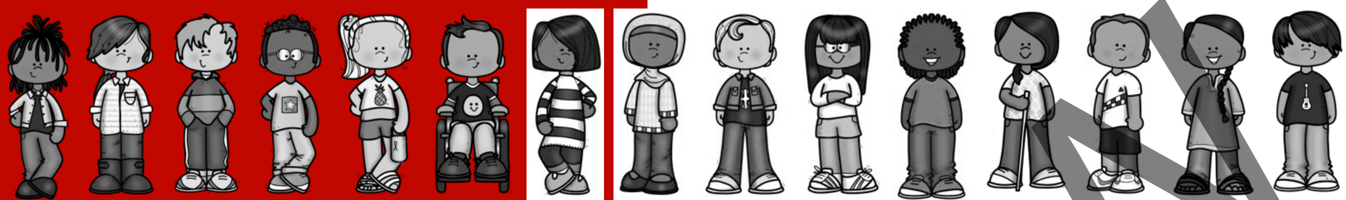
Name ______________________________ Period __________ Date ______________

UNIT 3
STUDENT PACKET

MathLinks
GRADE 6



RATIO REPRESENTATIONS

Parent (or Guardian) signature ______________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

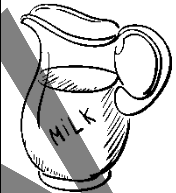
conversion rate	customary units metric units
double number line	equivalent ratios
ratio	tape diagram
unit price	unit rate

NANA'S CHOCOLATE MILK

Follow your teacher's directions for (1) – (2).

(1)

What I should have done:	What I did:



(2)

Use this space to revisit the problem after lesson 3.1.

TAPE DIAGRAMS AND TABLES

We will develop ratio language and notation. We will use ratios, tables, and tape diagrams to solve problems.

[6.RP.1, 6.RP.3ad; SMP1, 2, 3, 7]

GETTING STARTED

1. According to the directions on a can of frozen orange juice concentrate, 3 cans of water are to be mixed with 1 can of concentrate to make orange juice.

   Draw a picture to illustrate this mixture.

2. Complete the table.

	Draw a picture to illustrate this mixture.	**Will this mixture be "more orangey," "less orangey," or "the same" as what is recommended?**
Colin mixes 4 cans of water and 1 can of concentrate.		
Indy mixes 2 cans of water and 1 can of concentrate.		
Blue mixes $\frac{3}{4}$ cans of water and $\frac{1}{4}$ cans of concentrate.		

PAINT MIXTURES

Follow your teacher's directions for (1) – (3).

(1)

Mixture Card:	Mixture Card:	Mixture Card:	Mixture Card:

(2) Anita: "Mixture B and mixture C will be the same because they both have …
(3) Drew: "Mixture A and Mixture D will be the same because Mixture A has … and Mixture D has …

4. Choose a different pair of mixture cards than the ones discussed in (2) and (3) above and explain how you know which mixture represents the darker pink.

TAPE DIAGRAMS

Follow your teacher's directions for (1) – (5).

(1) Mixture B	(2) Mixture A
(3) Mixture C	(4) Mixture D

(5) Talia wants to make ______ ounces of Mixture B.

a. What number of ounces would each small rectangle represent?

b. How much of each color would there be?

Alex and Lily were asked how many gallons of red paint and white paint were needed to make 12 gallons of paint that is the same shade as the mixture on Mixture card B.

6a. Explain Alex's work:

R	*R*	*W*	*W*	*W*	*W*
R	*R*	*W*	*W*	*W*	*W*

4 gal red	8 gal white

6b. Use Alex's method to find the amount of each ingredient needed to make 9 quarts of Mixture C.

7a. Explain Lily's work:

12 gallons					
R	*R*	*W*	*W*	*W*	*W*
2	2	2	2	2	2
4 gal red		8 gal white			

7b. Use Lily's method to find the amount of each ingredient needed to make 72 ounces of Mixture D.

8. Record the meaning of <u>tape diagram</u> with examples in **My Word Bank**.

PRACTICE 1

Use tape diagrams to solve these problems.

Zachary likes to make fruit soda when he has friends over to his house. He uses 4 parts juice for every 3 parts sparkling water.

1. Make a tape diagram to illustrate this mixture.

2. How much juice and how much sparkling water will Zachary need if he wants to make 14 cups of fruit soda?

3. How much sparkling water should Zachary use if he has 12 cups of juice?

4. How much juice should Zachary use if he wants to make 70 cups of fruit soda?

5. Jane likes to make fruit soda too. Her recipe uses 2 parts juice and 1 part sparkling water. Who makes a fruitier soda, Zachary or Jane? Explain how you know.

RATIOS AND TABLES

Follow your teacher's directions for (1) – (3).

(1) Words	(2) Numbers	(3) Table

parts red	3				1
parts white		8	2	1	
total					

4. Record the meaning of <u>ratio</u> in **My Word Bank**.

5. Refer to Mixture cards B, C, and D that you copied when introduced to **Paint Mixtures**. Write the ratios for each of the mixtures.

	Mixture B	Mixture C	Mixture D
Red to White			
White to Red			
Red to Total			

6. Complete each table below based on mixtures cards B and D.

Mixture B					
cups red	2	1	10		
cups white				1	7

Mixture D					
cups red	4	2		8	
cups white			20	10	1

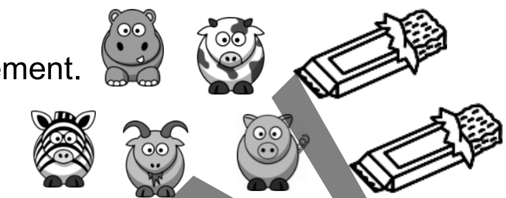
7. Use the entries in the table above to explain which is a darker pink: Mixture B or Mixture D?

8. Circle all the representations of paint mixtures that make sense to you.

pictures of mixtures tape diagrams ratios tables

PRACTICE 2

Blakely is making “goodie bags” to give out at her younger brother Graham’s birthday party. She wants to put 5 stickers and 2 granola bars in each one.



1. Describe ratios in different ways by completing each statement.

   For every 5 stickers, there will be _____ granola bars.

   The ratio of stickers to granola bars is _____ : _____.

   The ratio of granola bars to total items in the bags is _____ to _____.

2. Complete the table below based on the given ratio of stickers to granola bars in the goodie bags. Leave the last column blank for problem 3 below.

