

8-4 NONROUTINE PROBLEMS

OPEN MIDDLE PROBLEMS

Use any of the integers -9 to 9 at most once in each table below.

Answers may vary. Examples are shown.

Structure:

x	□	□	□	□
y	□	□	□	□

1. Create an input-output table for:

a. any linear function

x	0	1	2	3
y	-4	-3	-2	-1

b. any non-linear function

x	-2	0	2	4
y	-9	-1	7	63

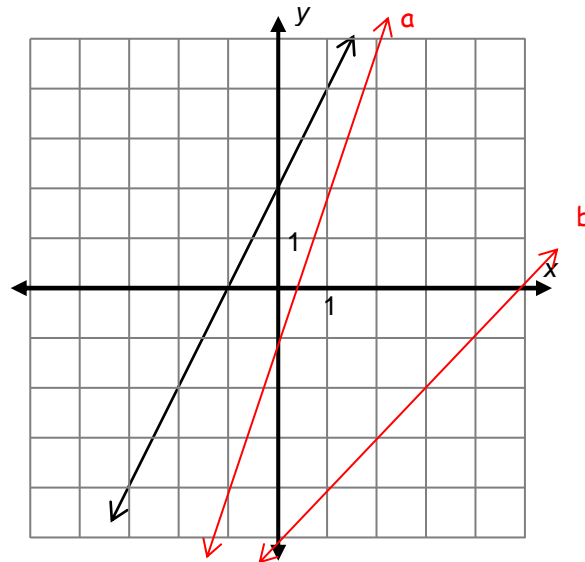
2. For the given graph, create input-output values as directed and graph the new lines.

a. a line that is “steeper” than the pictured line.

x	-2	-1	1	3
y	-7	-4	2	8

b. a line that is “flatter” than the pictured line.

x	-1	0	1	2
y	-6	-5	-4	-3



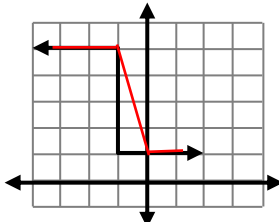
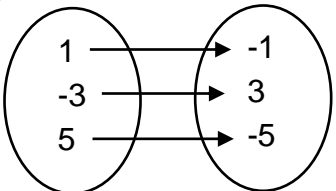
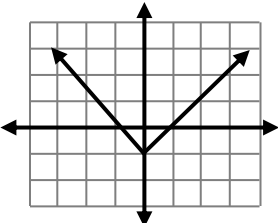
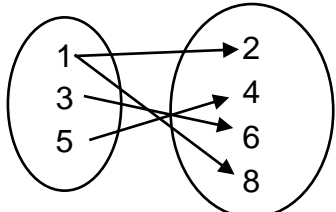
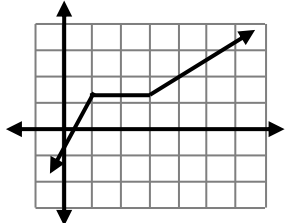
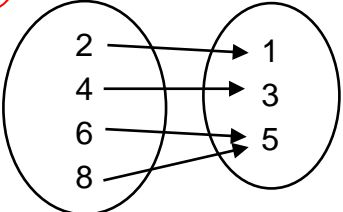
3. Can you create an input-output table that does not represent a function? Explain.

x	$\frac{1}{2}$	$\frac{3}{6}$	5	7
y	-2	-3	-4	-5

It is not possible with integers because they cannot be used more than once (specifically for x). However, using integers to create equivalent fractions for x-values makes a non-function possible (see table).

IS IT A FUNCTION?

Below are different representations of sets of ordered pairs. Circle the ones that could represent functions. If it is **not** a function, change it to represent one. *Answers will vary.*


