Name _____ Period _____ Date _____

UNIT 2
STUDENT PACKET

Math Links
GRADE 6

FACTORS AND MULTIPLES

Parent (or Guardian) signature _____

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

composite number	factor
greatest common factor	least common multiple lowest common denominator
multiple	prime number
relatively prime	square number

THE LOCKER PROBLEM

Follow your teacher's directions.

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

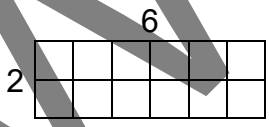
GREATEST COMMON FACTOR

We will learn how to find the greatest common factor (GCF) of two natural numbers and use the GCF to simplify fractions and solve problems. We will explore factors, primes, and composite numbers.

[6.NS.4; SMP3, 6, 7]

GETTING STARTED

Let each small square represent one square unit.



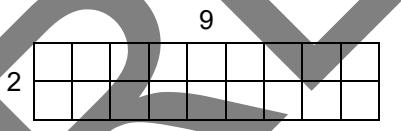
1. How many rows of 6 square units are in this rectangle? _____
2. How many columns of 2 square units are in this rectangle? _____
3. How many square units are in this rectangle? _____

For a rectangle's dimensions, we sometimes read $\times$ as "by." The above rectangle is "2 by 6" or "6 by 2." It does not matter which dimension is called "length" and which is called "width." Both of these terms describe the length of a side.

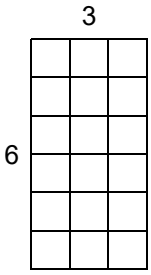
4. Write a multiplication sentence that relates length and width to the area of this rectangle.

5. Write multiplication sentences suggested by the rectangles.

a.



b.



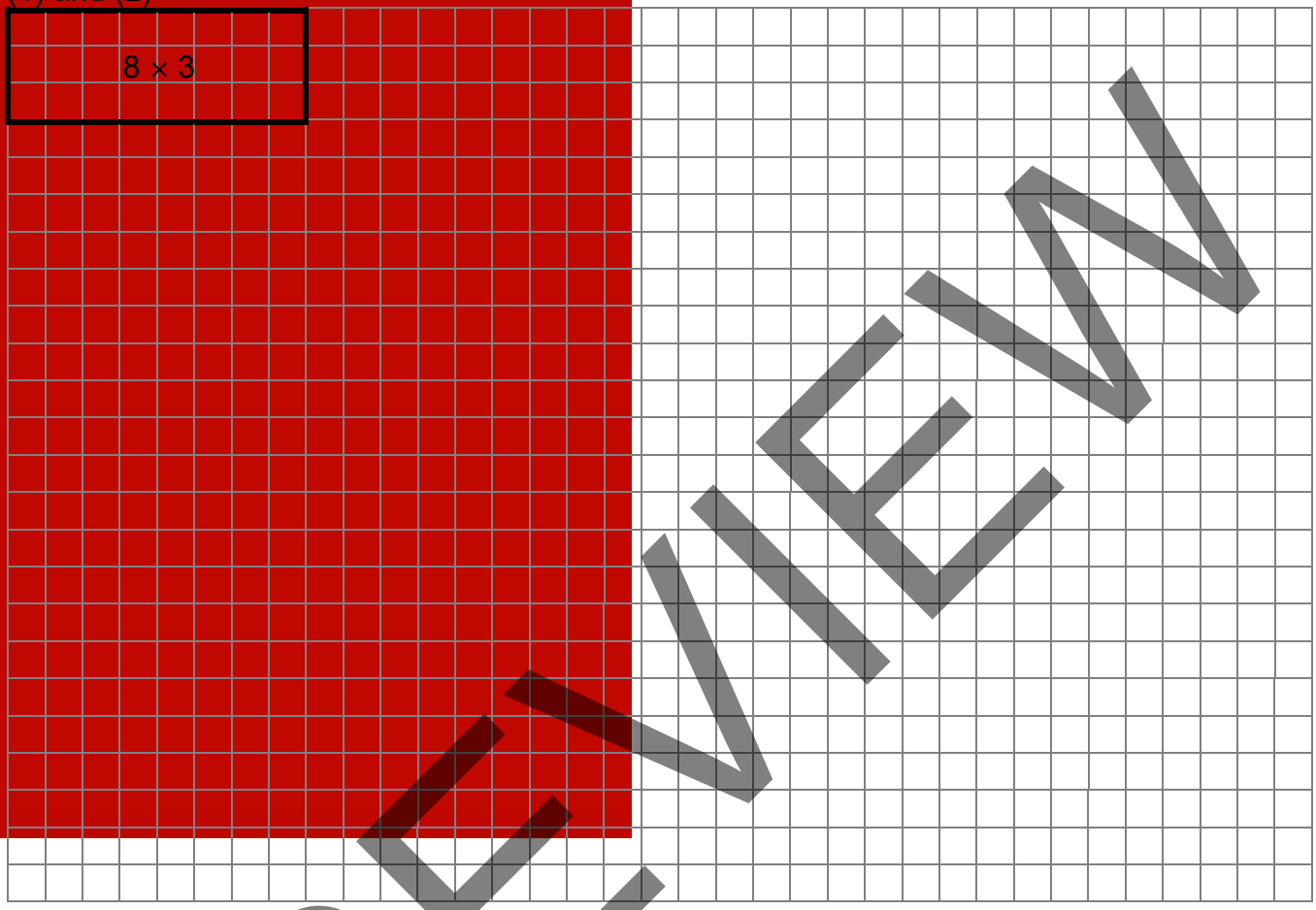
6. What is the same and what is different about the two rectangles in problem 5?