# of stickers	5		20		50	
# of granola bars	2					
# of goodie bags		2		8		

3. Blakely uses 60 stickers to fill bags.

   Enter this quantity in the last column in the table above.

   Explain how the table can be used to help determine how many granola bars she will need.

   How many bags does she plan to fill? _____ How many granola bars will she need? _____

4. Suppose there were 20 people coming to Graham’s party. How many stickers and granola bars would be needed? Show or explain how you know.

PRACTICE 3

1. A purple paint is made with 2 parts blue and 6 parts red.

    a. Make a tape diagram to illustrate this relationship.

    b. Express three different ratios that could be represented based on the tape diagram above. Try to use symbols for some and words for others

2. Sam makes tie-dyed shirts. Her most frequently used colors are orange and green.

    a. For the orange dye, she uses red and yellow in a ratio of 3 : 2. How many ounces of red and yellow dye will she need if she wants to make 80 ounces of orange dye? Use a tape diagram.

    b. For the green dye, she uses blue and yellow in a ratio of 5 : 2. How many ounces of yellow dye will she need if she is using 40 ounces of blue dye? Use a table.

THE ASSEMBLY

The school auditorium has 330 seats. When the 6th grade students went to assembly period 1, seats were filled at a ratio of 9 occupied to 2 unoccupied. When the 7th grade students went to assembly period 2, seats were filled at a ratio of 5 occupied to 1 unoccupied. Show work using tape diagrams.

1. How many 6th grade students went to the assembly?

2. How many 7th grade students went to the assembly?

3. If 300 8th grade students went to assembly period 3, what was the ratio of occupied to unoccupied seats?

NANA'S CHOCOLATE MILK…REVISITED

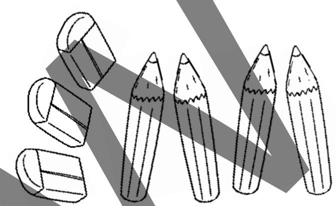
Recall that Nana likes her chocolate milk with 1 cup milk and 4 scoops chocolate. BUT… you mix 1 cup milk and 5 scoops chocolate. OH NO! Go back to the opening problem and revise your work. Use one or more of the representations you have learned to help you organize your thinking.

EQUIVALENT RATIOS AND TABLES

We will learn how to determine if ratios are equivalent. We will use arrow diagrams and tables to represent equivalent ratios. We will solve problems that involve ratios.

[6.RP.1, 6.RP.3a; SMP2, 3, 7]

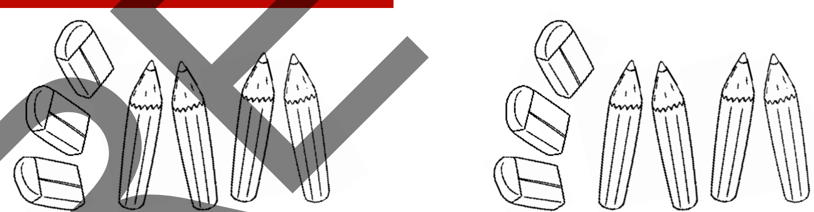
GETTING STARTED



1. Here is a picture of some erasers and pencils. Write the ratios for this diagram.

Number of pencils to number of erasers	4 to _____	or	4 : _____
Number of pencils to total number of objects	_____ to _____	or	_____ : _____
Number of total objects to number of pencils	__________	or	__________

2. The original picture is duplicated here. Write ratios for this diagram.

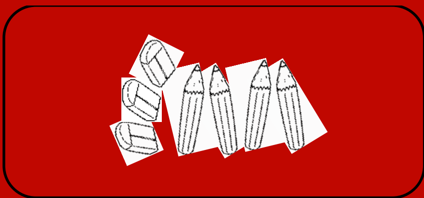


Number of erasers to number of pencils	6 to _____	or	6 : _____
Number of pencils to total number of objects	_____ to _____	or	_____ : _____
Number of pencils to number of erasers	__________	or	__________

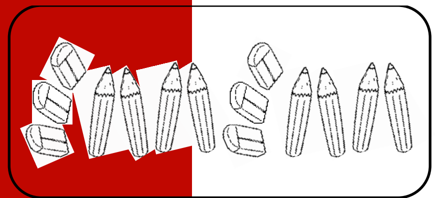
EQUIVALENT RATIOS

Follow your teacher's directions for (1) – (4).

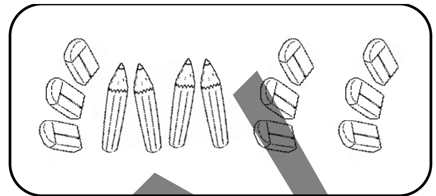
s



Picture A



Picture B



Picture C

(1) × ____ × ____	(2)	(3)

(4)

Show your work to determine if each pair of ratios is equivalent.

5. 3 to 5 and 12 to 30	6. 8 : 5 and 24 : 15	7. 8 : 6 and 12 : 9

8. Record the meaning of <u>equivalent ratios</u> in **My Word Bank**.

PRACTICE 4

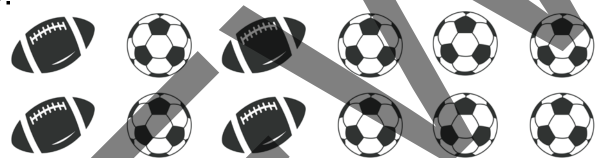
1. Write the ratios of soccer balls to footballs in each collection. Then draw arrows to match collections that represent equivalent ratios. Not every collection has a match.

A.

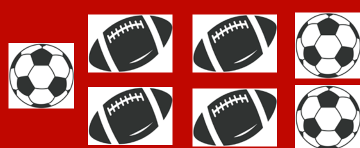
B.

C.

D.



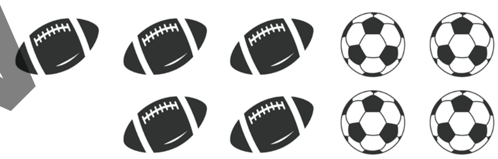
E.



F.

G.

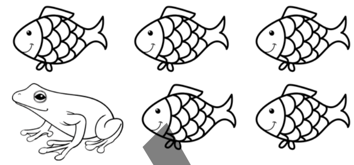
H.



2. Choose one pair of equivalent ratios above. Show or explain how you know they are equivalent.

3. Matteo says that collections A and H represent equivalent ratios because each one has one more football than soccer ball. Explain why Matteo is wrong.

