FOCUS, COHERENCE, AND RIGOR

A group of educators at Rutgers University led by Dr. Roberta Shorr (Young, 2009) conducted a seven-year study on problem solving with minority and low-income students in low performing schools. They found that giving conceptually challenging problems to students caused frustration, but at the same time students were engaged and motivated. Working through the frustration in an emotionally safe environment, students gained "satisfaction, pride, and a willingness to work harder next time." This work also resulted in higher standardized test scores (e.g., average scores for Newark fourth graders rose from 45 to 79 percent.

At the Center for Mathematics and Teaching, we know that all students have potential to achieve in mathematics, we believe that the development of mathematics should reflect the connectedness of Big Ideas into a coherent whole, and we aim to make mathematics inviting and inclusive to more students. One of the ways we put these beliefs and intentions into action is by attending to focus, coherence, and rigor in program development.

FOCUS

According to Achieve the Core (2014), "Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness."

Achieve the Core organizes mathematics clusters into three categories: major clusters, supporting clusters, and additional clusters. This table identifies the clusters for 7th grade and their alignment to *MathLinks* lessons.

FOCUS IN *MATHLINKS* BASED ON MAJOR, MINOR, AND SUPPORTING CLUSTERS

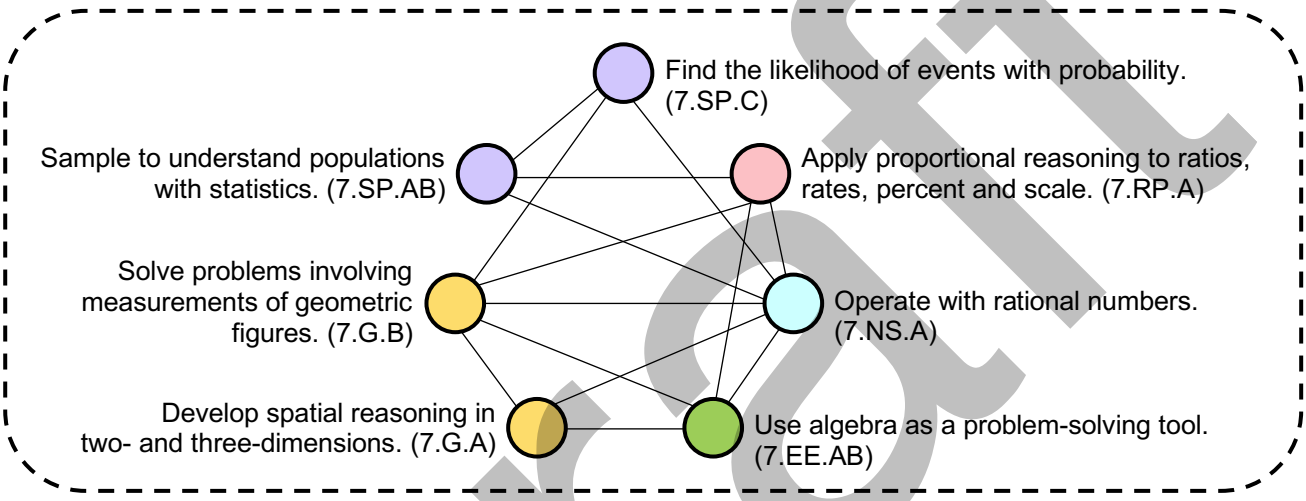
CLUSTER	Standards	Lesson Focus	Additional Lessons, Spiral Review Packets
Major Clusters			
• 7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.	7.RP.1,2,3	2.1, 2.2, 2.3, 3.1, 3.2, 3.3,	6.1, 6.2, 6.4, 7.2, 9.1, 9.2, 9.3, 10.3 Spiral Rev (3,4,5,7,8)
• 7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	7.NS.1,2,3	1.2, 1.3, 4.1, 4.2, 4.3, 5.1, 5.2, 5.3,	2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 6.4, 7.2, 7.3, 7.4, 9.1, 9.2, 9.3 Spiral Rev (2 – 10)
• 7.EE.A Use properties of operations to generate equivalent expressions.	7.EE.1,2	6.1, 6.3, 6.4, 7.2, 7.3, 7.4,	2.1 Spiral Rev (7, 9)
• 7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	7.EE.3,4	7.1, 7.2, 7.3, 7.4	8.1 Spiral Rev (8,9,10)
Supporting Clusters			
• 7.SP.A Use random sampling to draw inferences about a population.	7.SP.1,2	10.1, 10.3, 10.3	
• 7.SP.C Investigate chance processes and develop, use, and evaluate probability models	7.SP.5,6,7,8	1.1, 1.2, 1.3, 10.1	4.2
Additional Clusters			
• 7.G.A Draw, construct and describe geometrical figures and describe the relationship between them.	7.G.1,2,3	2.3, 3.1, 8.2, 8.3	Spiral Rev (10)
• 7.G. B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	7.G.4,5,6	9.1, 9.2, 8.3	Spiral Rev (10)
• 7.SP.B Draw informal comparative inferences about two populations	7.SP.3,4	10.1, 10.2, 10.3	Spiral Rev (8)