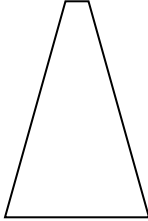
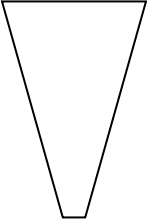
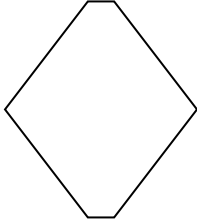
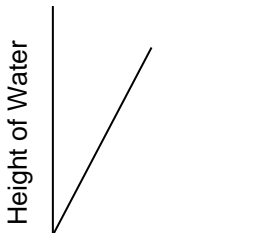
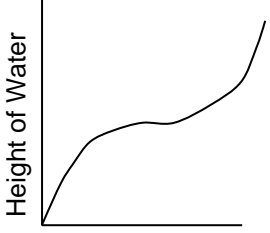
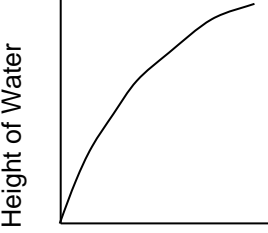
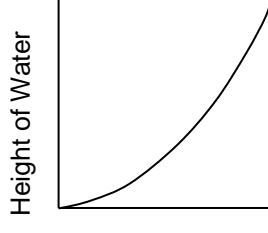
<p>1.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="padding: 2px 5px;">x</th> <th style="padding: 2px 5px;">y</th> </tr> </thead> <tbody> <tr> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">-2</td> </tr> <tr> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">-2</td> </tr> <tr> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">-2</td> </tr> <tr> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">-2</td> </tr> </tbody> </table>	x	y	1	-2	2	-2	3	-2	4	-2	<p>2.</p>  <p style="color: red; text-align: center;">connect (-1,5) to (0,1)</p>	<p>3.</p> 
x	y											
1	-2											
2	-2											
3	-2											
4	-2											
<p>4.</p> 	<p>5.</p>  <p style="color: red; text-align: center;">remove the arrow from 1 to 8</p>	<p>6.</p> <p>(6,3), (3,6), (-3, -6), (-6, -3)</p>										
<p>7.</p> <p>{(4,3), (4,4), (4,5), (4,6)}</p> <p style="color: red;">reverse the coordinates for x and y to get:</p> <p style="color: red;">(3,4), (4,4), (5,4), (6,4)</p>	<p>8.</p> 	<p>9.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="padding: 2px 5px;">x</th> <th style="padding: 2px 5px;">y</th> </tr> </thead> <tbody> <tr> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">4</td> </tr> <tr> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">4</td> </tr> <tr> <td style="padding: 2px 5px;">6</td> <td style="padding: 2px 5px;">7</td> </tr> <tr> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">8</td> </tr> </tbody> </table>	x	y	5	4	5	4	6	7	7	8
x	y											
5	4											
5	4											
6	7											
7	8											
<p>10.</p> 	<p>11.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="padding: 2px 5px;">x</th> <th style="padding: 2px 5px;">y</th> </tr> </thead> <tbody> <tr> <td style="padding: 2px 5px;">-2</td> <td style="padding: 2px 5px;">1</td> </tr> <tr> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">4</td> </tr> <tr> <td style="padding: 2px 5px;">-2</td> <td style="padding: 2px 5px;">3</td> </tr> <tr> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">-6</td> </tr> </tbody> </table> <p style="color: red; text-align: center;">change (-2,1) to (-1,1)</p>	x	y	-2	1	2	4	-2	3	0	-6	<p>12.</p> <p>(4,3), (2,5), (1,8), (0,3)</p>
x	y											
-2	1											
2	4											
-2	3											
0	-6											

POURING WATER

Imagine pouring water into each of these containers at a constant rate.

- Match each container with an appropriate graph below.
- Write one or two sentences to justify each choice.

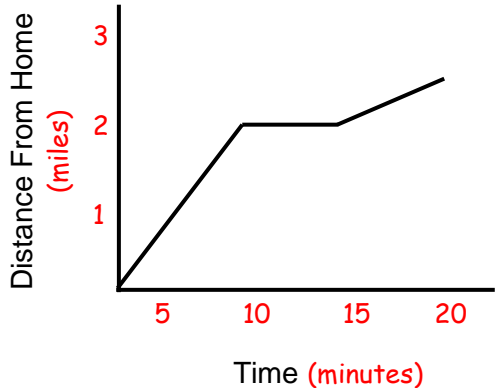
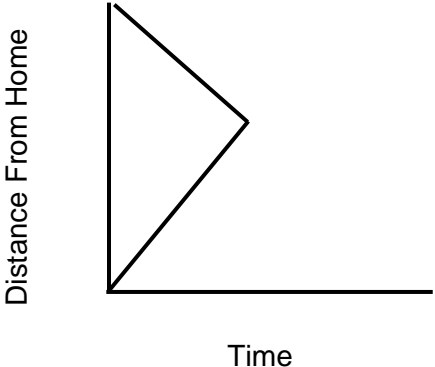
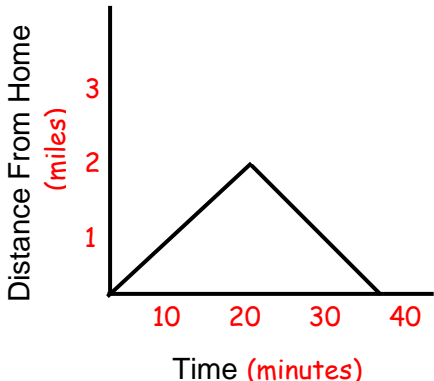
Example justifications are shown.