EQUIVALENT RATIOS IN TABLES



The ratio of the number of fishes to the number of frogs in the science lab is 5 to 1, or 5 fish for every 1 frog.

1. Complete the table below for possible numbers of fish and frogs that could be in the lab. Write in the multipliers for the arrows.

# of fish	5		30		
# of frogs	1	3		4	12

2. Explain how to use common multipliers to determine the number of fish if there are 3 frogs.

3. Use the table above. Choose two different pairs of entries and form ratios. Find the multiplier for each pair of ratios that can be used to justify that they are equivalent.

Example: frogs : fish

$1 : 5$

$\times 3$ $\times 3$

$3 : 15$

Suppose there are currently 15 fish in the lab. Then the science teacher adds one new frog and one new fish.

4. Complete the table below.

	Currently	After additions
# of fish	15	
# of frogs		

5. Is the ratio of fish : frogs the same after the additions? _______ Show with an arrow diagram on the table. Explain the meaning of your arrows.

PRACTICE 5

In a soccer tournament, the ratio of the number of 12-year-olds to the number of 11-year-olds is 1 to 2.

# of 11-year-olds	# of 12-year-olds	Total Players
	1	
20		
	25	
		150
	20	
350		

1. What is the ratio of 11-year-olds to 12-year-olds?

2. What is the ratio of 11 year-olds to total players?

3. Complete the table.

4. Choose two different ratios for the number of 11-year-olds to the number of total players. Then use an arrow diagram to show that these two ratios are equivalent.

5. Make a tape diagram to solve this problem: If there are 78 players in the tournament, how many 11-year-olds will there be?

6. Solve problem 5 using an arrow diagram or a table.

7. Circle all the representations that make sense to you. Put a star by the representation(s) you prefer.

tape diagram table arrow diagram

EQUIVALENT RATIOS AND DOUBLE NUMBER LINES

We will explore the connection between equivalent ratios and unit rates. We will create double number lines using tables that represent equivalent ratios. We will use double number lines to solve problems.

[6.RP.1, 6.RP.2, 6.RP.3a, b; SMP2, 6, 7, 8]

GETTING STARTED

Leo earns $32 for every four hours of babysitting.

1. Complete the following table.

dollars	32	64				
hours			1	2		5
dollars : hours					48 : 6	
$\frac{\text{dollars}}{\text{hours}}$	$\frac{32}{4} = 8$					

2. Explain how you found the number of dollars earned for 5 hours of babysitting.

3. Choose two different dollars to hours ratios from the table and show they are equivalent ratios with an arrow diagram.

4. How much does Leo earn per hour? How do you know from the table?

5. Sydelle babysat for 6 hours and earned $42. Did she earn the same rate of pay as Leo? How do you know?

EQUIVALENT RATIOS REVISITED

Refer to the table in **Getting Started**. Recall that Leo earned $32 for every four hours of babysitting.

1. The number in the bottom row $\left(\frac{\text{dollars}}{\text{hours}}\right)$ is called the value of the ratio or the unit rate of pay, expressed in dollars per hour. What is Leo's unit rate?

2. Jojo thinks that equivalent ratios have the same value. She also thinks the unit rates will be the same when the two ratios are equivalent. Does that appear to be true in this situation?

Domingo paid $24 for 6 gallons of gas. Jerold bought 9 gallons of gas for $36.

3. Find the cost per gallon (a special kind of unit rate called unit price) for each purchase.

4. Use an arrow diagram to show these are equivalent ratios. Where do unit rates (or unit prices) appear in the arrow diagram?

Two ways to determine if ratios are equivalent are with arrow diagrams and unit rates.

5. Josh bought 12 gallons of gas for $60. Show using arrow diagrams and unit rates that the cost per gallon is NOT equivalent to Domingo's.

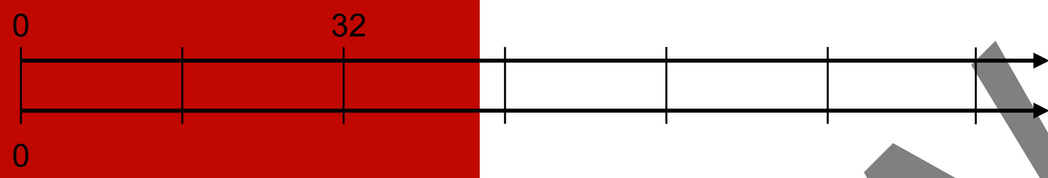
6. Which method for determining equivalent ratios do you prefer? Why?

7. Record the meanings of unit price and unit rate in **My Word Bank**.

DOUBLE NUMBER LINES

Follow your teacher's directions for (1) – (4). Use the **Getting Started** table to help you.

(1) Show ________________ on a double number line.



(2) Show the ______________ on the double number line for the ratio _______________.

(3) How are a ___________ and a ______________________ the same? Different?

Same	Different

(4) What patterns do you notice on the __________________________________?

Leo wants to buy a jacket. Add entries to the double number line to help answer these questions.

5. Suppose the jacket costs $88. How many hours will Leo need to work? How do you know?

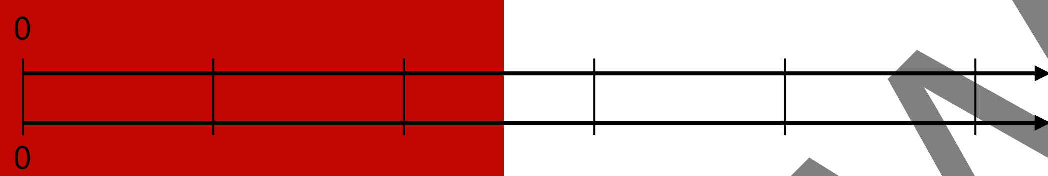
6. Suppose the jacket costs $84. How many hours will Leo need to work? How do you know?

7. Record the meaning of <u>double number line</u> in **My Word Bank**.

PRACTICE 6

Create double number lines to help you solve each problem. Assume constant rates (i.e. equivalent ratios) for each problem.