COHERENCE

According to a draft of the CA Math Framework (2022), "instructional materials should primarily involve tasks that invite students to make sense of big ideas, elicit wondering in authentic contexts, and necessitate mathematical investigation. Big ideas in mathematics are central to the learning of mathematics, link numerous mathematical understandings into a coherent whole, and provide focal points for students' investigations."

Inspired by the work of Jo Boaler (Boaler, Munson, Williams, What is Mathematical Beauty? Teaching through Big Ideas and Connections, youcubed.org, 2017) and based on the Content Domains, the *MathLinks* team synthesized the work of 7th grade into seven big ideas. These ideas and their connections within the entire program are shown here.

Grade 7: Big Ideas and Connections



The work in *MathLinks* is organized into ten packets. An expanded dive into the specific big ideas and connections are included in the front section of each Teacher Edition packet. Towards the end of each Student Packet, students identify the big ideas within the packet and reflect upon how they are connected to each other.

RIGOR

According to the Common Core State Standards Initiative (2022), "a rigorous math program will pursue with equal intensity conceptual understanding, procedural skill and fluency, and applications." We first examine the MathLinks: Grade 6 program through this lens.

Conceptual Understanding in *MathLinks*

Conceptual understanding, the bedrock of a *MathLinks* course, frequently drives the other two components of rigor. It's a *MathLinks* philosophy to make sure all students have the opportunity to make meaning for every concept presented, and we aim to focus on the conceptual development of big ideas in depth, and make them plausible through investigations, activities, and practice. This is commonly done throughout lessons in Student Packets, oftentimes with the help of teacher Lesson Notes and Slide Decks (or Slide Deck Alternatives).

This table identifies some examples of concept development in *MathLinks:* Grade 7 Student Packets (lesson location in parentheses). Typically, these lessons include guided instruction with a slide deck, lesson notes, class discussions, or group work. Opportunities for independent work within a Student Packet appear as "Practice" pages within lessons, in the "Review" section as activities, and as spiral review in subsequent packets. Packet Resources on the Teacher Portal also contain problems, tasks, and projects to support conceptual development.

EXAMPLES OF CONCEPT DEVELOPMENT LESSONS IN *MATHLINKS*: GRADE 7	
CLUSTER DESCRIPTION	**CONCEPT DEVELOPMENT WITH TEACHER OR GROUP**
Major Clusters	
• 7.RP.A (RP 1-3) Analyze proportional relationships and use them to solve real-world and mathematical problems.	(3.1) Proportional Relationships (3.3) Double Number Lines and Equations
• 7.NS.A (NS 1-3)Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	(1.2) Investigating One-Third (4.2a) Subtracting Integers with Counters 1 (4.2b) Subtracting Integers with Counters 2
• 7.EE.A (EE 1-2) Use properties of operations to generate equivalent expressions.	(6.1) How Many on the Border? (6.3a) Introduction to Cups and Counters (6.3b) The Upside Down Cup)
• 7.EE.B (EE 3-4) Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	(7.2a) Solving Equations with Balance (7.2b) Solving Equations Algebraically
Supporting and Additional Clusters	
• 7.SP.A (SP 1-2) Use random sampling to draw inferences about a population.	(10.1b) Populations and Sampling
• 7.SP.B (SP 3-4) Draw informal comparative inferences about two populations	(10.2) Math Score Samples
• 7.SP.C (SP 5-8) Investigate chance processes and develop, use, and evaluate probability models	(1.2) Flip and Roll
• 7.G.A (G 1-3) Draw, construct and describe geometrical figures and describe the relationship between them	(8.0) Tear it Up (8.2) Sketching Figures
• 7.G. B (G 4-6) Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	(9.1) Circles (9.2) An Area Investigation

