FOCUS, COHERENCE, AND RIGOR

A group of educators at Rutgers University led by Dr. Roberta Shorr (Young, 2009) conducted a seven-year study on problem solving with minority and low-income students in low performing schools. They found that giving conceptually challenging problems to students caused frustration, but at the same time students were engaged and motivated. Working through the frustration in an emotionally safe environment, students gained "satisfaction, pride, and a willingness to work harder next time." This work also resulted in higher standardized test scores (e.g., average scores for Newark fourth graders rose from 45 to 79 percent.

At the Center for Mathematics and Teaching, we know that all students have potential to achieve in mathematics, we believe that the development of mathematics should reflect the connectedness of Big Ideas into a coherent whole, and we aim to make mathematics inviting and inclusive to more students. One of the ways we put these beliefs and intentions into action is by attending to focus, coherence, and rigor in program development.

FOCUS

According to Achieve the Core (2014), "Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness."

Achieve the Core organizes mathematics clusters into three categories: major clusters, supporting clusters, and additional clusters. This table identifies the clusters for 8th grade and their alignment to *MathLinks* lessons.

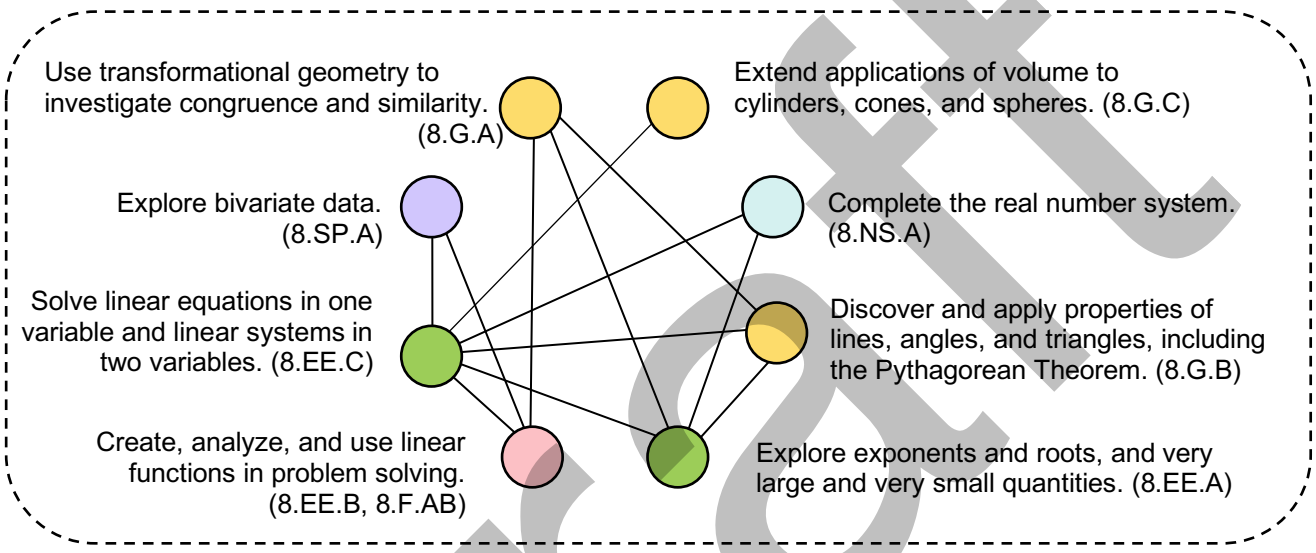
FOCUS IN *MATHLINKS* BASED ON MAJOR, MINOR, AND SUPPORTING CLUSTERS: GRADE 8

CLUSTER	Standards	Lesson Focus	Additional Lessons, Spiral Review Packets
Major Clusters			
• 8.EE.A Work with radicals and integer exponents	8.EE.1-4	3.1, 3.2, 3.3, 10.1, 10.2, 10.3	Spiral Rev: 2,4,6,8,10
• 8.EE.B Understand the connections between proportional relationships, lines, and linear equations.	8.EE.5-6	4.1, 4.3, 5.1, 5.2, 5.3, 10.3	Spiral Rev: 2,3,9
• 8.EE.C Analyze and solve linear equations and pairs of simultaneous linear equations	8.EE.7-8	7.1, 7.2, 7.3, 8.1, 8.2, 8.3	1,2,3,4,5,6,7,8,9,10
• 8.F.A Define, evaluate, and compare functions	8.F.1-3	4.1, 4.2, 4.3, 5.2, 5.3, 8.2, 8.3	7.1, 9.1, 10.3
• 8.F.B Use functions to model relationships between quantities.	8.F.4-5	4.1, 4.2, 4.3, 5.1, 5.2, 5.3, 6.2, 8.3	7.1 Spiral Rev: 8
• 8.G.A Understand congruence and similarity using physical models, transparencies, or geometry software.	8.G.1-5	1.3, 9.1, 9.2, 9.3	10.1 Spiral Rev: 8,9,10
• 8.G.B Understand and apply the Pythagorean theorem.	8.G.6-8	2.2, 10.3	9.1, 9.3, 10.1, 10.2,10.3 Spiral Rev: 4,7,8,9
Supporting Clusters			
• 8.NS.A Know that there are numbers that are not rational, and approximate them by rational numbers.	8.NS.1-2	2.1, 2.2, 2.3	Spiral Rev: 5,7,9
• 8.SP.A Investigate patterns of association in bivariate data.	8.SP.1-4	6.1, 6.2, 6.3	Spiral Rev: 8,9
Additional Clusters			
• 8.G.C Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	8.G.9	1.1, 1.2	Spiral Rev: 4,6,7,8

COHERENCE

According to a draft of the CA Math Framework (2022), "instructional materials should primarily involve tasks that invite students to make sense of big ideas, elicit wondering in authentic contexts, and necessitate mathematical investigation. Big ideas in mathematics are central to the learning of mathematics, link numerous mathematical understandings into a coherent whole, and provide focal points for students' investigations."

Inspired by the work of Jo Boaler (Boaler, Munson, Williams, What is Mathematical Beauty? Teaching through Big Ideas and Connections, youcubed.org, 2017) and based on the Content Domains, the *MathLinks* team synthesized the work of 8th grade into eight big ideas. These ideas and their connections within the entire program are shown here.