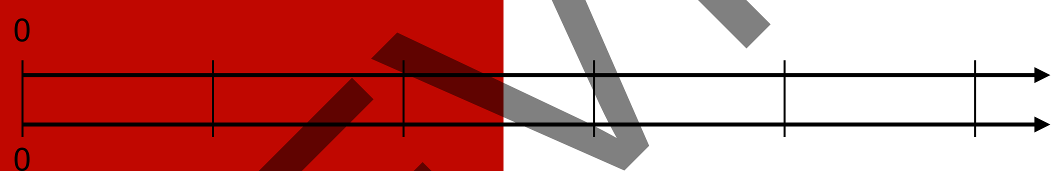
1. Alex pays $60 for 10 sandwiches.



   a. What is the price for 25 sandwiches at this rate?

   b. What is cost for 1 sandwich?

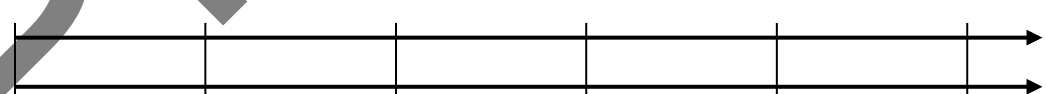
2. Max read 5 books in 2 weeks.



   a. At this rate, how many books will she read in 9 weeks?

   b. How many books did she read per week?

3. A workday is 8 hours. You earn $48 for one work day.



   a. What is the hourly pay rate?

   b. At this rate, how much would you earn in $5\frac{1}{2}$ hours?

THE GRAIN GROCER

The Grain Grocer sells rice in bulk.
The special of the day is to the right.

Grain Grocer
Special of the Day
5 pounds of rice for $4

Chelsea said, “The ratio of the number of dollars to the number of pounds is 4:5. That's $0.80 per pound.”

Lauren said, "The sign means that the ratio of the number of pounds to the number of dollars is 5:4. That's 1.25 pounds per dollar."

1. Represent this situation with a tape diagram, table, or double number line.

2. Explain why Chelsea and Lauren are both correct.

3. Allie needs two pounds of rice to make a casserole. Explain to Allie how much money she will need.

4. Lev has $10 and wants to stock up on rice. Explain to Lev how many pounds of rice he can buy.

5. Did your representation in problem 1 above help you answer problem 4? If so, how?

MEASUREMENTS AND RATES

We will classify customary and metric measurement units. We will use ratio reasoning to convert measurement units. We will convert between units of measure. We will solve problems involving measurements.

[6.RP.1, 6.RP.3a,d; SMP2, 4, 6]

GETTING STARTED

Use the measurement units in the box, or others you know, for problems 1-3.

centimeters	liters
cups	meters
feet	miles
fluid ounces	ounces
gallons	pints
grams	pounds
kilometers	quarts
inches	yards

1. What are some units used to measure length, such as your height or the distance from your home to school?

2. What are some units used to measure capacity or volume, such as the amount of water in a bottle or pool?

3. What are some units used to measure mass or weight, such as the weight of a package to be mailed or an elephant?

4. Use abbreviations to write 5 feet 6 inches in two different ways.

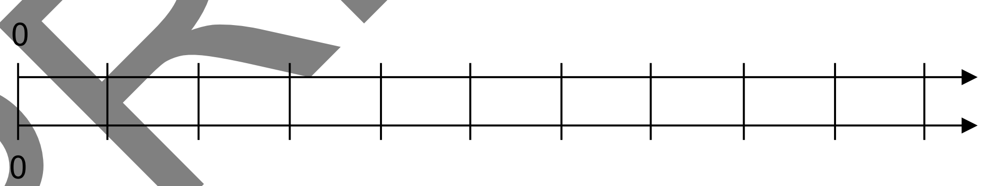
5. What is the difference between a fluid ounce and an ounce?

MEASUREMENT SYSTEMS

Follow your teacher's directions.

(1) – (4)	Customary Measurements	Metric Measurements
length		
volume (capacity)		
weight (mass)		

(5) Make a double number line to determine how many ounces are in _____ pounds



(6) Use your double number line. Determine how many pounds are in _____ ounces.

Complete the table below.

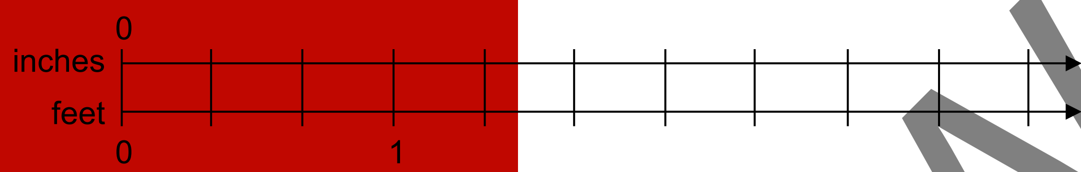
(7)

meters			
kilometers			

PRACTICE 7

1. Record the meanings of customary units and conversion rate in **My Word Bank**.

2. Complete this double number line that relates inches to feet.

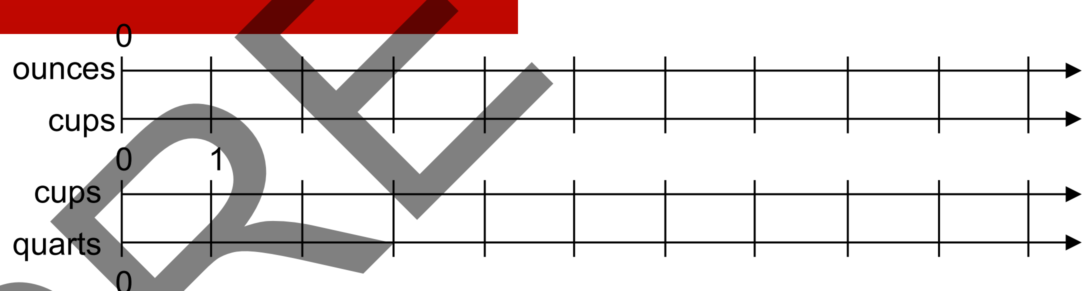


Use the double number line above to complete these conversion statements. You may want to insert values or extend lines to help you.

3. 4 in = ______ ft	4. 6 in = ______ ft	5. $3\frac{1}{2}$ ft = ______ in
6. 4 ft = ______ in	7. 6 ft = ______ in	8. 22 in = ______ ft

9. Explain how you found the number of inches in $3\frac{1}{2}$ feet.

10. There are ______ fluid ounces in a cup and ______ cups in a quart. Create two double number lines using the given overlapping scale for cups.



Convert the units below. Use the double number lines above for reference.

