Name _____ Period _____ Date _____

UNIT 4
STUDENT PACKET

Math Links
GRADE 6

DIVISION

Parent (or Guardian) signature _____

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

dividend, divisor, quotient	multiplication property of 1
reciprocal	unit rate

CHOCOLATE BARS

Follow your teacher's directions.
(1)

Name			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

WHOLE NUMBER DIVISION

We will solve division problems using "chunking." We will link chunking division to the standard algorithm. We will interpret solutions to division problems in context.

[6.NS.2; SMP2, 3, 6, 8]

GETTING STARTED

1. Fill in the products.

15 × 1	15 × 2	15 × 3	15 × 4

2. Show or explain two different ways to find 15 × 5 from the facts above.

Method 1:	Method 2:

3. Fill in the products.

15 × 10	15 × 20	15 × 30	15 × 40

4. Show or explain two different ways to find 15 × 50 from the facts above.

Method 1:	Method 2:

5. Write 24 divided by 6 using a "division house" $\overline{)\ \ }$, division symbol (÷), and fraction bar.

DIVISION STRATEGIES

Follow your teacher's directions for (1) – (8).

(1)

(2)

(3)
(4)

Multiplication Bank

(5)

(6)
(7)

Multiplication Bank

(8)

9. Record the meanings of divisor, dividend, and quotient, in **My Word Bank**.

PRACTICE 1

Use chunking to divide. Use a Multiplication Bank as needed.

1. Compute $\frac{855}{9}$ → _____

$9\overline{)855}$

Multiplication Bank

2. Use the quotient you found above to write the quotient for $\frac{860}{9}$. _____

3. Compute 792 ÷ 22 → _____

$22\overline{)792}$

Multiplication Bank

4. Use the quotient you found above to write the quotient for 799 ÷ 22. _____

PRACTICE 2

Use chunking to divide. Use a Multiplication Bank as needed.

1. Compute $\frac{4575}{15} \rightarrow$ _____

15) 4575

Multiplication Bank

2. Use the quotient you found above to write the quotient for $\frac{4580}{15}$. _____

3. How many miles per gallon did Mr. Garcia's car get if he drove 952 miles and used 34 gallons of gas?

   Solution:

Multiplication Bank

4. If Mr. Garcia could drive 966 miles using 34 gallons of gas, would he be getting more than 28 miles per gallon or less? Explain without showing more computations.

INTERPRETING QUOTIENTS

Use chunking division and interpret the meaning of quotients in context.

1. Compute 123 ÷ 5. Then use the result for problems 2 – 3.

Multiplication Bank

2. There are 123 packages of pencils. If 5 pencils fit in a box, how many full boxes can be filled? Interpret the quotient.

3. There are 123 soccer players that are going to a tournament. If 5 players can fit in each car, how many cars are needed to take all the players? Interpret the quotient.

4. There are 210 sixth graders at Math Academy. If the maximum number of students per class is 32, how many classes will the school need to hold all of the 6th graders? Compute and interpret the quotient.

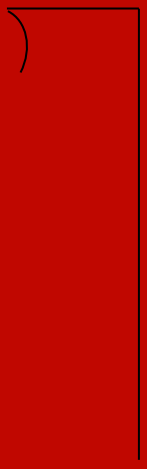
Multiplication Bank

Solution:

PRACTICE 3

Show each computation. Explain the meaning of the remainder. Answer the question using the context of the problem.

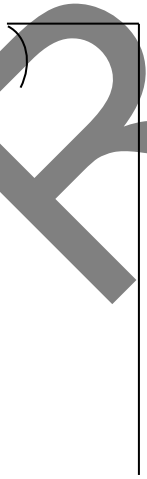
1. A bus holds 63 students. If 2,842 students are going on a field trip, how many buses are needed?



Multiplication Bank

Solution:

2. A softball team earns $1,250 to purchase uniforms. If the price of a uniform is $38, how many uniforms can the team purchase?



Multiplication Bank

Solution:

DIVISION PROCEDURES

Follow your teacher's directions for (1) – (2).

(1)

Chunking

Multiplication Bank:

18 × 1 = 18	18 × 10 = 180
18 × 2 = 36	18 × 20 = 360
18 × 3 = 54	18 × 30 = 540
18 × 4 = 72	18 × 40 = 720
18 × 5 = 90	

Standard Algorithm

(2)

Chunking

Multiplication Bank:

14 × 1 = 14	14 × 10 = 140
14 × 2 = 28	14 × 20 = 280
14 × 3 = 42	14 × 30 = 420
14 × 4 = 56	14 × 40 = 560
14 × 5 = 70	14 × 50 = 700
14 × 6 = 84	14 × 60 = 840
14 × 7 = 98	14 × 70 = 980
14 × 8 = 112	
14 × 9 = 126	

Standard Algorithm

Compute using the standard algorithm.

3. 678 ÷ 27	4. $\frac{8{,}640}{32}$	5. 1,496 ÷ 19

PRACTICE 4

Compute using the standard algorithm.

1. 791 ÷ 75	2. $\frac{1{,}332}{18}$	3. 9,856 ÷ 64
4. There are 256 students going on a field trip. Each bus can hold 70 students. a. How many buses are needed? b. If buses are filled one by one, how many students will be in the bus that is not full?	5. The Community Service Club is making blankets for a charity. Each blanket requires 7 feet of fabric. They have 450 feet of fabric. a. How many blankets can they make? b. How many feet of fabric will be left over?	6. A school enrolls 1,040 students, and they are assigned to 37 homerooms. How would you assign students to homerooms so that each one has about the same number of students?

DECIMAL DIVISION AND RATE PROBLEMS

We will build fluency for division of decimals. We will deepen our understanding of the structure of rate problems and solve them.

[6.RP.2, 6.RP.3bd, 6.NS.2, 6.NS.3; SMP2, 5, 6, 8]

GETTING STARTED

Use number sense, a diagram, or a table to find the following.

1. Four friends share $3 so that each one gets the same amount. How much will each friend get?

2. Four friends share $3.12 so that each one gets the same amount. How much will each friend get?

3. Why are the following numbers equivalent? 3 3.0 3.00

4. Write this division statement $\frac{3}{4}$ = 0.75 in three different ways.

_____ divided by _____ is _____)‾‾‾ _____ ÷ _____ = _____

5. In the division statement $4\overline{)3}$ $\overset{0.75}{}$, what is …

the dividend? ____ the divisor? _____ the quotient? _____

QUOTIENTS THAT INVOLVE DECIMALS

Follow your teacher's directions.