The work in *MathLinks* is organized into ten packets. An expanded dive into the specific big ideas and connections are included in the front section of each Teacher Edition packet. Towards the end of each Student Packet, students identify the big ideas within the packet and reflect upon how they are connected to each other.

RIGOR

According to the Common Core State Standards Initiative (2022), "a rigorous math program will pursue with equal intensity conceptual understanding, procedural skill and fluency, and applications." We first examine the MathLinks: Grade 8 program through this lens.

Conceptual Understanding in *MathLinks*

Conceptual understanding, the bedrock of a *MathLinks* course, frequently drives the other two components of rigor. It's a *MathLinks* philosophy to make sure all students have the opportunity to make meaning for every concept presented, and we aim to focus on the conceptual development of big ideas in depth, and make them plausible through investigations, activities, and practice. This is commonly done throughout lessons in Student Packets, oftentimes with the help of teacher Lesson Notes and Slide Decks (or Slide Deck Alternatives).

This table identifies some examples of concept development in *MathLinks:* Grade 8 Student Packets (lesson location in parentheses). Typically, these lessons include guided instruction with a slide deck, lesson notes, class discussions, or group work. Opportunities for independent work within a Student Packet appear as "Practice" pages within lessons, in the "Review" section as activities, and as spiral review in subsequent packets. Packet Resources on the Teacher Portal also contain problems, tasks, and projects to support conceptual development.

EXAMPLES OF CONCEPT DEVELOPMENT LESSONS IN *MATHLINKS*: GRADE 8	
CLUSTER DESCRIPTION	**CONCEPT DEVELOPMENT WITH TEACHER OR GROUP**
Major Clusters	
• 8.EE.A (EE 1-4) Work with radicals and integer exponents	(3.1) Investigating Two Exponent Patterns
• 8.EE.B (EE 5-6) Understand the connections between proportional relationships, lines, and linear equations.	(5.1) The Meaning of Slope (5.1) The Slope Formula
• 8.EE.C (EE 7-8) Analyze and solve linear equations and pairs of simultaneous linear equations	(7.1) 100-Mile Walking Challenge (7.2) Solving Equations with Balance 1, 2
• 8.F.A (F 1-3) Define, evaluate, and compare functions	(4.1) The Pool Problem
• 8.F.B (F 4-5) Use functions to model relationships between quantities.	(4.1) Saving vs. Spending
• 8.G.A (G 1-5) Understand congruence and similarity using physical models, transparencies, or geometry software.	(9.0) Slides, Turns, and Flips (10.0) A Rubber Band Experiment
• 8.G.B (G 6-8) Understand and apply the Pythagorean theorem.	(2.2) A Famous Theorem
Supporting and Additional Clusters	
• 8.NS.A (NS 1-2) Know that there are numbers that are not rational, and approximate them by rational numbers.	(2.1) A Radical Investigation (2.3) How Can 0.999… = 1?
• 8.SP.A (SP 1-4) Investigate patterns of association in bivariate data.	(6.1) Association and Causation
• 8.G.C (G 9) Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	(1.2) Volume of a Cone and Sphere

Applications in *MathLinks*

Problem solving is an important driver of instruction within *MathLinks* courses. In *MathLinks*, we aim to include engaging mathematical problems and applications with accessible entry points for all students, multiple approaches or solutions, and extensions to challenge and enrich. All Student Packets begin with an Opening Problem, which introduces a concept or establishes a "need to know." In many cases, students require more instruction throughout the packet before they are fully prepared to bring the problem to its conclusion. More "meaty" problems exist the packets as well.