<p>1. Container 1</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Graph: <u> A </u></p> <p>Explain: <i>Because of the rectangular shape, the container will fill at a constant rate (a straight-line graph).</i></p>	<p>2. Container 2</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Graph: <u> D </u></p> <p>Explain: <i>Because it is wider on the bottom, the container will fill more slowly at first and then speed up (the curve on the graph is flatter at first and then steeper).</i></p>		
<p>3. Container 3</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Graph: <u> C </u></p> <p>Explain: <i>Because it is narrower on the bottom, the container will fill more rapidly at first and then slow down (the curve on the graph is steeper at first and then flatter).</i></p>	<p>4. Container 4</p> <div style="text-align: center; margin: 10px 0;">  </div> <p>Graph: <u> B </u></p> <p>Explain: <i>Because it is smaller at the bases, the container will start to fill rapidly, slow down, and then fill more rapidly (the curve on the graph is steeper at first, then flatter, then steeper again).</i></p>		
<p>A.</p> <div style="text-align: center;">  </div> <p style="text-align: center;">Number of Pours</p>	<p>B.</p> <div style="text-align: center;">  </div> <p style="text-align: center;">Number of Pours</p>	<p>C.</p> <div style="text-align: center;">  </div> <p style="text-align: center;">Number of Pours</p>	<p>D.</p> <div style="text-align: center;">  </div> <p style="text-align: center;">Number of Pours</p>

RUNNING STORIES

Create stories about walking or running that each graph could represent. If the situation is not possible, explain. Include appropriate scales if numbers help to describe the story.

Answers will vary. Example stories are given.

Graph	Story
<p>A.</p>  <p style="text-align: center;">Time (minutes)</p>	<p><i>A runner ran 2 miles from home in 10 minutes, stopped to tie a shoe and rest for 5 minutes, then walked to cool down for another half mile in 5 minutes.</i></p>
<p>B.</p>  <p style="text-align: center;">Time</p>	<p><i>This one is not possible (in the real world) because the runner cannot be in two places at the same time. Another way to think about it is that time must be increasing (time cannot increase and then decrease) as distance from home is increasing.</i></p>
<p>C.</p>  <p style="text-align: center;">Time (minutes)</p>	<p><i>A runner ran at a constant pace to a location that was 2 miles away, and then ran back home at the same pace. She finished in about 37 minutes. She ran a pace of 9.25 miles per hour.</i></p>

MIXED PROBLEMS

1. The table below represents a function where x is the input value and y is the output value.

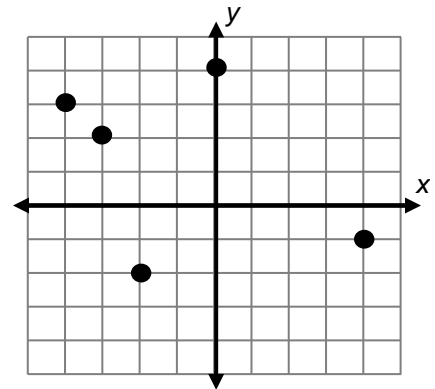
x	-3	-2	0	2
y	7	5	1	-3

Select all ordered pairs that could be included in the table so that it remains a function?

- a. (0, 0) **b.** (5, -2) **c.** (1, 1)
- d. (2, 3) **e.** (7, 0) f. (-3, 4)

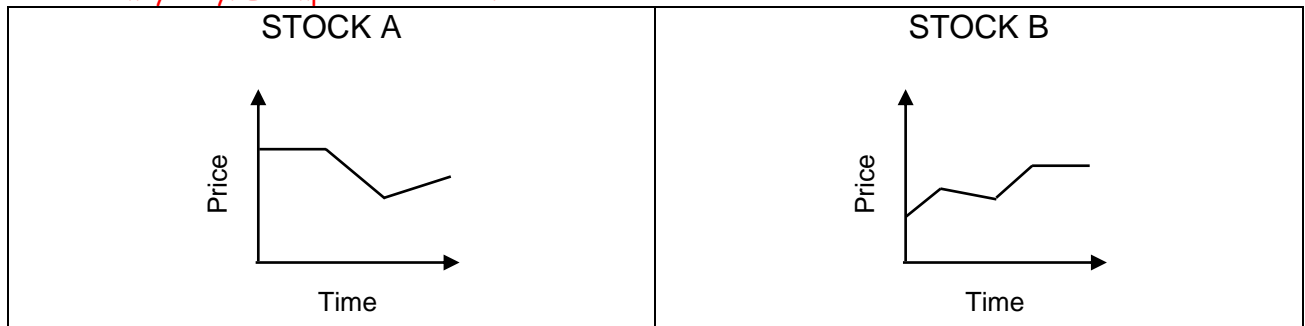
2. The graph represents y as a function of x . Which additional point can be plotted so that the graph continues to represent y as a function of x ?