(1)	(2) – (3)
(4)	(5) – (6)
(7)	(9)
(10)	

PRACTICE 5

1. Write the division statement "7 divided by 20 equals 0.35" in three different ways.

2. Five friends go to lunch and share the cost equally. If the lunch bill is $31.30, how much will each friend pay?

3. Write $\frac{3}{20}$ as a decimal.

$$\frac{3}{20}\left(\frac{5}{5}\right) = \frac{\square}{100} = \_\_\_\_\_$$

Verify the result with division.

$$20\overline{)3}$$

4. Write $\frac{5}{8}$ as a decimal.

$$\frac{5}{8}\left(\frac{\square}{\square}\right) = \frac{\square}{1000} = \_\_\_\_\_$$

Verify the result with division.

5. Write $\frac{7}{8}$ as a decimal.

6. Write $\frac{9}{20}$ as a decimal.

7. Circle the numbers that are equivalent to 14.3.

014.3 104.3 140.3 14.30 14.300 14.3000

Choose one of the circled numbers above and explain how you know it is equivalent to 14.3.

DIVISION BY A DECIMAL

Follow your teacher's directions for (1) – (9).

(1)	(2)	(3)
(4)	(5)	(6)
(7)	(8)	(9)

10. Explain the standard algorithm for decimal division in your own words.

11. Record the meaning of multiplication property of 1 in **My Word Bank**.

PRACTICE 6

1. Write this division statement $\frac{1.8}{0.06} = 30$ in three different ways.

2. Circle the numbers that are equivalent to 1.05.

   01.5 1.005 001.0500 1.050 01.05 1.5

   Choose one of the numbers circled above and explain how you know it is equivalent to 1.05.

3. Use the multiplication property of 1 (the big 1) to explain why $0.08\overline{)3.2}$ is not equivalent to $8\overline{)3200}$.

4. Use division to find how many nickels are in $8.75.

5. Compute. $6.48 \div 1.8$

6. Compute $\frac{301.52}{0.08}$.

7. Dee Harmon thinks that division makes things smaller. Is she always correct, sometimes correct, or never correct? Explain.

WHY DOESN'T IT BELONG?: DIVISION

Solve the problems. Write unit rates. Choose a rate equation that best matches each problem. Explain why each of these problems is mathematically different from the others.

rate × time = distance	$\frac{\text{distance}}{\text{rate}}$ = time	$\frac{\text{distance}}{\text{time}}$ = rate
unit rate × quantity = total	$\frac{\text{total}}{\text{unit rate}}$ = quantity	$\frac{\text{total}}{\text{quantity}}$ = unit rate

1. Caleb flew from Los Angeles to Seattle. The cities are about 1,000 miles apart and the plane flew at an average of 500 miles per hour. How long was the trip?

2. Jeremy rode his bike for $1\frac{1}{2}$ hours at an average rate of 15 miles/hour. How far did he go?

3. Ariana drove 100 miles from San Francisco to Sacramento in 2 hours. Then she drove 120 miles from Sacramento to Reno in 3 hours. What was her average speed for the whole trip?

4. At the gas pump, Kyle paid $35.40 for 12 gallons of gas. What was the cost per gallon?

5. Record the meaning of unit rate in **My Word Bank**.

PRACTICE 7

Solve each problem below. Possible strategies include tables, double number lines, and unit rates. For problems 1 and 2, all the burgers are the same size and quality.

1. Show which is the better buy:
   - 3 burgers for $7.50
   - 3 burgers for $9, or
   - or 3 burgers for $6.

2. Show which is the better buy:
   - 6 burgers for $25.50
   - 4 burgers for $18, or
   - 5 burgers for $21

3. 16 gallons of gas cost $61.60. A quart container of motor oil costs $3.55.
   a. What is the price per gallon of gas?
   b. What is the cost for 22 gallons?
   c. What is the cost for 16 gallons of gas and 2 quarts of motor oil?
   d. If you have $100 and get 16 gallons of gas, what is the greatest number of quarts of oil you can buy?

4. On Saturday Angela babysat for 5 hours and earned $62.50.
   a. How much did she get per hour?
   b. At this rate, how much would she earn in 9 hours?
   c. On Sunday she babysat again, getting the same pay rate, and earned $43.75.How many hours did she work?
   d. How much more did she earn Saturday compared to Sunday?

FRACTION DIVISION: DIVIDE ACROSS

We will use pictures to make sense of fraction division. We will use a fraction procedure to solve fraction division problems.

[6.NS.1; SMP2, 3, 7, 8]

GETTING STARTED

Compute each expression below.

1. $\frac{2}{5} + \frac{3}{4}$	2. $\frac{8}{9} - \frac{5}{6}$	3. $\frac{5}{8} \bullet \frac{4}{5}$

4. Use your knowledge of fraction multiplication to fill in the blank. $\square \bullet \frac{3}{11} = \frac{6}{55}$

Simplify. Show your work.

5. $\frac{7}{35}$	6. $\frac{18}{30}$

7. Circle all of the expressions below that could represent 6 ÷ 3.

a. $\frac{6}{3}$

b. How many 3's go into 6?

c. $\frac{3}{6}$

d. How many 3's does it take to make 6?

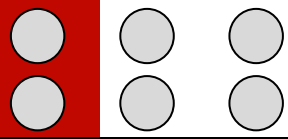
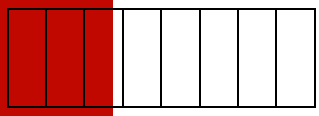
e. How many 6's are in 3?

f. How many groups of 3 are there in 6?

8. Write a short story that can be represented by 6 ÷ 3 = 2.

EXPLORING DIVIDE ACROSS

Follow your teacher's directions to explore fraction division problems (1) – (4).

Words	Diagram	Division Expression	Quotient
(1) How many groups of $\square$ are in $\square$?			
(2) How many groups of $\frac{\square}{\square}$ are in $\frac{\square}{\square}$?			
(3) How many groups of $\square$ are in $\square$?			
(4) How many groups of $\frac{\square}{\square}$ are in $\frac{\square}{\square}$?			