7. Record the meanings of factor and multiple in **My Word Bank**.

BUILDING RECTANGLES

Follow your teacher's directions for (1) – (4).

(1) and (2)



(3)

Area in sq units	Dimensions
24	
12	
5	
9	

(4)

Number	Factor Pairs	Number of Unique Factors

5. Record the meanings of prime number, composite number, and square number in **My Word Bank**.

PRACTICE 1

1. Without making any drawings, how many rectangles with natural number side lengths could be drawn with 20 square units? _____ List their dimensions in pairs to justify your answer.

2. A natural number that has exactly two unique factors, like 5 or 7, is called a _____ number. Write the next number of this kind after 7.

3. A natural number that has more than two factors, like 4 or 6 is called a _____ number. Write the next number of this kind after 6.

4. List all factors of each number, then classify each as prime, composite, or neither. Explain how you know.

a. 12	b. 7
c. 25	d. 1

5. List all of the prime numbers less than 20.

6. List all of the composite numbers less than 20.

7. Explain why 2 is the ONLY even prime number.

8. Explain why 25 is called a square number, while 7 and 12 are not?

9. List any three square numbers greater than 25.

FACTOR GAME

1. Read the directions carefully and play the game with a partner multiple times.
2. Do a first move analysis.
   a. Suppose you are starting a round of the Factor Game and you are about to make the first move. For each first number, list the opponent's factors and score.

First Number Picked	Opponent Gets (list the factors)	Sum
16		
17		
20		
24		
25		
29		

   b. Put a star next to the numbers above that are good first moves. Explain how a number is a good first move.

   c. Name a good first move that is not in the table above and show how you know.

   d. Name a bad first move that is not in the table and show how you know.

   e. What is the *worst* first move on your game board? _____ Explain.

   f. What is the *best* first move on your game board? _____ Explain.

FINDING THE GREATEST COMMON FACTOR

Follow your teacher's directions for (1) – (3).

(1)	(2)
Greatest Common Factor is _____	GCF: _____
(3)	

Use the process described above to find the GCF for each pair of numbers.

4. 20 and 15	5. 14 and 42
6. 3 and 8	7. 50 and 75

8. Megan has two pieces of cloth. One piece is 45 inches wide. The other is 60 inches wide. She wants to cut strips of equal width that are as wide as possible with no fabric left over. How wide should she cut the strips?

9. Record the meanings of greatest common factor and relatively prime in **My Word Bank**.

FRACTIONS IN "SIMPLEST FORM"

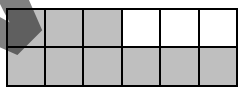
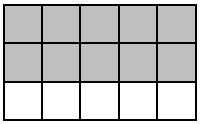
Follow your teacher's directions for (1).

(1)		

Use the GCF in a Big 1 calculation to rewrite each fraction in simplest form.

2. $\frac{12}{15}$	3. $\frac{24}{16}$	4. $\frac{35}{40}$

5. Four students have gardens of different sizes. Below are drawings of the gardens, where each small square represents one square yard. The shaded portions represent the part of each garden that is planted. Complete the table.

Student	Jamie	Li	Kertis
Gardens			
Fraction of each garden that is planted			

6. Write several different, equivalent fractions for each student's garden and clearly show who has the greatest fractional part planted.

Jamie	Li	Kertis

PRACTICE 2

Find the GCF of each pair of numbers.

1. 30 and 70	2. 27 and 17	3. 54 and 39

4. Write a pair of numbers that has 6 as their GCF.

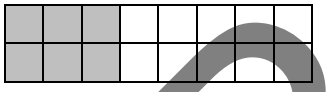
5. Write a pair of composite numbers that are relatively prime.

Simplify each fraction. Use the GCF and show the Big 1 calculation.

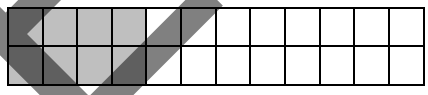
6. $\frac{60}{80}$	7. $\frac{18}{42}$	8. $\frac{22}{77}$

9. Write the fractional amount shaded for each rectangle. Then write several equivalent fractions for each. Clearly show which has the greatest fractional amount shaded.

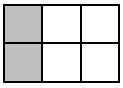
Rectangle A



Rectangle B



Rectangle C



LEAST COMMON MULTIPLE

We will review how to find multiples of natural numbers. We will learn how to find the least common multiple (LCM) of two numbers and apply the LCM to denominators to add and subtract fractions.
[6.NS.4; SMP6, 7, 8]

GETTING STARTED

1. Complete the multiplication chart.

1	2	3				7	8	9		11	
	4	6	8	10	12				20		24
	6	9			18	21	24		30		36
4	8				24	28	32		40		48
5	10	15	20	25	30				50		60
6	12	18	24	30	36				60		
	14			35					70		
	16	24							80		96
			36	45							108
10	20	30	40	50	60	70	80				
11					66		88	99			
12	24					84					

2. Explain how to use this table to find 12×7.

3. Explain how to use this table to find $63 \div 9$.

PRODUCT GAME

1. Read the directions carefully and play the game with a partner several times.
2. Which factors are needed to obtain a product of 35?

3. Which products can be obtained in more than one way? List all of them and show the ways.

4. Why is there no 11 on the game board?

5. Miguel wants to make two different 3×3 game boards using the factors given below. Create both game boards with all products, if possible. If one or both boards are not possible, explain why.

1 2 3 4

4 5 8 10

6. Find the factors used to make the Product Game boards below. List the factors in the spaces provided below each game board and fill in the missing products.