- a. (0,1) **b.** (1,0) c. (4,2) d. (-2, 4)



3. The graphs below show the prices of two stocks during one day.

Answers may vary. Examples are shown.



- a. Make two comparisons from the graphs.
The two stocks appear to be about the same price at the end of the day, though it's unclear when there is no scale. Both stocks B had some periods of growth, some loss, and some consistency over the course of the day.
- b. If you were to recommend one of the stocks, which one would you say is doing better? Explain.
One day of performance for stocks at (likely) similar prices is not enough information to make an educated decision on an investment. And even this one day does not seem to show one is performing better than the other.

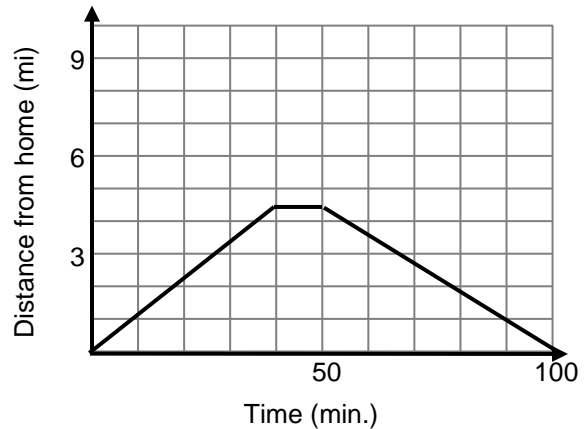
MIXED PROBLEMS Continued

4. Determine which equations define y as a linear function of x , and which equations define y as a non-linear function of x . Consider using a table of values or a graphing calculator.

Equation	Linear Function	Non-Linear Function
$y = 1.2x^2$		✓
$2y = x$	✓	
$x + y = 7$	✓	
$y = 4x + 3^2$	✓	
$y = 4 + 3x^2$		✓
$y = 2x^3 - 1$		✓
$y = \frac{x}{5}$	✓	

5. The graph shows a runner's distance from home over time.

Based on the graph, determine whether each statement is true or false.



a. The runner's distance from home is increasing between 40 minutes and 50 minutes.

false

b. The runner's distance from home is increasing from 10 minutes to 40 minutes.

true

c. The runner's distance from home is increasing from 50 minutes to 100 minutes.

false

d. The runner ran at a constant speed the entire time.

false

FROM THE MATH OLYMPIAD

1. A cricket chirps 6 times every 8 seconds. At that rate, how many times does the cricket chirp in 2 minutes?

90 chirps (2 minutes = 120 seconds)

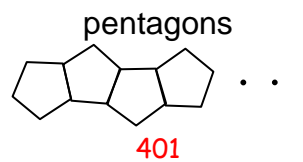
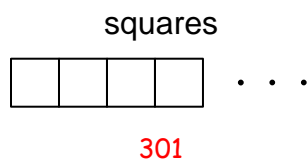
Number of Chirps	6	3	30	60	90
Time (seconds)	8	4	40	80	120

2. If 4 people can paint 2 fences in 5 hours, how many hours in all will it take for 8 people to paint 8 fences?

10 hours; one explanation: If 2 fences can be painted in 5 hours by 4 people, at that rate, 8 fences can be painted by those same 4 people in 4 times the number of hours, which is 20 hours. If it takes 4 people 20 hours to paint 8 fences, it will take 8 people half as long to paint those same 8 fences, which would be 10 hours.

3. Suppose that toothpicks are used to make rows of 100 figures. There is one toothpick per side, and the first 4 figures of each pattern are pictured below.

How many toothpicks are needed for each row of shapes?



Start with 4 toothpicks to make the first square. Then add 3 toothpicks to make each additional square. Since 100 figures will be made with 4 toothpicks (1st square) plus 3 times some number (the 99 additional squares), $4 + 3(99) = 301$.

Start with 5 toothpicks to make the first pentagon. Then add 4 toothpicks to make each additional pentagon. Since 100 figures will be made with 5 toothpicks (1st pentagon) plus 4 times some number (the 99 additional pentagons), $5 + 4(99) = 401$.

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