Applications in *MathLinks*

Problem solving is an important driver of instruction within *MathLinks* courses. In *MathLinks*, we aim to include engaging mathematical problems and applications with accessible entry points for all students, multiple approaches or solutions, and extensions to challenge and enrich. All Student Packets begin with an Opening Problem, which introduces a concept or establishes a "need to know." In many cases, students require more instruction throughout the packet before they are fully prepared to bring the problem to its conclusion. More "meaty" problems exist the packets as well.

This table identifies some examples of mathematical problems and applications in *MathLinks:* Grade 7 Student Packets (lesson location in parentheses). Some of these problems include guided instruction with a slide deck and lesson notes. Others are appropriate for independent work. Packet Resources on the Teacher Portal (e.g. Tasks, Projects) also contain mathematical problems and applications.

EXAMPLES OF PROBLEM SOLVING LESSONS IN *MATHLINKS*: GRADE 7	
CLUSTER DESCRIPTION	**MATHEMATICAL PROBLEMS AND APPLICATIONS**
Major Clusters	
• 7.RP.A (RP 1-3) Analyze proportional relationships and use them to solve real-world and mathematical problems.	(2.0, 2.1) Using Coupons, Using Coupons Revisited (3.1) Twinkie the Dog, Practice 2 (3.2) Capt'n Sherman's Shrimp Shop
• 7.NS.A (NS 1-3)Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	(3.3) Jenna's Cornbread, Practice 6 (4.2) A Zero-Sum Game
• 7.EE.A (EE 1-2) Use properties of operations to generate equivalent expressions.	(6.0, 6.1) Crossing the Lake, Crossing the Lake Revisited (6.1) Practice 1-2
• 7.EE.B (EE 3-4) Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	(6.3) Trousers for Sale (7.1) The Hundred Chart Puzzle (7.2) Joan's Phones
Supporting and Additional Clusters	
• 7.SP.A (SP 1-2) Use random sampling to draw inferences about a population.	(10.3) Estimating Fish Populations
• 7.SP.B (SP 3-4) Draw informal comparative inferences about two populations	(10.2) Practice 3, Practice 4
• 7.SP.C (SP 5-8) Investigate chance processes and develop, use, and evaluate probability models	(1.0, 1.2) Race to the Top, Race to the Top Revisited (1.2) Flip and Roll (1.3) Cereal Box Simulation
• 7.G.A (G 1-3) Draw, construct and describe geometrical figures and describe the relationship between them	(2.3) The Birdhouse (8.2) A Polygon Investigation
• 7.G.B (G 4-6) Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	(9.0, 9.2) Felix the Sheep, Practice 4 (9.2) Penny Drop Probabilities

Procedural Skill and Fluency in *MathLinks*

Procedural skill and fluency are more than just computational speed and accuracy. In a 2014 position paper, NCTM described procedural fluency as "the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another.

In *MathLinks*, we aim to thoughtfully develop new procedural skills and provide opportunities for students to gain fluency throughout the year. Skills practice in Student Packets is located in:

- Practice pages – These pages support concept development.
- Review activities – These pages often include skills practice.
- Spiral Review – These pages begin with a Math Path Fluency Challenge in the form of a maze and fraction-decimal-percent practice in the form of a table. Then follows distributed practice of prior skills.

While CCSS-M does not identify any specific standards for fluency in Grade 7, Achieve the Core's Instructional Materials Evaluation Tool (2021) suggests attention be given to three Standards: 7.NS.1, 7.EE.1, and 7.EE.4a. The table below shows some examples in 7th grade Student Packets where these procedural skills are developed and students have opportunities to gain fluency through independent practice.