4	6	9
12		27
36	54	81

4	6	8	9
12	16		24
27		36	48
54	64	72	81

FINDING THE LEAST COMMON MULTIPLE

Follow your teacher's directions for (1) – (5).

(1)
(2)
(3)
(4)
(5)

6. Record the meanings of multiple and least common multiple in **My Word Bank**.

PRACTICE 3

1. List the first ten multiples of 4: _____
2. List the first ten multiples of 6: _____
3. Circle all the multiples that 4 and 6 have in common.
4. The least common multiple (LCM) of 4 and 6 is _____.
5. Describe in your own words how to find the LCM of 4 and 6.

List several multiples of each number to find the LCM of each pair of numbers.

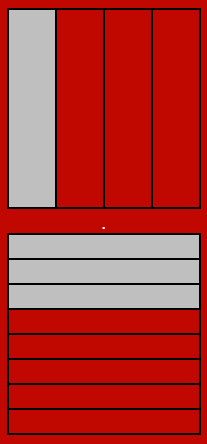
6. 8 and 6	7. 3 and 7
8. 8 and 10	9. 6 and 5

10. The drama club meets in the school auditorium every 6 days and the choir meets there every 9 days. If the groups are both meeting in the auditorium today, how many days from now will be the next day that they have to share the auditorium?

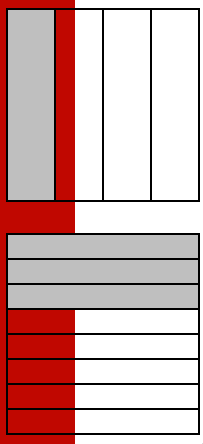
LCM AND FRACTIONS

Follow your teacher's directions for (1) – (6).

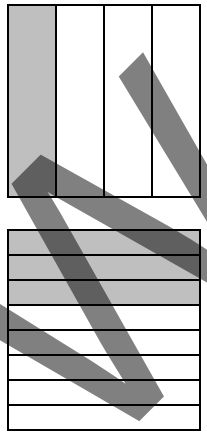
(1)



(2)



(3)



(4) Fractions:

LCD:

Add:

Subtract:

(5) Fractions

LCD:

Add:

Subtract:

(6) Fractions

LCD:

Add:

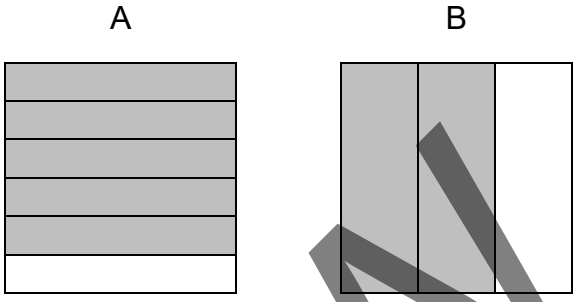
Subtract:

7. Record the meaning of lowest common denominator (LCD) in **My Word Bank**.

PRACTICE 4

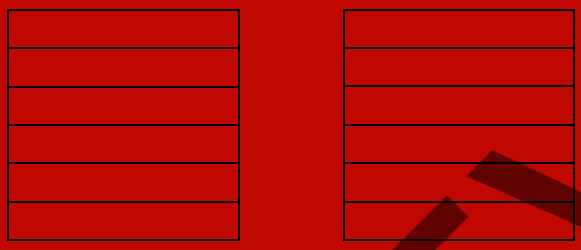
1. Write the fractional amount shaded for each whole in the figures to the right.

   A: B:

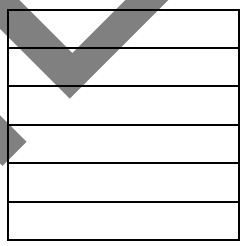


2. Split the parts of figure B so that the fractional pieces of both figures are the same size. They need not be the same shape.

3. Draw a picture of the sum of the shaded parts of pictures A plus B.



4. Draw a picture of the difference of the shaded parts of pictures A minus B.



5. Write each fraction with common denominators.

   A: B:

6. Write an addition equation for A + B.

7. Write a subtraction equation for A – B.

PRACTICE 5

Compute. Show work using the LCD.

1. $\frac{1}{4}+\frac{5}{12}$	2. $\frac{1}{2}+\frac{2}{3}$
3. $\frac{5}{6}-\frac{2}{3}$	4. $\frac{9}{4}+\frac{5}{6}$
5. $1\frac{3}{4}+\frac{7}{8}$	6. $3\frac{1}{2}-\frac{3}{4}$

7. Answer the following without mentioning common denominators.
   Barkley thinks that $\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$.

   a. What mistake did he make?

   b. Why does the sum not make sense, given the values of the two addends?

GCF AND LCM EXTENDED

We will explore an alternative method for finding the GCF and LCM of numbers. We will use GCF and LCM to solve problems.

[6.NS.4; SMP2, 3, 8]

GETTING STARTED

Find the GCF and LCM of the following pairs of numbers.

	GCF (make a list of all factors of both numbers)	**LCM** (make a list several multiples of both numbers)
1. 10, 25		
2. 24, 32		

3. What are the pros and cons of listing all the factors for finding the GCF and listing many multiples for finding the LCM?

USING FACTOR LADDERS FOR GCF AND LCM

Follow your teacher's directions. Factorizations using the "factor ladder" may vary. Sample steps:

(1)	(2)
(3)	

Find the GCF and LCM using the ladder method.

4.	5.
10 25	24 32

6. Compare the make a list methods for finding GCF and LCM in **Getting Started** to the ladder method for problems 4 and 5 above. Which do you prefer? Why?