Use your knowledge of the relationship between multiplication and division to fill in the blanks.
For each problem, the same number must go into $\square$ or $\bigcirc$.

	Multiplication Problem	Related Division Problem	Divide numerators and denominators	Equal Quotients?
5.	$\square \cdot 4 = 8$	$8 \div 4 = \square$	$\frac{8 \div 4}{1 \div 1} = \frac{\square}{1}$	
6.	$\frac{\square}{\bigcirc} \cdot \frac{4}{10} = \frac{8}{10}$	$\frac{8}{10} \div \frac{4}{10} = \frac{\square}{\bigcirc}$	$\frac{8 \div 4}{10 \div 10} = \frac{\square}{\bigcirc}$	
7.	$\frac{\square}{\bigcirc} \cdot \frac{5}{5} = \frac{5}{10}$	$\frac{5}{10} \div \frac{5}{5} = \frac{\square}{\bigcirc}$	$\frac{5 \div 5}{10 \div 5} = \frac{\square}{\bigcirc}$	
8.	$\frac{\square}{\bigcirc} \cdot \frac{4}{3} = \frac{8}{15}$	$\frac{8}{15} \div \frac{4}{3} = \frac{\square}{\bigcirc}$	$\frac{8 \div 4}{15 \div 3} = \frac{\square}{\bigcirc}$	

It appears that dividing across works. We will call this conjecture the "divide across rule."

THE DIVIDE ACROSS RULE

Follow your teacher's directions.

(1) – (3) Connor eats _____ of a small cake. A serving is _____ of the cake. How many servings does Connor eat? How many servings of $\frac{\square}{\square}$ are in $\frac{\square}{\square}$? Division Problem: Diagram: Computation: Answer question:	(4) – (6) Mia eats _____ cup of cereal. A serving size is _____ cup. How many servings does Mia eat? How many servings of $\frac{\square}{\square}$ are in $\frac{\square}{\square}$? Division Problem: Diagram: Computation: Answer question:

(7)

Words:

Symbols:

(8)	(9)

PRACTICE 8

1. Ryan has $1\frac{1}{2}$ sandwiches leftover from yesterday's party. A serving size is $\frac{3}{4}$ of a sandwich. How many servings does he have?

   Represent this situation with a picture and a division expression.
   Then perform the divide across procedure.
   Clearly show your work, and the result.

Compute.

2. $\frac{1}{3} \div \frac{5}{9}$	3. $\frac{1}{2} \div \frac{3}{5}$	4. $2\frac{1}{8} \div \frac{3}{4}$
5. $1\frac{3}{4} \div \frac{1}{2}$	6. $1\frac{1}{8} \div 4\frac{1}{2}$	7. $\frac{1}{2} \div 4$

8. Taylor tried to calculate $2\frac{2}{3} \div \frac{4}{5}$ as illustrated below and got stuck.

$$2\frac{2}{3} \div \frac{4}{5} = \frac{8}{3} \div \frac{4}{5} = \frac{2}{\frac{3}{5}}$$

   Even though she did nothing wrong, show a different approach that might be more successful for her.

PRACTICE 9

1. A 2-foot-long sandwich is cut into portions that are $\frac{3}{4}$ feet long each.

   a. Write a division expression that represents this situation.

      Words:

      Numbers:

   b. Use a diagram to show the full portions that can be cut and any leftover part.

   c. Solve using the divide across rule.

   d. How many full portions can be cut?

   e. How long is the piece that is leftover?

   f. What fraction of a portion is leftover?

   g. Check your solution by multiplication.

2. A 4-foot-long board is cut into shelves that are $1\frac{1}{4}$ feet long each.

   a. Write a division expression that represents this situation.

      Words:

      Numbers:

   b. Use a diagram to show the full shelves that can be cut and any leftover part.

   c. Solve using the divide across rule.

   d. How many full shelves can be cut?

   e. How long is the piece that is leftover?

   f. What fraction of a shelf is leftover?

   g. Check your solution by multiplication.

FRACTION DIVISION: MULTIPLY BY THE RECIPROCAL

We will use the inverse relationship between multiplication and division and the divide across rule to make sense of a common fraction division rule and solve problems.

[6.NS.1; SMP1, 2, 4, 7]

GETTING STARTED

1. Record the meaning of reciprocal in **My Word Bank**.

2. Write the reciprocals of each of the following numbers.

a. 3	b. $\frac{1}{6}$	c. $\frac{4}{5}$

3. The following pairs of numbers are reciprocals of one another. Multiply each pair of reciprocals.

a. 5 and $\frac{1}{5}$	b. $\frac{5}{7}$ and $\frac{7}{5}$

c. What is the result when a number is multiplied by its reciprocal? 1

4. Describe an easy way to find the reciprocal of a fraction.

5. What is the reciprocal of $\frac{a}{b}$?

6. Why is $\frac{2}{3}$ the reciprocal of $1\frac{1}{2}$?

7. What is the reciprocal of $2\frac{3}{5}$?

EXPLORING MULTIPLY BY THE RECIPROCAL

1. Compute.

a. $12 \div 4$	b. $\frac{1}{4}$ of 12	c. $12 \bullet \frac{1}{4}$

2. Does dividing by 4 and multiplying by $\frac{1}{4}$ produce the same result?

3. Compute.

	Column I Use the divide across rule dividend ÷ divisor = quotient	**Column II** Use the multiply across rule first factor × second factor = product	**Equal Results?**
a.	$\frac{10}{21} \div \frac{2}{7}$	$\frac{10}{21} \bullet \frac{7}{2}$	
b.	$\frac{7}{8} \div \frac{1}{4}$	$\frac{7}{8} \bullet \frac{4}{1}$	
c.	$\frac{2}{3} \div \frac{1}{6}$	$\frac{2}{3} \bullet \frac{6}{1}$	
d.	$\frac{1}{6} \div \frac{2}{3}$	$\frac{1}{6} \bullet \frac{3}{2}$	

4. For each pair in problem 3 above, compare Column I and Column II.
   a. How do the dividends compare to the first factors?
   b. How do the divisors compare to the second factors?
   c. How do the quotients compare to the products?
   d. Based on these examples, it appears that dividing by a number gives the same result as multiplying by the _____ of that _____.