EXAMPLES OF FLUENCY WORK IN *MATHLINKS*: GRADE 7		
Standard / Description	**Development of Procedure**	**Opportunities to gain fluency**
7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.	(4.1) Counters and Adding Integers (4.2) Counters and Subtracting Integers (4.3) Rational Number Addition and Subtraction (5.1) Multiplying and Dividing Integers (5.2) Multiplying and Dividing Rational Numbers (5.3) Order of Operations	(4.1, 4.2, 4.3) Practices 1-9, Review (5.1, 5.2, 5.3) Practices 1-9, Review Spiral Review (Packets 2-10)
7.EE.1 Apply properties of operations as strategies to add, subtract, and expand linear expressions with rational coefficients.	(6.3) Expressions with Cups and Counters (6.4) Fluency with Expressions	(6.3, 6.4) Practices 9, 10, Review (7.1) Hundred Chart Puzzle (7.1, 7.2) Practices 1-4) (8.1) Using Algebra to Find Angle Measures Spiral Review (Packet 9)
7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	(7.1) Solving Equations Using Mental Math (7.2) Solving Equations Using Algebra	(7.1) Hundred Chart Puzzle (7.1, 7.2) Practices (1-5) (8.1) Using Algebra to Find Angle Measures (8.1) Practice 3 Spiral Review (Packets 9, 10)

In addition to Student Packets, resources in the Teacher Portal offer opportunities to practice grade level skills and to fill in gaps where needed. Look in Packet Resources for:

- Essential Skills – this entire section reviews skills and concepts important for success in a given packet. Activity Routines such as Big Square Puzzles, Open Middle Problems, and Four-in-a-Row Games are also in these sections for some packets. They provide a practice alternative to "drill and kill."
- Extra Problems – Problems are available for every lesson.
- Nonroutine Problems – Some Big Square Puzzles, Open Middle Problems, and Four-in-a-Row Games are also located in this section for some packets.

Finally, for students who need to fill major gaps in skills, consider Skill Boosters, which are in the Teacher Portal. This Resource helps students catch up on below grade level work without losing extensive instructional time on grade level work. A Skill Booster routine is intended to take about 10 minutes per day.

Connecting the Three Aspects of Rigor

Interpretations of "rigor" are evolving. In 2022, the CA Framework defined rigor to mean that "conceptual understanding can be used to analyze a novel situation encountered in the world. Rigor means that students can flexibly apply methods to different situations, and connect mathematical ideas, approaches, and representations." A closer look at how algebra builds in 7th grade illustrates how this is done.

Algebra in *MathLinks*: Grade 7

Algebra topics primarily appear in the CCSS-M Expressions and Equations and Ratios and Proportional Relationships domains. These areas are the focus of four packets in *MathLinks*: Grade 7, and they extend work introduced in 6th grade.

- In Packet 2, **Percent and Scale**, students analyze and solve problems involving numerical and algebraic expressions and involving percent.

- In Packet 3, **Proportional Relationships**, students connect different representations (i.e., visual contexts, tables, graphs, equations, word descriptions) as they solve problems involving proportional relationships. Special attention is paid to whether two quantities are in a proportional relationship by analyzing tables, graphs, and equations. Students continue to develop flexibility when working with variables, expressions, and equations. Double number lines facilitate the learning of how to solve proportions (i.e., equations in the form $\frac{x}{a} = \frac{b}{c}$).

- In Packet 6, **Expressions**, students use a visual context to write numerical and algebraic expressions, paving the way to greater flexibility working with variables and expressions. Equations of the $y = mx + b$ are explored without formally addressing function, slope, and vertical intercept, which is done in 8th grade.

  The counter manipulative used for developing integer operations in Packets 4 and 5 (**Rational Number Addition and Subtraction** and **Rational Number Multiplication and Division**) is extended to include cups to represent an unknown in an equation. This model gives students a tool for exploring and rewriting more difficult expressions.

- In Packet 7, **Solving Equations and Inequalities,** students extend the use of substitution to solve equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Cups and counters help to facilitate the learning of the procedures. Students also learn to solve inequalities with negative coefficients and open/closed boundary points.

Additionally, in Packets 8, 9, and 10 (**Plane and Solid Figures**; **Length, Area, and Volume**; **and Sampling**), students apply their knowledge of proportional relationships and algebra to solve problems in other domains.

In Conclusion

The authors of MathLinks carefully researched and thoughtfully operationalized focus, coherence, and rigor as we designed the program. We aimed to integrate mathematical concepts, skills, and applications through contexts and problems in a natural way. The result is an efficient core program that contains just 10 Student Packets with 33 lessons that can be completed in about 100 days! This leaves ample time for review, intervention, enrichment, and choice for both teachers and students, using the resources available in Student Packets and on the Teacher Portal.