PRACTICE 6

Before solving, circle whether the problem is solved by using the GCF or LCM.

Problem	circle one	Solve (use any method)
1. Sadie has 16 apple slices and 24 grapes. If she wants to make identical snack packs using all of the fruit, what is the greatest number of packs she can make? How many of each type of fruit is in those packs?	GCF LCM	
2. Karen exercises at Jim's Gym every 4 days and David exercises at Jim's Gym every 10 days. Karen and David both exercised today. How many days from today will be the next day they exercise on the same day?	GCF LCM	
3. The drama club meets in the school auditorium after school every 6 days, and the choir meets there after school every 2 days. If the groups are both meeting in the auditorium today, then how many days from now will they have to share the auditorium again?	GCF LCM	
4. A family is preparing backpacks filled with school supplies to donate to children in need. They have 15 pencils and 20 notebooks. If they want to make all the backpacks the same, with no school supplies left over, what is the greatest number of backpacks they can fill? What will each contain?	GCF LCM	
5. Mrs. Stern has 120 markers and 30 pieces of paper to give to her students. What is the greatest number of students she can have in her class so that each student gets equal numbers of markers and equal number of pieces of paper? What supplies will each student get?	GCF LCM	

REVIEW

POSTER PROBLEMS: FACTORS AND MULTIPLES

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
36 and 54	40 and 60	48 and 64	80 and 100

A. Copy the two numbers onto your poster. Write ALL factors of the first number only. Write the FIRST SIX multiples of the first number only.

B. Write ALL the factors of the second number only. Write the FIRST SIX multiples of the second number only.

C. Use the lists of factors to find the GCF and the lists of multiples to find the LCM.

D. Check the GCF and the LCM using a factor ladder.

Part 3: Return to your seats. Work with your group and show all work.

Mystery numbers

- There are two even, two-digit numbers.
- They have GCF = 6
- They have LCM = 90
- What are these numbers?

THE LOCKER PROBLEM REVISITED

At Centennial Middle School, there are 100 students and 100 lockers. They are numbered in order from 1 to 100. All the lockers start out closed.

- Student 1 enters the building and opens every locker.
- Student 2 enters the building and closes every even numbered locker.
- Student 3 enters and changes the position of every third locker, closing those that are open and opening those that are closed.
- This pattern continues with all 100 students.

For 1-8, explain using mathematical concepts learned in this unit. Use vocabulary from **My Word Bank** in your explanations and underline it.

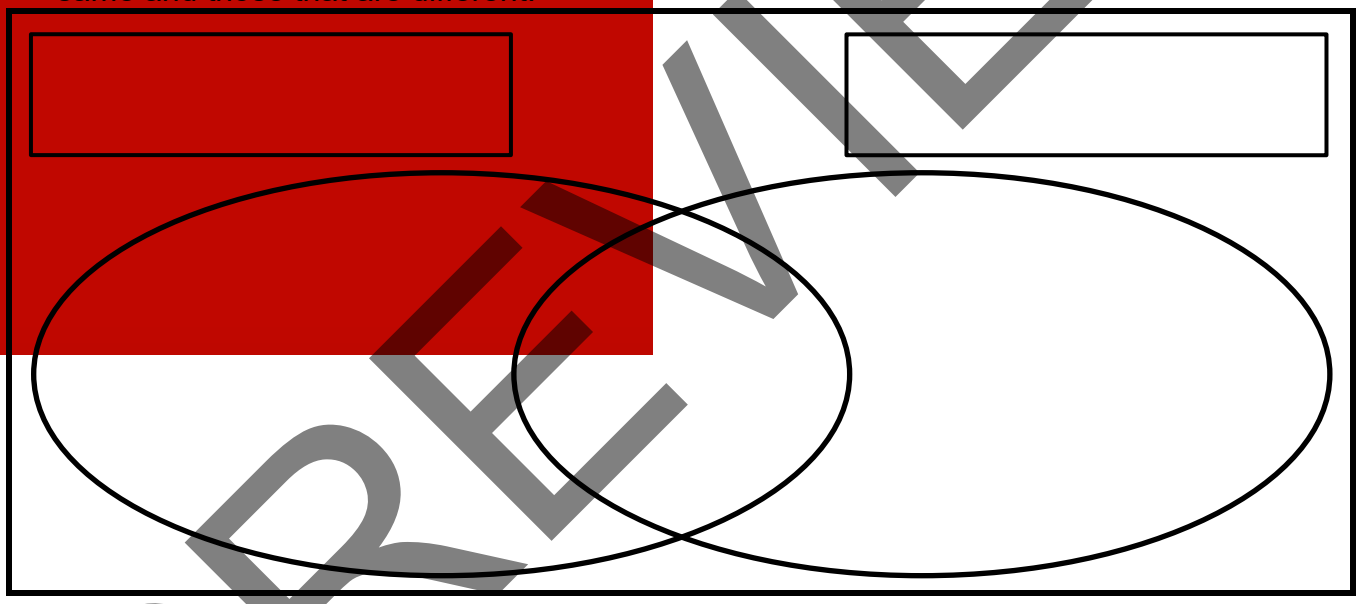
1. Which students change lockers 5, 17, and 31? What kind of numbers are these?	2. Which students change lockers 24 and 30? What kind of numbers are these?
3. Which students change both lockers 24 and 30? Explain the relationship between student numbers and locker numbers.	4. Who was the last student to change both lockers 24 and 30? Explain the relationship.
5. Which lockers are changed by students 18 and 36? Describe these numbers.	6. What is the first locker changed by both students 18 and 36? Explain relationship.
7. Which students change lockers 25 and 36? What kind of numbers are these?	8. Which lockers are open after all 100 students pass through? Explain.