MULTIPLY BY THE RECIPROCAL RULE

On the previous page you observed that dividing by a number gives the same result as multiplying by the reciprocal of that divisor. We will call this conjecture the "multiply by the reciprocal rule."

Compute. Use the divide across rule for Column I and test the multiply by the reciprocal rule for Column II.

	Column I **Divide across**	**Column II** **Multiply by the reciprocal of the divisor**	**Equal Results?**
1.	$\frac{3}{4} \div \frac{5}{8}$		
2.	$\frac{2}{3} \div \frac{1}{2}$		
3.	$5 \div \frac{1}{6}$		
4.	$3\frac{1}{2} \div 4$		

5. What is the multiply by the reciprocal rule for fractions?

   Words:

   Symbols:

6. Explain in words how to apply this rule to compute $3 \div 1\frac{1}{2}$.

PRACTICE 10

1. Write the reciprocal of each number.

a. 8	b. $\frac{1}{3}$	c. $\frac{5}{9}$	d. $2\frac{3}{4}$

Compute. Use the divide across rule for Column I and the multiply by the reciprocal rule for Column II.

	Column I Divide across	**Column II Multiply by the reciprocal of the divisor**	**Equal Results?**
2.	$\frac{5}{6} \div \frac{1}{8}$		
3.	$3 \div \frac{2}{3}$		
4.	$1\frac{1}{4} \div 2$		

Compute using any method.

5. $\frac{9}{10} \div \frac{3}{5}$	6. $6 \div \frac{3}{4}$	7. $2\frac{1}{4} \div 1\frac{1}{6}$

8. Hector runs 3 miles around the perimeter of a park. One lap around is $\frac{2}{3}$ miles. How many full laps does he run? What fraction of a lap does he run at the end? Show with a diagram and with computations.

9. Create a story problem for $6 \div \frac{3}{4}$ and solve it.

PRACTICE 11: EXTEND YOUR THINKING

Use your computational skills, along with strategies and representations you have learned to solve these problems.

1. Robert spent $\frac{2}{3}$ of his money on new ear buds and half of what remained on a new wallet. If the wallet cost $15.50, how much did he have at the start?

2. Students were surveyed about their favorite fruit. $\frac{1}{4}$ preferred apples, $\frac{1}{8}$ preferred oranges, and $\frac{4}{5}$ of the remaining students preferred grapes. If 16 students preferred grapes, how many students were surveyed?

3. 15 gallons of water fill a tank to $\frac{3}{5}$ capacity. How many 8-oz cups of water can be filled with a full tank?

REVIEW

COMPUTATIONAL FLUENCY CHALLENGES

This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

1. Begin with any single digit whole number. Multiply your number by 2. Multiply the result by 3. Multiply that result by 4. Multiply that result by 5. Multiply that result by 6. Multiply that result by 7. Multiply that result by 8. Multiply that result by 9.	2. Start with your **big number** from the first problem. Divide it by 2. Divide that result by 3. Divide that result by 4. Divide that result by 5. Divide that result by 6. Divide that result by 7. Divide that result by 8. Divide that result by 9.
I began with the number _____. After multiplying, my **big number** is _____.	After dividing, my result is _____.

COMPUTATIONAL FLUENCY CHALLENGES
Continued

3. Start with your big number from the first problem on the previous page. Divide it by 18. Divide that result by 24. Divide that result by 28. Divide that result by 30.

After dividing, my result is _____.

4. Look back at the first problem on the previous page. Write your **big number** as a product of one-digit numbers.

5. Did you get the same result for problem 2 and problem 3? _____

(If not, go back and check your work!)

6. Explain why the results should be the same.

POSTER PROBLEMS: DIVISION

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
Murphy has $2\frac{1}{3}$ feet of fabric. She wants to make pillows that each require $\frac{1}{2}$ feet of fabric.	Olivia has 10.5 feet of fabric. She wants to make pillows that each require 0.75 feet of fabric.	Manuel has 5.25 feet of fabric. He wants to make pillows that each require 1.25 feet of fabric.	Michael has $4\frac{1}{3}$ feet of fabric. He wants to make pillows that each require $1\frac{2}{3}$ feet of fabric.

A. Copy the main facts of the problem, and draw a picture to represent the actions required to cut the fabric.

B. Use the picture to answer: How many full pillows can be made? How long is the leftover fabric? What fraction of a pillow does the leftover fabric represent?

C. Compute using one method.

D. Compute using a different method than was used in Part C.

Part 3: Return to your seats. Work with your group and show all work.

Check your start problem by multiplication.

RATE PROBLEMS

Your teacher will give you some Rate Cards. Cut them up.

1. Sort the cards. Discuss how you sorted them with the class.

2. List 3 cards that represent rates.	3. List 3 cards that represent quantities.	4. List 3 cards that represent totals.

5. Organize the cards to create multiplication or division rate equations. Discuss your equations with the class.

6. Select three cards to make a rate multiplication equation and record them here. Write a problem to fit the equation and solve it. Include at least one decimal in your problem or answer.