This table identifies some examples of mathematical problems and applications in *MathLinks:* Grade 8 Student Packets (lesson location in parentheses). Some of these problems include guided instruction with a slide deck and lesson notes. Others are appropriate for independent work. Packet Resources on the Teacher Portal (e.g. Tasks, Projects) also contain mathematical problems and applications.

EXAMPLES OF PROBLEM SOLVING LESSONS IN *MATHLINKS*: GRADE 8	
CLUSTER DESCRIPTION	**MATHEMATICAL PROBLEMS AND APPLICATIONS**
Major Clusters	
• 8.EE.A (EE 1-4) Work with radicals and integer exponents	(2.0, 2.2) A Rectangle Paradox (2.2) The Club and the Box
• 8.EE.B (EE 5-6) Understand the connections between proportional relationships, lines, and linear equations.	(5.0, 5.3) The Rope Investigation, The Rope Revisited (5.3) Rectangle Paradox: A Fresh Look (7.1) Saving for a Skateboard
• 8.EE.C (EE 7-8) Analyze and solve linear equations and pairs of simultaneous linear equations	(8.3) Watering Cans
• 8.F.A (F 1-3) Define, evaluate, and compare functions	(4.1) The Pool Problem
• 8.F.B (F 4-5) Use functions to model relationships between quantities.	(7.1) 100-Mile Walking Challenge (8.3) Training for a Marathon
• 8.G.A (G 1-5) Understand congruence and similarity using physical models, transparencies, or geometry software.	(9.3) Swimming at the River
• 8.G.B (G 6-8) Understand and apply the Pythagorean theorem.	(2.0, 2.2) A Rectangle Paradox (2.2) The Club and the Box (10.3) A Mathematical Surprise
Supporting and Additional Clusters	
• 8.NS.A (NS 1-2) Know that there are numbers that are not rational, and approximate them by rational numbers.	(2.3) A Rational Numbers Investigation (3.2) What in the World?, Practice 7 (Question 10)
• 8.SP.A (SP 1-4) Investigate patterns of association in bivariate data.	(6.1, 6.2) Practice 2, Obesity Rates by State (6.3) Two-Way Tables
• 8.G.C (G 9) Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	(1.0, 1.1) Paper Solids, Practice 2 (1.1) A Coin Problem

Procedural Skill and Fluency in *MathLinks*

Procedural skill and fluency are more than just computational speed and accuracy. In a 2014 position paper, NCTM described procedural fluency as "the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another.

In *MathLinks*, we aim to thoughtfully develop new procedural skills and provide opportunities for students to gain fluency throughout the year. Skills practice in Student Packets is located in:

- Practice pages – These pages support concept development.
- Review activities – These pages often include skills practice.
- Alge-Grids – These puzzles, created by Carole Greenes and Tanner Wolfram, give students practice with expressions and exponents. They appear in half of the Spiral Reviews.
- READY-X – These puzzles, created by Carole Greenes and Tanner Wolfram, give students practice with one-and multi-variable equations. They appear in half the Spiral Reviews.
- Spiral Reviews – in Addition to Alge-Grids and READY-X, these pages include always include equation solving practice and distributed practice of prior skills.

While CCSS-M does not identify any specific standards for fluency in Grade 8, Achieve the Core's Instructional Materials Evaluation Tool (2021) suggests attention be given to two Standards: 8.EE.7 and 8.EE.8b. The table below shows some examples in 8th grade Student Packets where these procedural skills are developed and students have opportunities to gain fluency through independent practice.