MATCH AND COMPARE SORT: FACTORS AND MULTIPLES

1. Individually, match your word cards to your description cards, discuss with your partner(s), and record all of your results in the table.

Card set △			Card set ◯		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I			**I**		
II			**II**		
III			**III**		
IV			**IV**		

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.

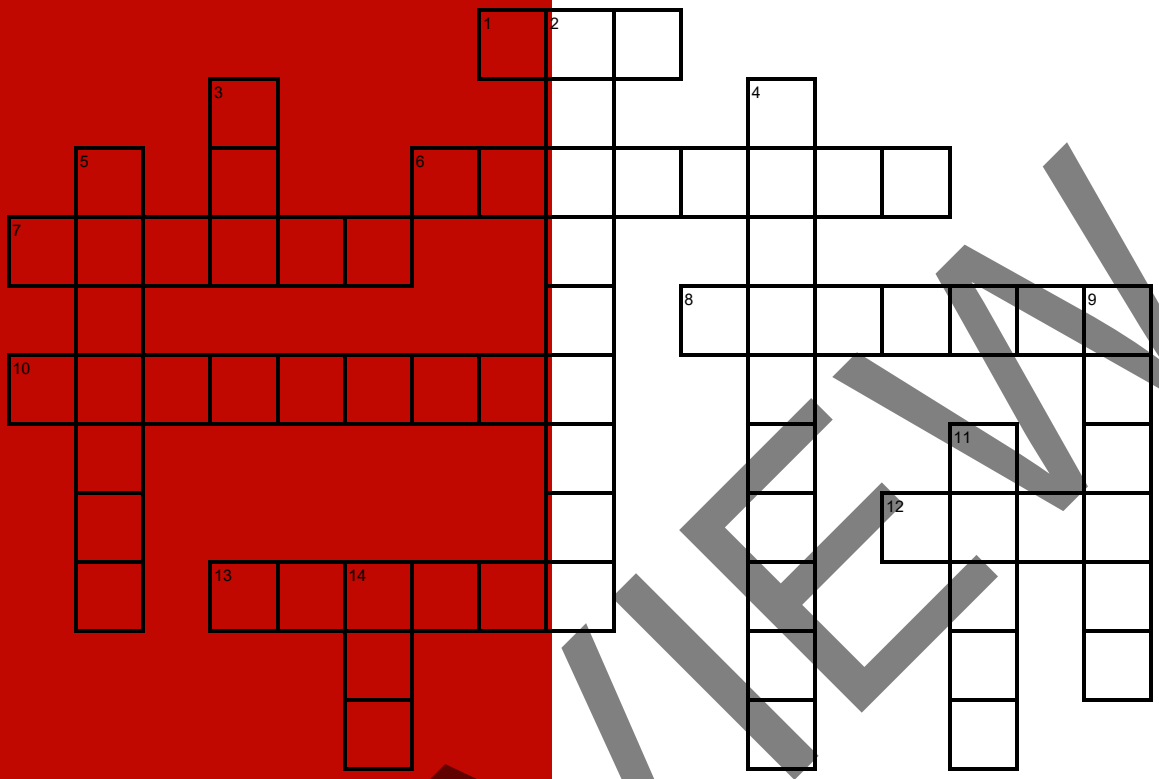


WHY DOESN'T IT BELONG?: FACTORS AND MULTIPLES

Choose one of the numbers below and explain why it doesn't belong with the others. Then choose one more and explain why it doesn't belong.

A	B
2	11
C	**D**
16	27

VOCABULARY REVIEW



Across

1 The ___ of 4 and 12 is 12. (abbreviation)

6 form of a fraction whose numerator and denominator are relatively prime

7 a shortcut method for finding LCM and GCF

8 1, 2, 3, 4, and 6 are ___ of 24.

10 14 and 28 are two ___ of 7.

12 product of the length and width of a rectangle

13 a nickname for the multiplication property of 1 in fraction form (two words)

Down

2 a ___ number with more than two factors

3 the least common multiple of denominators (abbreviation)

4 The GCF of 4 and 9 is 1, so 4 and 9 are ___ prime.

5 another name for counting numbers

9 The first five ___ numbers are 1, 4, 9, 16, 25.

11 a numbers whose only factors are itself and 1

14 The ___ of 4 and 12 is 4 (abbreviation).

SPIRAL REVIEW

1. Start **Computational Fluency Challenge**: This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

   a. Start with 8. Multiply by 3. Multiply the result by 5. Multiply the result by 6. Multiply the result by 8. Now you have a "big number." My big number is _____.

   b. Start with your big number. Divide it by 2. Divide the result by 4. Divide the result by 5. Divide the result by 18. What is the final result? _____.

2. Four friends had lunch together. The different lunches cost $12.50, $11.80, $14.32, and $12.86. They decided to split the total equally between the four of them.

   a. Write a numerical expression for the amount each person will pay?

   b. Simplify the expression.

SPIRAL REVIEW
Continued

3. Find each equivalent fraction or decimal in the table below.

Fraction	$\frac{2}{5}$			$\frac{6}{5}$		$\frac{7}{10}$	
Decimal		0.8	0.15		0.05		0.45

4. Mason is throwing a birthday party for Ramsay. There is a total of 12 people coming to the party. For each person, it will cost $8.95 for lunch and $4.32 for dessert from Connor's Catering Company. What is the total amount Mason will have to spend, if he also tips the catering company $35?

   a. Write a numerical expression for the total cost.

   b. Simplify the expression.