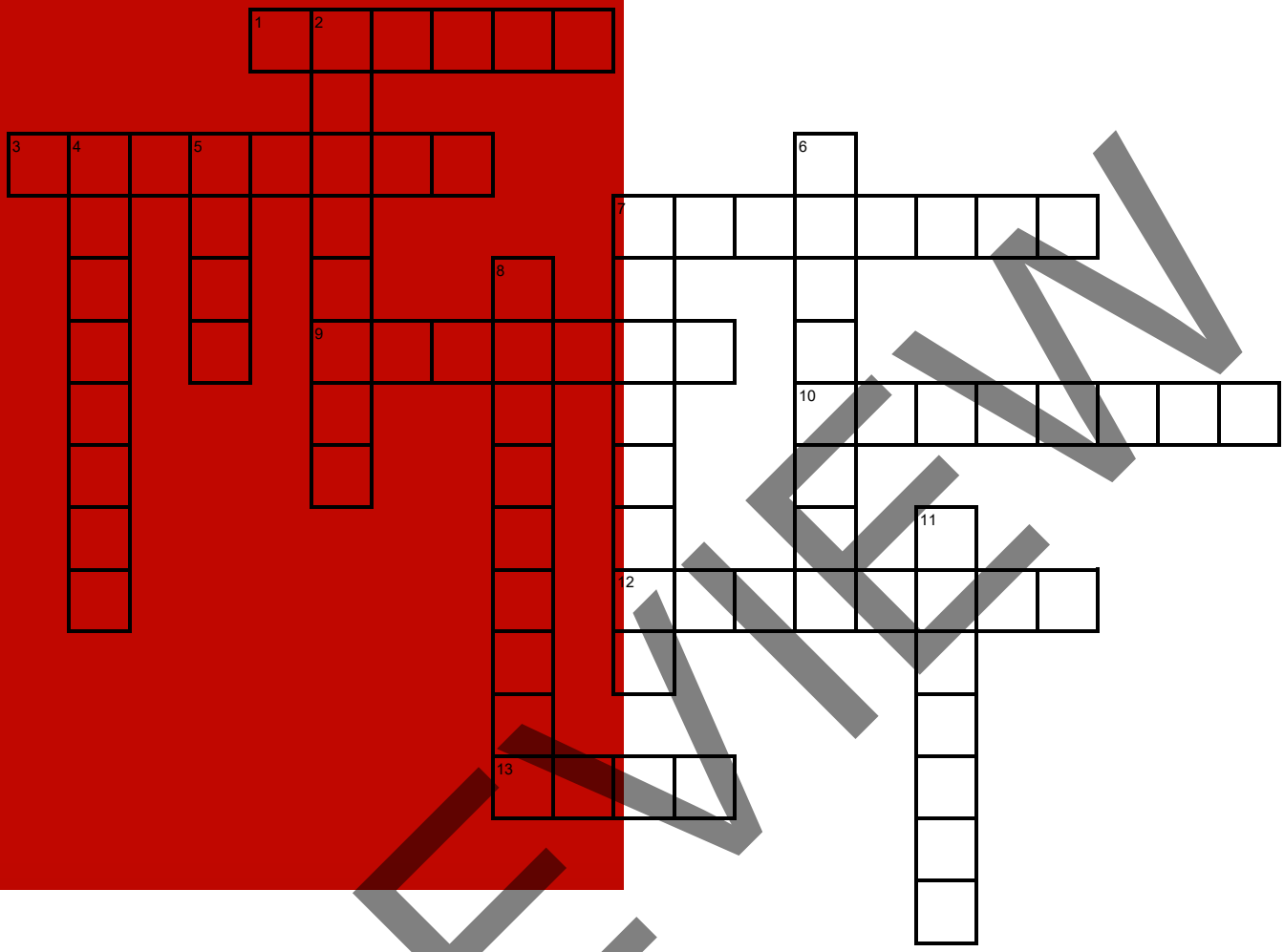
$$\square \times \square = \square$$

7. Select three different cards to make a rate division equation and record them here. Write a problem to fit the equation and solve it. Include at least one decimal in your problem or answer.

$$\frac{\square}{\square} = \square$$

8. Improve your problems and answers with feedback. Write your name and favorite problem on the front of a 3 x 5 card. Write the solution on the back of the 3 x 5 card. Exchange cards with classmates. Solve problems written by others.

VOCABULARY REVIEW



Across

1 nickname for the multiplicative identity

3 In the division problem 525 ÷ 15, 35 is the ___.

7 In the division problem 965 ÷ 12, 965 is the ___.

9 Multiplication is the ____ operation to division.

10 traditional division algorithm

12 division method that removes groups of the divisor

13 If Rocco drives 45 miles per hour for 7 hours, then 45 is the ___ in mph.

Down

2 1 is the multiplicative ___

4 a rate for one unit of measure (2 words)

5 If Rocco drives 45 miles per hour for 7 hours, then 7 is the ___ in hours.

6 the inverse of multiplication

7 If Rocco drives 45 miles per hour for 7 hours, then 315 is the ___ in miles.

8 In the division problem 965 ÷ 12, 5 is the ___.

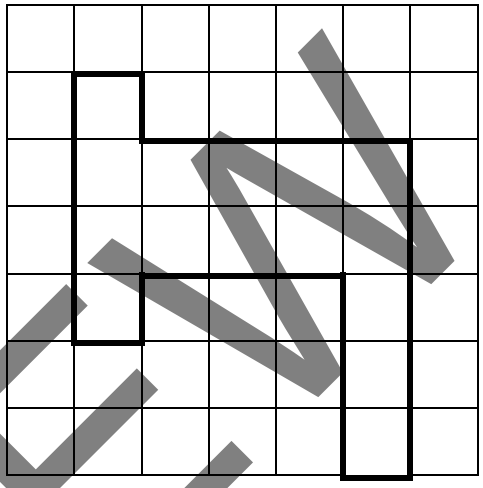
11 In the division problem 965 ÷ 12, 12 is the ___.

SPIRAL REVIEW

1. Blair is building a play area in her backyard for her kids. The design for her play area is below. Each small square is 1 yard by 1 yard.

   a. She is putting a fence around the entire play yard. How much fence does she need, in feet?

   b. Blair will fill the play area with grass. How many square feet of grass does she need?