11. 16 fl oz = ______ c	12. 24 c = ______ qt	13. $\frac{1}{2}$ c = ______ fl oz
14. 2 fl oz = ______ c	15. 6 c = ______ qt	16. 16 fl oz = ______ qt

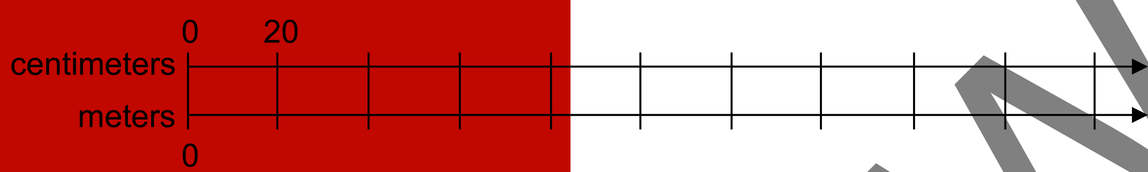
17. Explain how you found the number of cups in 2 fl oz.

PRACTICE 8

1. Record the meaning of <u>metric units</u> in **My Word Bank**.

Complete the double number lines, tables, and conversion statements below that relate metric measurements.

2.



3.

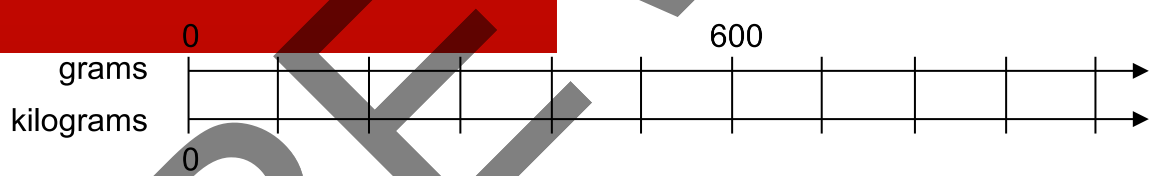
centimeters	500	10,000			15,000	1
meters			1	0.1		0.01

4. For every _____ cm, there is 1 m.

   To convert from meters to centimeters, ________________ by __________.
   multiply/divide number

   To convert from centimeters to meters, ________________ by __________.
   multiply/divide number

5.



6.

grams			1			
kilograms	1	0.25		0.00001	0.01	

7. There are _____ grams in 1 kilogram or _____ grams per kilogram.

   To convert from grams to kilograms, ________________ by __________.
   multiply/divide number

   To convert from kilograms to grams, ________________ by __________.
   multiply/divide number

CONVERTING BETWEEN SYSTEMS

A “wavy equal sign” symbol, ≈, means “approximately equal to.”

1. Refer to **Student Resources** as needed. Write several statements in symbols or words that convert between customary measurements and metric measurements. Approximations are expected.

Type of Measurement	Conversion Statements
length	
volume (capacity)	
weight (mass)	

2. Use approximations to complete the double number line.

For problems 3 – 5, convert so that the measurements are approximately equal.

3. 4 in ≈ _____ cm	4. 40 in ≈ _____ cm	5. 1 ft ≈ _____ cm
6. 4 cm ≈ _____ in	7. 3 cm ≈ _____ in	8. 6 cm ≈ _____ in

9. Approximate conversion rates: There are _____ $\frac{\text{pounds}}{\text{kilogram}}$. There are _________ $\frac{\text{kilograms}}{\text{pound}}$.

10. About how many pounds are in 50 kilograms?

11. About many kilograms are in 50 pounds?

SLIME

Slime is an example of a non-Newtonian fluid, a liquid whose viscosity (thickness/stickiness) changes depending on pressure. You can form slime temporarily into a shape, but if you let it rest, it will become gooey.

SLIME RECIPE

Ingredients:
- $\frac{1}{2}$ tsp borax powder*
- 1 cup clear or white PVA school glue
- 2 cups of water, divided into two 1-cup portions
- Food coloring (optional)
- Oil (to put on hands if slime is too sticky)
- Vinegar (to clean up accidental spills on clothing or furniture)

Supplies:
- 2 paper cups
- 1 plastic spoon
- Measuring cups and spoons
- 1 plastic baggy for storage
- Paper towels for cleanup

*Quantity may vary depending on glue used

Step 1: In one of the cups, dissolve borax powder into 1 cup of water.

Step 2: In the second cup, mix the glue with 1 cup of water (and food coloring if desired).

Step 3: Slowly pour the borax/water mixture into the glue/water and stir it up. You will see it come together right away. You may not need all the borax mixture.

Step 4: Knead the mixture with your hands until it is smooth and stretchy. Store in a baggy.

Ms. Ryann plans to give each student 2 ounces of glue in a cup to make slime. She has 24 students in her class.

1. Here are some prices for glue. What should Ms. Ryann purchase for her class? How much will it cost? Explain your reasoning.

2 oz
$1.50

16 oz
$8.00

1 gallon
$20.00

2. In addition to the glue in a cup, list all other ingredients and supplies that each student will need to make slime. Explain your reasoning.

3. What ingredients and supplies do you think Ms. Ryann should have available if needed?

REVIEW

BIG SQUARE PUZZLE: RATIO REPRESENTATIONS

Your teacher will give you a puzzle to assemble.

1. Choose a set of equivalent ratios from the big square. Show they are equivalent with an arrow diagram.

   Pairs of ratios: _______ and _______

2. Choose a different set of equivalent ratios from the big square. Show they are equivalent with unit rates.

   Pairs of ratios: _______ and _______

I HAVE, WHO HAS: RATIO REPRESENTATIONS

Your teacher will give you one (or more) "I Have, Who Has" cards.

1. Copy one of the cards here.

I have ________________________
Who has ______________________

2. Write a "Who has" question that could be a prompt to your "I have" statement.

3. Write an "I have" statement that could be an answer to your "Who has" question?

(After one round I Have, Who Has)

4. Write two conversion statements that slowed down or stumped the class.

POSTER PROBLEMS: RATIO REPRESENTATIONS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is ________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
A recipe calls for 2 cups sugar and 4 cups flour.	You can buy 3 cookies for $1.00.	Greg ran 10 kilometers in 50 minutes.	A project requires 5 colored pencils and 10 index cards.