SPIRAL REVIEW
Continued

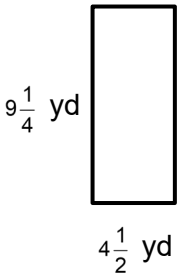
5. Camille's mom started a college account. Her mom started with $1,000 in the account on Camille's 12th birthday. Each year after, she placed $450 in the account on each birthday up to her 18th birthday. As a bonus, Camille's grandfather added $6,830 into her account on her 17th birthday. On her 18th birthday, Camille took out $368 to spend on college applications.

   a. Write an expression for the amount of money she will have for college.

   b. Camille plans to go to college for 4 years. How much money will she have for each year, if split evenly?

6. Graham is building an area in his yard for his dog. It is sketched below. He needs to cover the ground with sod (grass). Sod costs $2.25 per square foot. He also wants to put a fence around the area, which costs $10.35 per yard.

   a. How many square feet are in a square yard? (Hint: It is not 3.)



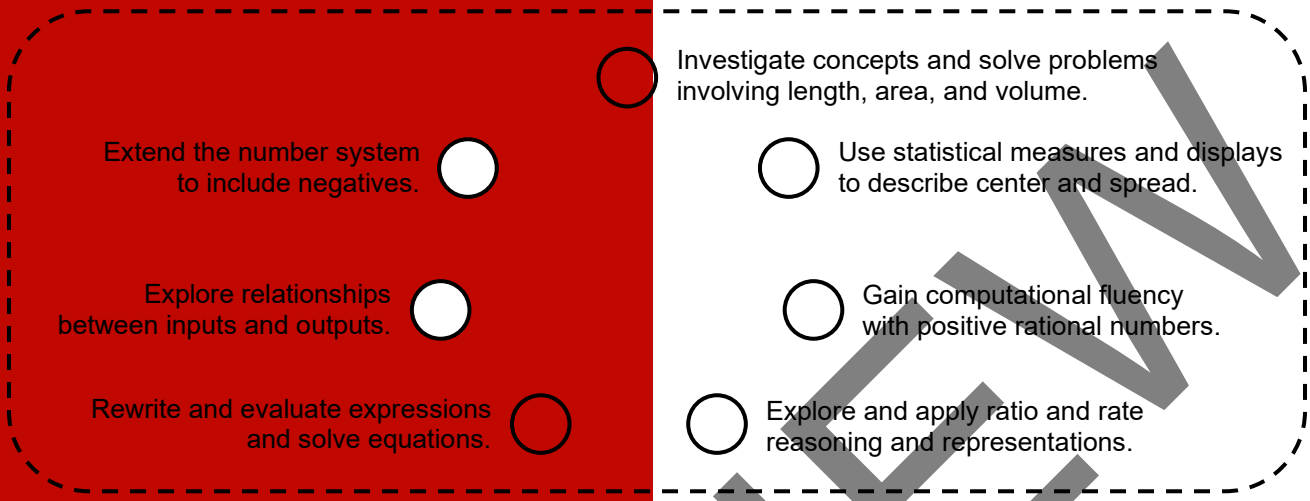
   b. Write a numerical expression for the cost of the sod.

   c. Write a numerical expression for the cost of the fence.

   d. Compute the cost of the sod and fence together.

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this unit. Draw lines to show connections that you noticed.



   Give an example from this unit of one of the connections above.

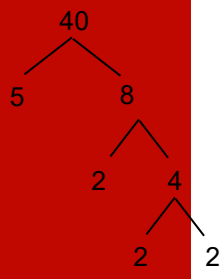
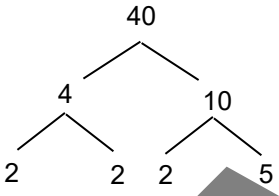
2. **Unit Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Explain how analyzed situations by breaking them into cases [SMP7, 8]. Then circle one more SMP on the back of this packet that you think was addressed in this unit and be prepared to share an example.

4. **More Connections.** Explain a procedure that is more efficient when you know the LCM or GCF, and give an example.

STUDENT RESOURCES

Word or Phrase	Definition
area	The area of a two-dimensional figure is a measure of the size of the figure, expressed in square units. The area of a rectangle is the product of its length and its width. If a rectangle has a length of 12 inches and a width of 5 inches, its area is (5)(12) = 60 square inches. Area = Length × Width = $L \bullet W$ 
composite number	A number is composite if it has more than two divisors or factors. 12 has six factors: 1, 2, 3, 4, 6, 12, because $12 = 1 \bullet 12$, $12 = 2 \bullet 6$, and $12 = 3 \bullet 4$. Since 12 has more than two factors, 12 is a composite number.
factor (of a number)	A factor of a number is a divisor of the number. The factors of 12 are 1, 2, 3, 4, 6, and 12.
greatest common factor	The greatest common factor (GCF) of two numbers is the greatest factor that divides the two numbers. The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 18 are 1, 2, 3, 6, 9, and 18. Therefore, the GCF of 12 and 18 is 6.
least common multiple	The least common multiple (LCM) of two numbers is the least number that is a multiple of both numbers. The multiples of 8 are 8, 16, 24, 32, 40, ... The multiples of 12 are 12, 24, 36, 48, … . Therefore, the LCM of 8 and 12 is 24.
lowest common denominator	The lowest common denominator of two fractions is the least common multiple of their denominators. The lowest common denominator of $\frac{3}{8}$ and $\frac{5}{12}$ is 24.
multiple (of a number)	A multiple of a number m is a number of the form $k \bullet m$ for any integer k. The numbers 5, 10, 15, and 20 are multiples of 5, since $1 \bullet 5 = 5$, $2 \bullet 5 = 10$, $3 \bullet 5 = 15$, and $4 \bullet 5 = 20$.
natural number	The natural numbers are the numbers 1, 2, 3, 4, ….