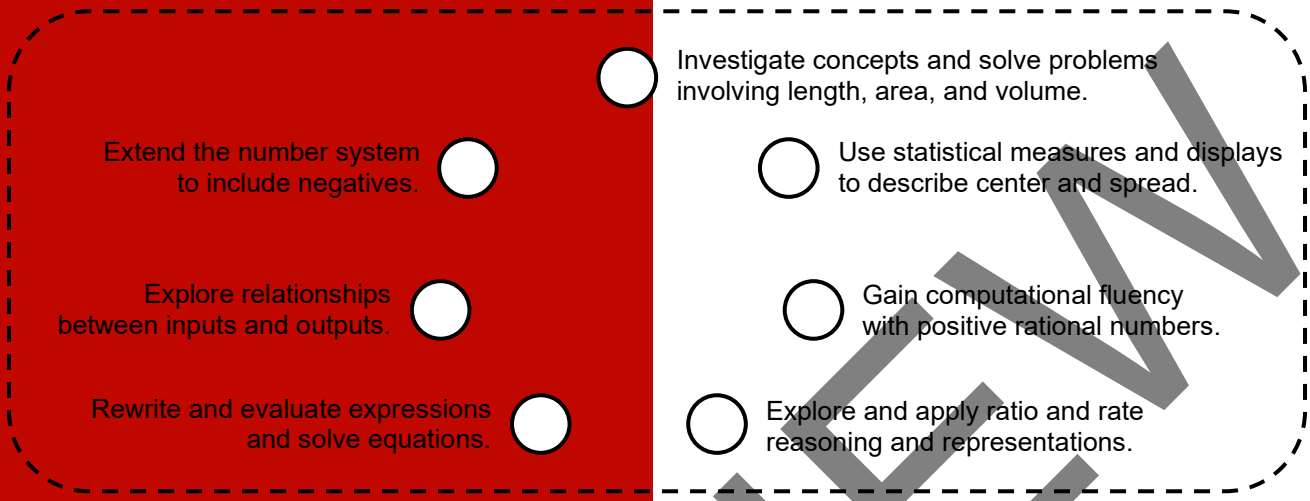
2. Liza, Sienna and Everett were deciding how to split up a candy bar between the three of them. Liza says, “Let’s split the chocolate into 6 equal pieces so we each get 2 pieces.” Sienna says, “That not right. Let’s split it into 12 equal pieces so we each get 3 pieces. If Everett gets the final decision, what should he do to split the candy bar fairly?

3. Josue had $1,218.19 in his checking account. After an automatic payment of $9.99 for his movie streaming service and a debit card payment for 3 pairs of socks at $5.45 each, how much money remained in his account?

4. Hot dogs come in packs of 10. Hot dog buns come in packs of 8. What is the least number of each that Graham should buy so that each hot dog has a bun with no hot dogs and no buns left over?

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.



   Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** In what ways are multiplication and division related [SMP7]? Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **More Connections.** Do you think it is important to develop fluency in division?

STUDENT RESOURCES

Word or Phrase	Definition
conjecture	A conjecture is a statement that is proposed to be true, but has neither been proven to be true nor to be false.
dividend	In a division problem, the dividend is the number being divided. In $12 \div 3 = 4$, the dividend is 12. dividend ÷ divisor = quotient
divisor	In a division problem, the divisor is the number by which another is divided. In $12 \div 3 = 4$, the divisor is 3. $\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$
multiplication property of 1	The multiplication property of 1 states that $a \bullet 1 = 1 \bullet a = a$ for all numbers a. In other words, 1 is a multiplicative identity. The multiplicative property of 1 is sometimes called the multiplicative identity property. $4 \bullet 1 = 4$, $1 \bullet (\frac{3}{8}) = \frac{3}{8}$, $\frac{3}{4} \bullet \frac{5}{5} = \frac{15}{20} = \frac{3}{4}$
quotient	In a division problem, the quotient is the result of the division. In $12 \div 3 = 4$, the quotient is 4. $\text{divisor}\overline{)\text{dividend}}^{\text{quotient}}$
reciprocal	For $b \neq 0$, the reciprocal of b is the number, denoted by $\frac{1}{b}$, that satisfies $b \bullet \frac{1}{b} = 1$. The reciprocal of b is also called the multiplicative inverse of b. The reciprocal of 3 is $\frac{1}{3}$. The reciprocal of $\frac{1}{6}$ is 6. The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.
unit rate	The unit rate associated to a ratio $a : b$, where a and b have units attached, is the number $\frac{a}{b}$, with the units "a-units per b-unit" attached. The ratio of 400 miles for every 8 hours corresponds to the unit rate 50 miles per hour.

Notation for Division

The quotient of 8 and 4 can be written as:

8 divided by 4 $\quad 8 \div 4 \quad 4\overline{)8} \quad \frac{8}{4} \quad 8/4$

In algebra, the preferred way to show division is with fraction notation.

A Chunking Division Procedure

This chunking division procedure keeps the dividend intact as we "close in" on the quotient. If you do not know all your multiplication facts, this procedure may be easier than the standard division algorithm because you subtract out groups of the divisor more flexibly, but still arrive at the correct quotient. If the largest amount possible is chosen to subtract at each step, this procedure is very efficient.

Divide 761 highlighters into 3 boxes.

Step 1: Rewrite problem

$3\overline{)761}$

Step 2: Make a Multiplication Bank that may be useful for this problem.

3 × 1 = 3	3 × 10 = 30	3 × 100 = 300
3 × 2 = 6	3 × 20 = 60	3 × 200 = 600
3 × 3 = 9	3 × 30 = 90	3 × 300 = 900
3 × 4 = 12	3 × 40 = 120	3 × 400 = 1200

Step 3: Select a fact from the Multiplication Bank that is less than or equal to the dividend, and record. Continue the routine until the remainder is less than the divisor.

3	761		3	761		3	761			253	R 2
−	600	200	−	600	200	−	600	200	3	761	
	161			161			161		−	600	200
			−	120	40	−	120	40		161	
				41			41		−	120	40
						−	30	10		41	
							11		−	30	10
										11	
									−	9	+ 3
										2	253

The last calculation shows that the quotient is (200 + 40 + 10 + 3) = 253, and the remainder is 2.

The Standard Division Algorithm for Whole Numbers		
The standard division algorithm is an efficient process for dividing. It involves a cyclical process: divide, multiply, subtract, "bring down"… until the remainder is less than the divisor.		
$14\overline{)\,9\,6\,3}$	Determine where to start	Look at the divisor. Choose digits in the dividend so that the quotient using these digits is between 1 and 9.
$\begin{array}{r} 6 \\ 14\overline{)\,9\,6\,3} \end{array}$	Divide	How many 14s in 96? Write this number above the 96. Place value reminder: The 96 in the dividend represents 960. The 6 in the quotient represents 60.
$\begin{array}{r} 6 \\ 14\overline{)\,9\,6\,3} \\ -\,8\,4 \end{array}$	Multiply	Find the product of 6 and 14. Write this below the 96. Place value reminder: $6 \times 14 = 84$ is compact notation for $60 \times 14 = 840$.
$\begin{array}{r} 6 \\ 14\overline{)\,9\,6\,3} \\ -\,8\,4 \\ \hline 1\,2 \end{array}$	Subtract	Find the difference between 96 and 84. Write this below the 84. Place value reminder: $96 - 80 = 12$ is compact notation for $960 - 840 = 120$.
$\begin{array}{r} 6 \\ 14\overline{)\,9\,6\,3} \\ -\,8\,4\downarrow \\ \hline 1\,2\,3 \end{array}$	Bring down	Bring down the next digit.
$\begin{array}{r} 6\,8 \\ 14\overline{)\,9\,6\,3} \\ -\,8\,4\downarrow \\ \hline 1\,2\,3 \\ -\,1\,1\,2 \\ \hline 1\,1 \end{array}$	Divide Multiply Subtract Bring down (remainder)	Repeat the divide, multiply, subtract, bring down (if necessary) process until the remainder is less than the divisor.