A. Copy the statement onto your poster and use the information to write two different ratios.

B. Make a table that displays different equivalent ratios for the statement.

C. Make a double number line or a tape diagram to display different equivalent ratios. Include some of the data from your table.

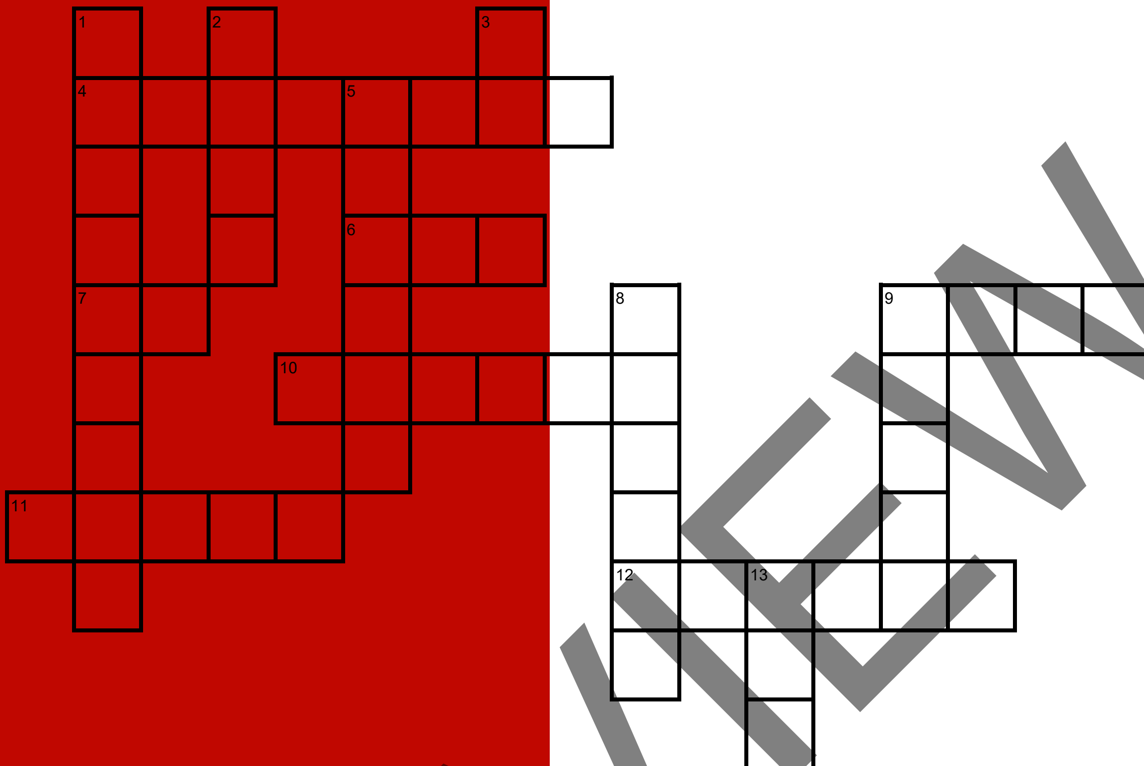
D. Use an arrow diagram to show that two of the ratios are equivalent.

Part 3: Return to your seats. Work with your group. Use your "start problem."

1. Write a story problem based on the fact statement, and find its solution.

2. Exchange problems with another group. Answer the other group's problem and check solutions.

VOCABULARY REVIEW



Across

4 value of a ratio (two words)

6 2,000 pounds is one of these

7 ounce (abbreviation)

9 a diagram where rectangles represent equal amounts

10 A ratio representation on scaled number lines is called a ______ number line.

11 a simple diagram used to show two equivalent ratios

12 5 cm ≈ 2 ___

Down

1 measurements such as feet, ounces, and pounds

2 5280 feet = 1 ______

3 4 cups = 1 ___ (abbreviation)

5 an arrow diagram and multiplier shows whether these are equivalent.

8 measurement system that includes meters, grams, and liters

9 a chart to organize data

13 one-fourth of a quart is one of these

SPIRAL REVIEW

1. **Computational Fluency Challenge**: This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

   a. Start with 4. Multiply by 3. Multiply the result by 4. Multiply the result by 2. Multiply the result by 10. Now you have a "big number". My big number is _____.

   b. Start with your big number. Divide it by 5. Divide the result by 8. Divide the result by 6. What is the final result? _____

2. Mia and her friends were having an ice cream party. Alex brought 2 containers of $2\frac{3}{4}$ pints each, Spencer brought $1\frac{2}{5}$ pints, Roberto brought 3 containers of $\frac{4}{5}$ pints each, and Mia brought $3\frac{1}{4}$ pints. How much ice cream did they have at the party?

   a. Write a numerical expression for the amount of ice cream at the party.

   b. How much ice cream did they have at the party?

SPIRAL REVIEW
Continued

3. Simplify each expression

a. $4 + 5(3)$	b. $7(2) – 5 + 12$	c. $15 + 4(5) – 2$
d. $12 \div (4 + 8)$	e. $4(10 + 2) \div 12$	f. $(13 + 17) – 7(2)$

4. Rewrite each of the expressions using the distributive property. Then evaluate each expression.

a. $3(5 + 7) = 3($______$) + 3($______$)$	b. $8(9) – 8(3) = 8($______ – ______$)$
c. $4(10 + 5) =$	d. $4(10) – 4(3) =$

5. Fill in the blank to create equivalent fractions.

a. $\frac{4}{5} = \frac{\boxed{}}{15}$	b. $\frac{\boxed{}}{30} = \frac{2}{5}$	c. $\frac{48}{12} = \frac{\boxed{}}{3}$

6. Find the missing number.

a. $14 +$ _____ $= 32$	b. _____ $\times 15 = 90$	c. $40 =$ _____ $– 32$

7. Nathan went shopping to get ready for the first day of school. He bought 4 shirts for \$8.34 each, a belt for \$10.50, 3 pairs of pants for \$14.15 each and one pair of shoes for \$30.80. He had a coupon for \$15.99 off.

   a. Write a numerical expression for the cost of all the items.

   b. How much did Nathan pay for all of his items?

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.

- Investigate concepts and solve problems involving length, area, and volume.
- Extend the number system to include negatives.
- Use statistical measures and displays to describe center and spread.
- Explore relationships between inputs and outputs.
- Gain computational fluency with positive rational numbers.
- Rewrite and evaluate expressions and solve equations.
- Explore and apply ratio and rate reasoning and representations.