Word or Phrase	Definition
prime factorization	The prime factorization of a number is an expression of that number as a product of primes. There is a unique way to express any number as a product of primes, except for order.   $40 = 5 \bullet 2 \bullet 2 \bullet 2$ and $40 = 2 \bullet 2 \bullet 2 \bullet 5$ The two prime factorization trees above illustrate that even though the order of the prime factors is different, the products are the same.
prime number	A prime number is a natural number that has exactly two factors, namely 1 and itself. The first six prime numbers are 2, 3, 5, 7, 11, and 13. 1 is *not* a prime number. It has exactly one factor.
relatively prime	Two numbers are relatively prime if their greatest common factor is 1. The factors of 6 are 1, 2, 3, and 6. The factors of 11 are 1 and 11. Since the greatest common factor of 6 and 11 is 1, the two numbers are relatively prime.
square number	A square number, or perfect square, is a number that is a square of a natural number. The area of a square with natural number side-length is a square number. The square numbers are $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$, … .

Symbols for Multiplication

The product of 8 and 4 can be written as:

8 times 4 8×4 $8 \bullet 4$ $(8)(4)$

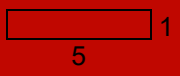
$$\begin{array}{r} 8 \\ \times\ 4 \\ \hline \end{array}$$

In algebra, we generally avoid using the × for multiplication because it could be misinterpreted as the variable *x*, and we cautiously use the symbol • for multiplication because it could be misinterpreted as a decimal point.

Using Rectangles to Visualize Prime and Composite Numbers

Building rectangles whose sides have natural number lengths is a geometric way to describe factors and multiples of numbers. If the area of the rectangle represents the product, then the side lengths of the rectangle represent the factors of the number.

A prime number p corresponds to only one rectangle, since p can be factored as a product in only one way, $p = 1 \bullet p$. (Here we regard the factorization $p = 1 \bullet p$ as the same as $p = p \bullet 1$, and we regard a $1 \times p$ rectangle as being the same as a $p \times 1$ rectangle.)



$5 = 1 \times 5$

1 and 5 are factors of 5.

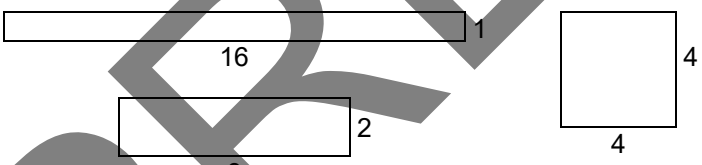
A composite number n always corresponds to more than one rectangle.

$14 = 1 \times 14$

$14 = 2 \times 7$

1, 2, 7, and 14 factors of 14.

A number such as 16 is called a square number (or perfect square) because one of the rectangles it corresponds to is a square (4×4).



$16 = 1 \times 16$

$16 = 2 \times 8$

$16 = 4 \times 4$

1, 2, 4, 8, and 16 factors of 16.

Greatest Common Factor (GCF)

The greatest common factor (GCF) of two numbers is the greatest factor that divides the two numbers. Here are two ways to find the GCF of two numbers.

Tensaye has 12 bottles of water and 18 granola bars. She wants to use all of the bars and bottles to make care packages for the homeless. How many care packages can Tensaye make so that there are the same number of bottles of water and granola bars in each package?

Method 1: Use a list to find the GCF of 12 and 18:

List all the factors of 12: 1, 2, 3, 4, 6, and 12.
List all the factors of 18: 1, 2, 3, 6, 9, and 18.

We can see that the factors 1, 2, 3, and 6 appear in both lists. Since 6 is the greatest factor from both lists that divides 12 and 18, the greatest common factor (GCF) of 12 and 18 is 6.

Method 2: Use prime factorization to identify the GCF of 12 and 18.

$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$

We see that 2 and 3 are common factors of both numbers. Therefore $2 \times 3 = 6$ is the GCF.

Since the GCF of 12 and 18 is 6, Tensaye can make 6 care packages for the homeless, and each care package will contain 2 bottles of water and 3 granola bars.

Least Common Multiple (LCM)

The least common multiple (LCM) of two numbers is the least number that is a positive multiple of both numbers. Here is one way to find the LCM of two numbers.

Tensaye wants buy bottles of water and granola bars to make care packages for the homeless. Bottles of water come in packages of 12, and granola bars are sold in packages of 18. How many bottles of water and how many granola bars should Tensaye buy so that she has the same number of each item? Note; She can only afford to buy the smallest amount to make this happen.

Use a list to find the LCM of 12 and 18:

The multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120,… .

The multiples of 18 are: 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, ...

The multiples that 12 and 18 have in common are 36, 72, 108, ... We can see that 36 is the least multiple the two numbers have in common. Therefore, the LCM of 12 and 18 is 36.

Since the LCM of 12 and 18 is 36, Tensaye should buy 36 bottles (or 3 packages) of water and 36 granola bars (or 2 packages) so that she has the same number of each item.

The "Big One"

The "Big 1" is a notation for 1 (multiplicative identity) in the form of a fraction $\frac{n}{n}$ $(n \neq 0)$.