Some ways to represent the dividend, divisor, quotient, and remainder:

$\text{divisor}\overline{)\text{dividend}}$ $\quad \text{quotient} \quad \text{remainder}$

dividend = (divisor)(quotient) + remainder

$14\overline{)963} = 68$ R11

$14\overline{)963} = 68\ \frac{11}{14}$

$963 = (14)(68) + 11$

Why Do We Move the Decimal Point when Dividing Decimals?

The procedure for dividing decimals involves "moving the decimal point." The reason this is done is because we usually consider dividing by a whole number to be an easier process.

Consider $12.5 \div 0.25$, which can be written as $0.25\overline{)12.5}$ or $\frac{12.5}{0.25}$.

Since $12.5 \div 0.25$ may be multiplied by 1 in the form of $\frac{100}{100}$, it is equal to $\frac{12.5}{0.25} \bullet \frac{100}{100} = 1{,}250 \div 25$.

Now we can divide by a whole number. This process often is depicted this way:

$$0.25\overline{)12.5} \rightarrow 0.25\overline{)12.50} \rightarrow 025.\overline{)1250.} \rightarrow 25.\overline{)1250.}^{50.}$$

Division of Decimals: Examples

- Multiply the divisor and dividend by the same power of 10 (10, 100, 1000, etc.) so that the divisor is a whole number.
- Divide as usual, lining up the digits of the quotient above the dividend so that the tens line up with tens, ones with ones, tenths with tenths, and so on. Place the decimal in the quotient in the same location as the dividend.

To obtain more decimal place accuracy, attach zeroes to the right of the final place in the decimal part and continue dividing until the remainder is zero (example 2) or the quotient pattern repeats (example 3).

Example 1

$0.02\overline{)0.358} \rightarrow 2\overline{)35.8}$

$$\begin{array}{r} 17.9 \\ 2\overline{)35.8} \\ -2\downarrow \\ \hline 15 \\ -14 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$$

Example 2

$8\overline{)3} \rightarrow 8\overline{)3.000}$

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -24\downarrow \\ \hline 60 \\ -56 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

Example 3

$11\overline{)4} \rightarrow 11\overline{)4.0000}$

$$\begin{array}{r} 0.3636... \\ 11\overline{)4.0000} \\ -33\downarrow \\ \hline 70 \\ -66 \\ \hline 40 \\ -33 \\ \hline 70 \\ -66 \\ \hline 4 \end{array}$$

Standard Algorithms for Decimal Operations

Addition

- Set up the problem in columns, with place values lined up to add tens with tens, ones with ones, tenths with tenths, etc. When the digits are properly lined up, the decimal points will also align.
- (Optional) Include trailing zeroes to the right of the decimal points as place holders if needed, as in this problem where 1 thousandth is added to 0 thousandths.
- Add with regrouping as usual. Since the place values in the sum line up with the place values in the two addends, the decimal point in the sum will align with the decimal points in the addends.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ \ } \\ 48.560 \\ +36.521 \\ \hline 85.081 \end{array}$$

Subtraction

- Set up the problem in columns, with place values lined up to subtract tens from tens, ones from ones, tenths from tenths, etc. When the digits are properly lined up, the decimal points will also align.
- Include trailing zeroes to the right of the decimal point as place holders in the minuend (top number) as needed to line up with any trailing nonzero digit in the subtrahend (bottom number).
- Subtract as though the decimal points are not there. When done calculating, place the decimal point in the difference directly below the decimal points in the problem.

$$\begin{array}{r} {\scriptstyle 6\ \ 13\ 10} \\ 7.4\,0 \\ -3.5\,1 \\ \hline 3.8\,9 \end{array}$$

Multiplication

- Set up the problem in columns, with digits right justified.
- Ignore decimal placement and multiply.
- Place decimal in the product. The number of digits to the right of the decimal point in the product is equal to the *sum* of the number of digits to the right of the decimal point of each factor.

$$\begin{array}{rl} 3\,0.5 & \text{(1 decimal place)} \\ \times\ 0.0\,0\,3 & \text{(3 decimal places)} \\ \hline 0.0\,9\,1\,5 & \text{(4 decimal places)} \end{array}$$

Division

- Multiply the divisor and dividend by the same power of 10 (10, 100, 1000, etc.) so that the divisor is a whole number.
- Divide as usual, lining up the digits of the quotient above the dividend so that the tens line up with tens, ones with ones, tenths with tenths, and so on. Place the decimal in the quotient in the same location as the dividend.

To obtain more decimal place accuracy, attach zeroes to the right of the final place in the decimal part and continue dividing until the remainder is zero or the quotient pattern repeats.

$$0.25\overline{)12.5} \quad \rightarrow$$

$$0.25\overline{)12.50} \quad \rightarrow$$

$$025.\overline{)1250.} \quad \rightarrow$$

$$\begin{array}{r} 50. \\ 25.\overline{)1250.} \end{array}$$

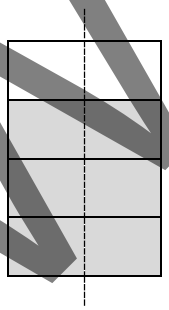
Visualizing Fraction Division as "Divvy Up"

A "divvie up" division problem poses the question:

"How can we divide ___ into ___ equal groups?

Suppose we want to divide $\frac{3}{4}$ cups of grape juice equally among two people. This division problem $\frac{3}{4} \div 2$, can be interpreted as "how can we divide $\frac{3}{4}$ into 2 equal parts?

Let the rectangle represent 1 full cup. It is filled with $\frac{3}{4}$ cups of grape juice.



From the diagram we see that each person will get $\frac{3}{8}$ cup of juice.