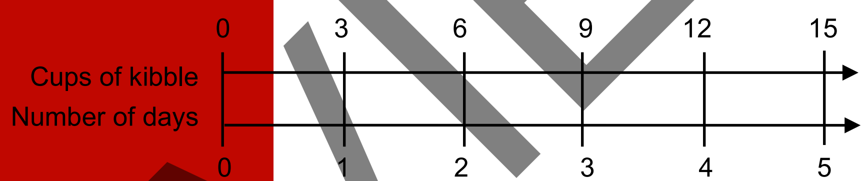
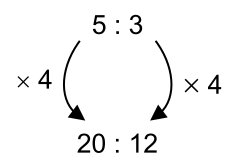
Give an example from this unit of one of the connections above.

2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Explain how one of the ratio representations gave you a process (structure) for solving different kinds of proportional reasoning problems [SMP7, 8]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **More Connections.** Give an example of how you might use proportional reasoning in an everyday situation.

STUDENT RESOURCES

Word or Phrase	Definition
conversion rate	A <u>conversion rate</u> is a unit rate expressing the number of units of one measure equal to one unit of another. Two conversion rates are 1.3 dollars per euro and 60 minutes per hour.
customary units	In the United States, <u>customary units</u> are a system of units of measurement that includes ounces, pounds, and tons to measure weight; inches, feet, yards, and miles to measure length; pints, quarts, and gallons to measure capacity; and degrees Fahrenheit to measure temperature.
double number line	A <u>double number line</u> is a diagram made up of two parallel number lines that visually depict the relative sizes of two quantities. Double number lines are often used when the two quantities have different units, such as miles and hours. The proportional relationship "Wrigley eats 3 cups of kibble per day" can be represented in the following double number line diagram. 
equivalent ratios	Two ratios are <u>equivalent</u> if each number in one ratio is obtained by multiplying the corresponding numbers in the other ratio by the same positive number. 5 : 3 and 20 : 12 are equivalent ratios because both numbers in the ratio 5 : 3 are multiplied by 4 to get to the ratio 20 : 12. An arrow diagram can be used to show equivalent ratios. 
metric units	<u>Metric units</u> are a system of units of measurement that includes grams and kilograms to measure weight; millimeters, centimeters, meters, and kilometers to measure length; milliliters and liters to measure capacity; and degrees Celsius to measure temperature.
ratio	A <u>ratio</u> is a pair of positive numbers in a specific order. The ratio of a to b is denoted by $a : b$ (read "*a* to *b*," or "*a* for every *b*"). The ratio of 3 to 2 is denoted by 3 : 2. The ratio of dogs to cats is 3 to 2. There are 3 cups of water for every 2 cups of juice. The fraction $\frac{3}{2}$ does not represent this ratio, but it does represent the *value of the ratio* (or the <u>unit rate</u>).

Word or Phrase	Definition
tape diagram	A <u>tape diagram</u> is a graphical representation that uses length to represent relationships between quantities. We draw rectangles with a common width to represent quantities, and rectangles with the same length to represent equal quantities. Tape diagrams are typically used to represent quantities expressed in the same unit. This tape diagram represents a drink mixture with 3 parts grape juice for every 2 parts water. | *G* | *G* | *G* | *W* | *W* |
unit price	A <u>unit price</u> is a price for one unit of measure. If 4 apples cost \$1.00, then the unit price is $\frac{\$1.00}{4}$ = \$0.25 for one apple, or 0.25 dollars per apple or 25 cents per apple.
unit rate	The <u>unit rate</u> associated with a ratio a : b of two quantities *a* and *b*, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. This is sometimes referred to as the <u>value of the ratio</u>. The ratio of 40 miles for every 5 hours has a unit rate of $\frac{40}{5}$ = 8 miles per hour.
value of a ratio	See <u>unit rate</u>.

Ratios: Language and Notation

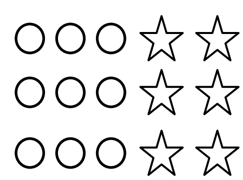
The <u>ratio</u> of *a* to *b* is denoted by *a* : *b* (read "*a* to *b*," or "*a* for every *b*").

Note that the ratio of *a* to *b* is not the same as the ratio of *b* to *a* unless *a* = *b*.

We can identify several ratios for the objects in the picture to the right.

- There are 3 circles for every 2 stars.
- The ratio of stars to circles is 2 to 3.
- The ratio of circles to total shapes is 3 : 5
- The ratio of total shapes to stars is 5 : 2.

Three copies of the figure above are pictured to the right. Here the ratio of circles to stars is 9 : 6. The ratio 9 : 6 is obtained by multiplying each number in the ratio 3 : 2 by 3 (called the multiplier).



circles : stars
3 : 2
× 3 × 3
9 : 6

The arrow diagram to the left shows that 3 : 2 and 9 : 6 are <u>equivalent ratios</u>.

A fraction formed by a ratio is called the <u>value of the ratio</u> (or <u>unit rate</u>). Equivalent ratios have the same value. In our example, $\frac{3}{2} = \frac{9}{6} = 1.5$.

Tables of Number Pairs

Tables are useful for recording number pairs that have equivalent ratios. In the case of a ratio of three circles for every two stars, there are two ways that number pairs with equivalent ratios might be recorded in a table. Table 1 is aligned horizontally. Table 2 is aligned vertically. Entries may be in any order.

○○○☆☆

Table 1

Circles	3	9	6
Stars	2	6	4

Table 2

Circles	**Stars**
3	2
9	6
6	4

Tape Diagrams

A tape diagram is a visual model consisting of strips divided into rectangular segments whose areas represent relative sizes of quantities. Tape diagrams are typically used when quantities have the same units.

This tape diagram shows that the ratio of grape juice to water in some mixture is 2 : 4.

G	*G*	*W*	*W*	*W*	*W*

Suppose we want to know how much grape juice is needed to make a mixture that is 24 gallons. Here are two methods:

Method 1:

G	*G*	*W*	*W*	*W*	*W*
G	*G*	*W*	*W*	*W*	*W*
G	*G*	*W*	*W*	*W*	*W*
G	*G*	*W*	*W*	*W*	*W*

Replicate the tape diagram, making 24 rectangles. Each rectangle now represents 1 gallon. This shows that:

2 gallons grape : 4 gallons water (6 total gallons)

is the same ratio as

8 gallons grape : 16 gallons water (24 total gallons)

24 gallons of mixture will require 8 gallons of grape juice.