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \ldots$$

We can use the following picture to help remind us that these fractions are equivalent to 1:

$$1 = \boxed{\frac{8}{8}}$$

The Big 1 can be used to help find equivalent fractions. For example,

$$\frac{2}{5} \times \boxed{\frac{10}{10}} = \frac{20}{50} \quad \text{or} \quad \frac{20}{50} \div \boxed{\frac{10}{10}} = \frac{2}{5}.$$

Diagrams that Show Equivalent Fractions

These diagrams illustrate that $\frac{1}{2} = \frac{4}{8}$. In the second diagram, each half is split into four parts, but the size of the whole does not change, nor does the amount shaded.

Using the Big 1, this equivalence can be written:

$$\frac{1}{2} \bullet \boxed{\frac{4}{4}} = \frac{4}{8} \quad \text{or} \quad \frac{4}{8} \div \boxed{\frac{4}{4}} = \frac{1}{2}$$

Fractions in "Simplest Form" with the GCF

To write a fraction in its simplest form, divide the numerator and denominator by the greatest common factor. Though it is not required to use the GCF, doing so is the most efficient way, because it only takes one step. Use the Big 1 when dividing.

To simplify $\frac{12}{30}$, first use any method to determine that the GCF of 12 and 30 is 6. Then divide the numerator and denominator by 6, in the form of the Big 1.

$$\frac{12}{30} \div \boxed{\frac{6}{6}} = \frac{2}{5}$$

Renaming Fractions with their Lowest Common Denominator (LCD)

To rename fractions with their LCD, first find the least common multiple (LCM) of the denominators. Then change each fraction to an equivalent fraction by multiplying each of them by the appropriate forms of the Big 1.

To write two fractions, $\frac{3}{4}$ and $\frac{5}{6}$, with their LCD, first find the LCM of the denominators. After using any method to determine that the LCD of 4 and 6 is 12, rename the fractions so that they both have a denominator of 12 using the Big 1.

$$\frac{3}{4} \times \boxed{\frac{3}{3}} = \frac{9}{12} \qquad \frac{5}{6} \times \boxed{\frac{2}{2}} = \frac{10}{12}$$

This computation results in lesser numerators and denominators in the fractions because 12 is the least mutiple that 4 and 6 share in common. In other words, 12 is the LCM of 4 and 6, or the LCD of the fractions.

Using "Factor Ladders" to Find the GCF and LCM of Two Numbers

Factor ladders are a useful tool for finding the GCF and LCM of two numbers.

Use repeated division to find the GCF and LCM of 12 and 18:

Divide each number by any common factor greater than 1. In this case we have choices, so let's begin by dividing both numbers by 2. The resulting quotients are 6 and 9.

Keep dividing until both resulting quotients have no factors in common greater than 1. In this case, we can still divide by 3. The resulting quotients are now 2 and 3, and they have no common factors greater than 1. They are relatively prime.

2	12	18
3	6	9
	2	3

The GCF is the product of the factors along the side. Therefore, the GCF of 12 and 18 is $2 \bullet 3 = 6$.

The LCM is the product of the factors along the side and the bottom. Therefore, the LCM of 12 and 18 is the GCF multiplied by 2 and 3, or $6 \bullet 2 \bullet 3 = 36$.

Fraction Addition with Diagrams

The standard procedure for adding fractions requires that the fractions have common denominators. An area model supports why this is reasonable.

Example 1: $\frac{1}{5} + \frac{3}{5}$

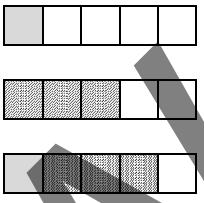
The fractions already have a common denominator.

$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$

$\frac{1}{5}$

$\frac{3}{5}$

$\frac{1}{5} + \frac{3}{5}$



Example 2: $\frac{1}{2} + \frac{1}{3}$

Find a common denominator.

$\frac{1}{2} + \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{2}{2}$

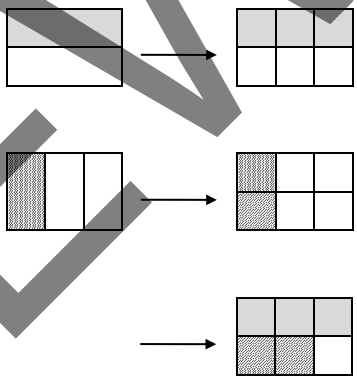
$= \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$

$\frac{1}{2} = \frac{3}{6}$

$\frac{1}{3} = \frac{2}{6}$

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$



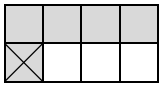
Fraction Subtraction with Diagrams

The standard procedure for subtracting fractions requires that the fractions have common denominators. An area model supports why this is reasonable.

Example 1: $\frac{5}{8} - \frac{1}{8}$

The fractions already have a common denominator.

$\frac{5}{8} - \frac{1}{8} = \frac{4}{8}$.



Example 2: $\frac{1}{2} - \frac{1}{3}$

Find a common denominator.

$\frac{1}{2} - \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{3} - \frac{1}{3} \cdot \frac{2}{2}$

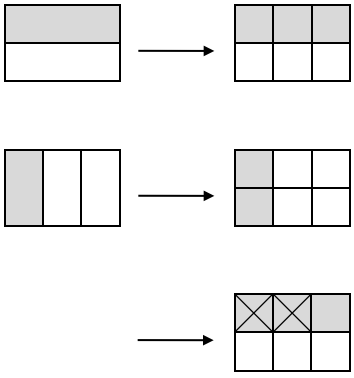
$= \frac{3}{6} - \frac{2}{6}$

$= \frac{1}{6}$

$\frac{1}{2} = \frac{3}{6}$

$\frac{1}{3} = \frac{2}{6}$

$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$



COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.NS.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. ~~Use the distributive property to express a sum of two whole numbers 1 – 100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9 + 2).*~~

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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