Therefore, $\frac{3}{4} \div 2 = \frac{3}{8}$.

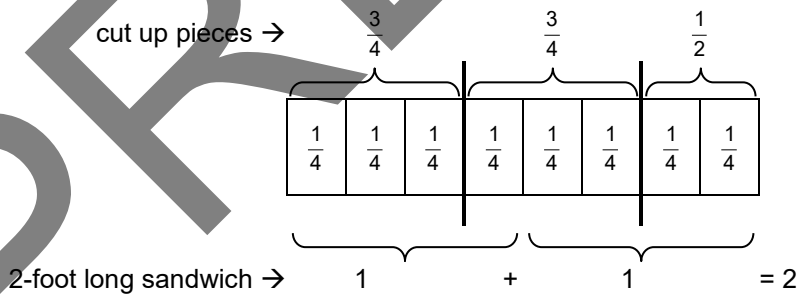
Visualizing Fraction Division as "Measure Out"

A "measure out" division problem poses the question:

"How many ___ are in ___?"

Suppose a two-foot sandwich is cut into pieces that are $\frac{3}{4}$ foot long each. This division problem $2 \div \frac{3}{4}$ can be interpreted as "how many $\frac{3}{4}$ ft. are in 2 ft.?" The unit of measure is $\frac{3}{4}$ ft. From the diagram, we see that there are TWO $\frac{3}{4}$ ft. sandwiches in the 2 ft. sandwich. We see further that there is $\frac{1}{2}$ ft. of sandwich leftover. Since $\frac{1}{2} = \frac{2}{3}$ of $\frac{3}{4}$, the leftover represents $\frac{2}{3}$ of the unit of measure.

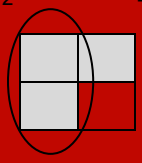
Therefore, $2 \div \frac{3}{4} = 2\frac{2}{3}$.



A Closer Look at the Unit in Fraction Measurement Division

Consider the problem: How many $\frac{1}{2}$s are in $\frac{3}{4}$?

$$\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$$



What is the whole? $\frac{3}{4}$

What is the unit of measure? $\frac{1}{2}$

Is there a full $\frac{1}{2}$ ***in*** $\frac{3}{4}$***?*** Yes.

How much is leftover? $\frac{1}{4}$

What part of the unit is leftover? $\frac{1}{2}$ because $\frac{1}{4}$ is $\frac{1}{2}$ of $\frac{1}{2}$.

How many $\frac{1}{2}$***s are in*** $\frac{3}{4}$***?*** $1\frac{1}{2}$

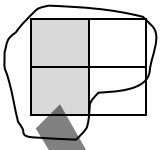
$\frac{1}{2}$ is circled and $\frac{1}{4}$ is left over.

$\frac{1}{4}$ is $\frac{1}{2}$ of a $\frac{1}{2}$.

In this case, a larger positive number is being divided by a smaller positive number. The result is a quotient greater than 1.

Consider the problem: How many $\frac{3}{4}$s are in $\frac{1}{2}$?

$$\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$$



What is the whole? $\frac{1}{2}$

What is the unit of measure? $\frac{3}{4}$

Is there a full $\frac{3}{4}$ ***in*** $\frac{1}{2}$***?*** No.

How many $\frac{3}{4}$***s are in*** $\frac{1}{2}$***?*** $\frac{2}{3}$

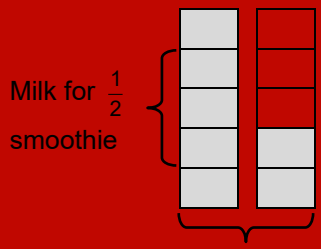
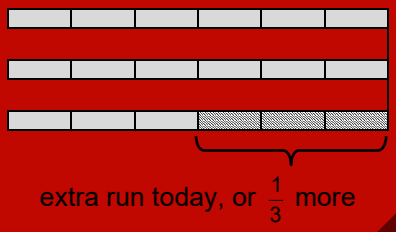
$\frac{2}{3}$ of $\frac{3}{4}$ is shaded.

In this case, a smaller positive number is being divided by a larger positive number. The result is a quotient less than 1.

Rules for Dividing Fractions

Divide Across	Multiply by the Reciprocal
$\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d}$	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$
$b \neq 0, d \neq 0$	$b \neq 0, d \neq 0$

Examples: Dividing Fractions

Words or Diagrams	Divide Across	Multiply by the Reciprocal
Millenium needs $1\frac{1}{2}$ cups of milk to make a smoothie. How much smoothie can Millenium make with $\frac{3}{4}$ cup of milk?  	$\frac{3}{4} \div 1\frac{1}{2}$ $= \frac{3}{4} \div \frac{3}{2}$ $= \frac{3 \div 3}{4 \div 2}$ $= \frac{1}{2}$	$\frac{3}{4} \div 1\frac{1}{2}$ $= \frac{3}{4} \div \frac{3}{2}$ $= \frac{3}{4} \times \frac{2}{3}$ $= \frac{3 \times 2}{4 \times 3}$ $= \frac{6}{12} = \frac{1}{2}$
Helen usually runs $2\frac{1}{2}$ miles a day. Today, she ran $3\frac{1}{3}$ miles. How much of her usual run did Helen run today?  	$3\frac{1}{3} \div 2\frac{1}{2} = \frac{10}{3} \div \frac{5}{2}$ $= \frac{20}{6} \div \frac{15}{6}$ $= \frac{20 \div 15}{6 \div 6}$ $= \frac{\frac{20}{15}}{1} = \frac{20}{15}$ $= 1\frac{5}{15} = 1\frac{1}{3}$	$3\frac{1}{3} \div 2\frac{1}{2} = \frac{10}{3} \div \frac{5}{2}$ $= \frac{10}{3} \times \frac{2}{5}$ $= \frac{10 \times 2}{3 \times 5}$ $= \frac{20}{15}$ $= 1\frac{5}{15} = 1\frac{1}{3}$

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.2	Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with *b* ≠0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations:
b.	Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
d.	~~Use ratio reasoning to convert measurement units~~; manipulate and transform units appropriately when multiplying or dividing quantities.
6.NS.A	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
6.NS.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.NS.2	Fluently divide multi-digit numbers using the standard algorithm.
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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