EXAMPLES OF FLUENCY WORK IN *MATHLINKS*: GRADE 7		
Standard / Description	**Development of Procedure**	**Opportunities to gain fluency**
8.EE.7 Solve linear equations in one variable.	(7.2) Solving Equations Using Cups and Counters (7.3) Solving Equations Algebraically (8.1) Solving Equations Involving Rational Numbers	(8.3) Algebra Applications Spiral Rev: 1,2,3,4,5,6,7,8,9
8.EE.8b Solve systems of two linear equations in two variables, and estimate solutions by graphing the equations. Solve simple cases by inspection.	(7.1) Solving Linear Systems by Graphing (8.2) Solving Systems Using Algebra	Spiral Rev: 1,3,5,7,9,10

In addition to Student Packets, resources in the Teacher Portal offer opportunities to practice grade level skills and to fill in gaps where needed. Look in Packet Resources for:

- Essential Skills – this entire section reviews skills and concepts important for success in a given packet. Activity Routines such as Big Square Puzzles, Open Middle Problems, and Four-in-a-Row Games are also in these sections for some packets. They provide a practice alternative to "drill and kill."
- Extra Problems – Problems are available for every lesson.
- Nonroutine Problems – Some Big Square Puzzles, Open Middle Problems, and Four-in-a-Row Games are also located in this section for some packets.

Finally, for students who need to fill major gaps in skills, consider Skill Boosters, which are in the Teacher Portal. This Resource helps students catch up on below grade level work without losing extensive instructional time on grade level work. A Skill Booster routine is intended to take about 10 minutes per day.

Connecting the Three Aspects of Rigor

Interpretations of "rigor" are evolving. In 2022, the CA Framework defined rigor to mean that "conceptual understanding can be used to analyze a novel situation encountered in the world. Rigor means that students can flexibly apply methods to different situations, and connect mathematical ideas, approaches, and representations." A closer look at algebra in 8th grade illustrates how this is done.

The Algebra Progression in *MathLinks*: Grade 8

Algebra topics primarily appear in the CCSS-M Expressions and Equations and Functions domains. They are also in the Statistics and Probability domain. These areas are the focus of six packets in *MathLinks*: Grade 8.

- Packets 1 and 2 (**Applying Algebra to Geometry**; **Real Numbers and the Pythagorean Theorem**) apply 7th grade algebra to new 8th grade topics.
- In Packet 3, **The Algebra of Exponents and Roots**, students observe patterns in numerical expressions with exponents and generalize them to obtain the product, power, and quotient rules for exponents. They also use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number.

- In Packet 4, **Introduction to Functions**, students continue the work started in 6th and 7th grades with multiple representations by connecting real-life or visual contexts to tables, graphs, and algebraic equations. Functions are formally defined and students represent them in tables, graphs, and mapping diagrams. They interpret different characteristics of functions (e.g., increasing or decreasing, linear or nonlinear).

- In Packet 5, **Linear Functions**, slope is formally defined as students find slopes of lines by counting on a grid, and by using the slope formula, which then leads into using the slope-intercept form of a line, and applying this knowledge to solving various problems.

- In Packet 6, **Bivariate Data**, students graph bivariate numerical data as scatter plots, describe patterns of association (if they exist), estimate lines of best fit to points showing linear associations, write equations of these estimated lines from which predictions are made, and draw conclusions from the data.

- In Packet 7, **Linear Equations and Systems 1**, students solve linear systems of equations by graphing, noting that these systems have exactly one solution, no solutions, or infinitely many solutions. Estimating solutions creates the need for a more precise solution method. The process of substitution is introduced as a way to take a system of two equations in slope-intercept form and creating one equation in one variable. This creates a need to learn to solve equations with variables on both sides.

  Students then revisit the "cups and counters" model (introduced in Grade 7) for solving equations. The model aids the transition to formal procedures. Students see a parallel to the work done with systems, as linear equations in one variable may have one, infinite, or no solutions.

- In Packet 8, **Linear Equations and Systems 2**, students revisit the use of procedures to solve equations in one variable, though made harder by the introduction of non-integer values. With newly acquired equation-solving skills, students use the substitution method to solve systems algebraically. The elimination method is introduced as an alternative algebraic method, but not rigorously pursued, being left to high school mathematics. The packet culminates with applications that utilize skills learned in Packets 7 and 8.

- The concept of a function is revisited in Packets 9 (**Congruence**) and 10 (**Similarity**) as a mapping diagram, and transformations are defined as functions that map points in the plane to points in the plane. Algebraic rules are used to describe translations on coordinate planes

In Conclusion

The authors of MathLinks carefully researched and thoughtfully operationalized focus, coherence, and rigor as we designed the program. We aimed to integrate mathematical concepts, skills, and applications through contexts and problems in a natural way. The result is an efficient core program that contains just 10 Student Packets with 33 lessons that can be completed in about 100 days! This leaves ample time for review, intervention, enrichment, and choice for both teachers and students, using the resources available in Student Packets and on the Teacher Portal.