and 3 bags of peanuts for $1.25 each. Write an expression for this situation, and simplify the expression to find the total cost.

Expression: $$\underbrace{2 \bullet (1.50)}_{3.00} + \underbrace{3 \bullet (1.25)}_{3.75} = 6.75$$

In this problem, it is natural to find the cost of the 2 bottles of water and then the cost of the 3 bags of peanuts prior to adding these amounts together. In other words, we perform the multiplication operations before the addition operation.

However, if we were to perform the operations in order from left to right (as we read the English language from left to right), we would obtain a different result:

$$2(1.50) = 3 \;\longrightarrow\; 3 + 3 = 6 \;\longrightarrow\; 6(1.25) = 7.50, \text{ wrong answer!}$$

Using Order of Operations to Simplify Expressions		
Order of Operations	**Expression**	**Comments**
	$\frac{2^3 \div 2(5-2)}{4+2\bullet 10}$	
1. Simplify expressions within grouping symbols.	$\frac{2^3 \div 2(3)}{4+2\bullet 10}$	Parentheses are grouping symbols: Therefore, 5 – 2 = 3. The fraction bar is also a grouping symbol, so the first step here is to simplify the numerator and denominator.
2. Calculate powers.	$\frac{8 \div 2(3)}{4+2\bullet 10}$	$2^3 = 2 \bullet 2 \bullet 2 = 8$
3. Perform multiplication and division from left to right.	$\frac{12}{4+20}$	In the numerator: Divide 8 by 2, then multiply by 3. In the denominator: Multiply 2 by 10.
4. Perform addition and subtraction from left to right.	$\frac{12}{24} = \frac{1}{2}$	Perform the addition: 4 + 20 = 24. Now the groupings in both the numerator and denominator have been simplified, so the final division can be performed.

Writing Expressions
The notation used for algebra is sometimes different from the notation used for arithmetic. For example:
• 54 means the sum of five tens and four ones, that is, 5(10) + 4.
• $5\frac{1}{2}$ means the sum of five and one-half. that is, $5 + \frac{1}{2}$.
• 5*x* means the product of 5 and *x*, which can also be written 5(*x*) or $5 \bullet x$. We typically do not write $5 \times x$ because the multiplication symbol '×' is easily confused with the variable *x*.

Evaluate or Simplify?
We use the word "evaluate" when we want to calculate the value of an expression. Example: To evaluate 16 – 4(2), follow the rules for order of operations and compute: 16 – 4(2) = 16 – 8 = 8. To evaluate 6 + 3x when x = 2, substitute 2 for x and calculate: 6 + 3(2) = 6 + 6 = 12. We use the word "simplify" when rewriting a number or an expression in a form more easily readable or understandable. Example: To simplify 2x + 3 + 5x, combine like terms: 2x + 3 + 5x = 7x + 3. Sometimes it may not be clear what the simplest form of an expression is. For instance, by the distributive property, 4(x + 2) = 4x + 8. For some applications, 4(x + 2) may be considered simpler than 4x + 8, but for other applications, 4x + 8 may be considered simpler than 4(x + 2).

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
6.NS.4	Find the greatest common factor of two whole numbers less than or equal to 100 ~~and the least common multiple of two whole numbers less than or equal to 12.~~ Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
6.EE.A	**Apply and extend previous understandings of arithmetic to algebraic expressions.**
6.EE.1	Write and evaluate numerical expressions involving whole-number exponents.
6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers:
a.	Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
b.	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
c.	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length s = 1/2.*
6.EE.3	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
6.EE.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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