Notice here that each rectangle (piece of tape) represents 1 unit (1 gallon of liquid.)

Method 2:

G	*G*	*W*	*W*	*W*	*W*

24 gallons					
4	4	4	4	4	4
8 gal		16 gal			

Six rectangles in the tape diagram represent 24 gallons of mixture.

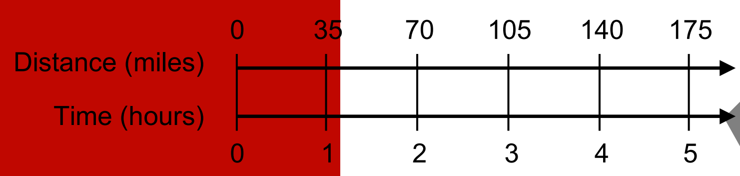
Since 24 ÷ 6 = 4, one rectangle in the tape diagram represents 4 gallons of liquid.

24 gallons of mixture will require 8 gallons of grape juice.

Notice here that each rectangle (piece of tape) represents more than 1 unit (4 gallons, in this case). Pieces of tape in the diagrams do not always need to represent 1 unit.

Double Number Lines
A double number line diagram is a graphical representation of two quantities in which corresponding values are placed on two parallel number lines for easy comparison. Double number lines are often used to compare two quantities that have different units.

The double number line below shows corresponding ratios if a car travels 70 miles every 2 hours.



We can see from the double number line diagram above that at the given rate, the car goes 35 miles in 1 hour (which is the unit rate of 35 miles per hour), 105 miles in 3 hours, etc. Notice the same tick marks on the number line are used to represent different quantities, and values are scaled in numerical order.

Unit Rate and Unit Price

The unit rate associated with a ratio is the value of the ratio, to which we usually attach units for clarity. In other words, the unit rate associated with the ratio $a : b$ is the number $\frac{a}{b}$, to which we may attach units. For this to make sense, we must assume that $b \neq 0$.

Suppose a car travels 70 miles every 2 hours.

- This may be represented by the ratio $70 : 2$.
- The number $\frac{70}{2} = \frac{35}{1} = 35$ is the value of the ratio.
- The unit rate is then the value 35, to which we attach the units “miles per hour.” Thus, the unit rate may be written:

$35 \frac{\text{miles}}{\text{hour}}$ or 35 miles per hour or 35 miles/hour

A unit price is the price for one unit.

Suppose it costs \$1.50 for 5 apples.

- This may be represented as the ratio $1.50 : 5$.
- The number $\frac{1.50}{5} = 0.30$ is the value of the ratio.
- The unit price is then the value 0.30, to which we attach the units “dollars per apple.” The unit price can be written in any of the forms below.

$0.30 \frac{\text{dollars}}{\text{apple}}$ 0.30 dollars per apple \$0.30 per apple

Metric Measurements	
Common metric units	**Examples (sizes approximate)**
Length	
1 millimeter (mm)	the thickness of a dime
1 centimeter (cm)	the width of a small finger
1 meter (m)	the length of a baseball bat
1 kilometer (km)	the length of 9 football fields
Capacity / Volume	
1 milliliter (mL)	an eyedropper
1 liter (L)	a juice carton
1 kiloliter (kL)	four filled bathtubs
Mass / Weight	
1 milligram (mg)	a grain of sand
1 gram (g)	a paperclip
1 kilogram (kg)	a textbook

U.S. Customary Measurements	
Common customary units	**Examples (sizes approximate)**
Length	
1 inch (in)	the length of a small paperclip
1 foot (ft)	the length of a sheet of notebook paper
1 yard (yd)	the width of a door
1 mile (mi)	the length of 15 football fields
Capacity / Volume	
1 fluid ounce (fl oz)	a serving of honey
1 cup (c)	a small cup of coffee
1 pint (pt)	a bowl of soup
1 quart (qt)	an engine oil container
1 gallon (gal)	a jug of milk
Mass / Weight	
1 ounce (oz)	a slice of bread
1 pound (lb)	a soccer ball
1 bushel (bsh)	a block of hay
1 ton (T)	a walrus

Conversion Statements

Length	Capacity / Volume	Mass / Weight
1 foot = 12 inches	1 cup = 8 fluid ounces	1 pound = 16 ounces
1 yard = 3 feet	1 pint = 2 cups	1 bushel = 60 pounds
1 mile = 5,280 feet	1 quart = 4 cups	1 ton = 2,000 pounds
1 kilometer = 1,000 meters	1 gallon = 4 quarts	1 kilogram = 1,000 grams
1 meter = 100 centimeter	1 liter ≈ 1.06 quarts	1 kilogram ≈ 2.2 pounds
1 centimeter ≈ 0.4 inches		
1 meter ≈ 39 inches		
1 kilometer ≈ 0.6 mile		
Area		
1 acre = 43,560 square feet		

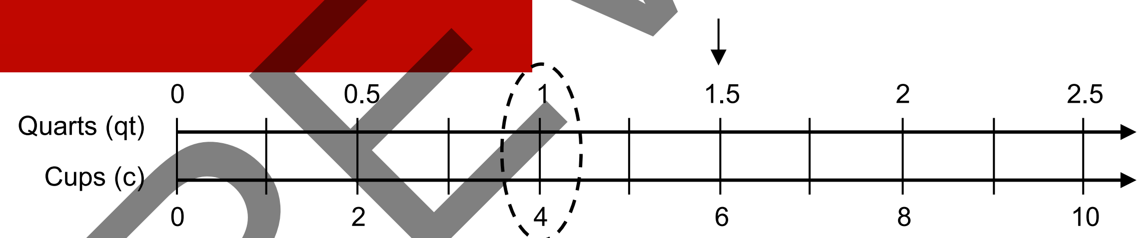
Conversions

Double number lines can be used to organize measurement conversion calculations.

How many cups are in 1.5 quarts?

Create a double number line that shows 1 quart = 4 cups.

Then fill in other numbers on the line to answer the question.



There are 6 cups in 1.5 quarts.

Information from a double number line may also be organized into a table.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
6.RP.2	Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with *b* ≠0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or ~~equations~~:
a.	Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, ~~and plot the pairs of values on the coordinate plane~~. Use tables to compare ratios.
b.	Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
d.	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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