PACKET 3
TEACHER EDITION

MathLinks GRADE 7

PROPORTIONAL RELATIONSHIPS

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
7.RP.A	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*
7.RP.2	Recognize and represent proportional relationships between quantities:
a	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c	Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.
d	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
7.EE.B	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
7.G.A	**Draw, construct, and describe geometrical figures and describe the relationships between them.**
7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

PACKET PLANNING

Starred (*) resources can be accessed on the Teacher Portal.

Packet Pacing* Up to 13 class periods	3.0 Length and Area Patterns (1 period) 3.1 An Introduction to Proportional Relationships (3 periods) 3.2 Digging Deeper into Proportional Relationships (2 periods) 3.3 Equations and Problems (3 periods) Review (3 periods) Assessment (1 period)
Packet Resources* Up to 3 class periods	• Extra Problems • Essential Skills • Math Talks • Nonroutine Problems • Tasks • Projects (in some packets) • Technology Activities
Assessment Options*	• On the Teacher Portal ✓ Packet Quizzes ✓ Comprehensive Tests ✓ Tasks ✓ Projects ✓ Using the *MathLinks* Rubric (an Activity Routine) • In the Student Packet ✓ Monitor Your Progress ✓ Packet Reflection • In the Teacher Edition ✓ References to Journals ✓ Suggested problems for the *MathLinks* Rubric
Materials	• Poster paper, butcher paper, or board space [Review] • Markers [Review] • General supplies (e.g., colored pencils, rulers, tape, scissors, graph paper, calculators, chart paper)
Slide Decks*	S3.0 Length and Area Patterns S3.1a Proportional Relationships S3.1b Twinkie the Dog S3.2 Cap'n Sherman's Shrimp Shop S3.3 Double Number Lines and Equations
Reproducibles*	R3-1 Matching Activity: Nuts! [Review] (1/pair or group) R3-2 Match and Compare Sort Cards: Proportional Relationships [Review] (1/pair)
Prepare Ahead	• Cut up R3-1, R3-2 • See Activity Routines in the Teacher Portal for directions for Using the *MathLinks* Rubric, **Poster Problems** and **Match and Compare Sort** [3.1, Review]
Other Resources*	• Parent Letter (English and Spanish) • Videos

COMPONENTS FOR DIFFERENT USERS

These resources can be accessed on the Teacher Portal.

For all students	• Student Packet (copyright protected, for viewing and projecting) • Resource Guide (Complete) • Extra Problems • Math Talks • Nonroutine Problems • Tasks • Projects (in some packets) • Technology Activities • Carole’s Puzzles and Games
For English learners	• Student Packet Text File for Translation
For struggling learners	• Essential Skills • Extra Problems • Skill Boosters (Options: Fraction Addition and Subtraction, Fraction Multiplication and Division)
For advanced learners	• Student Packet (speed up instruction when possible) • Nonroutine Problems
For teachers	• Teacher Edition (this document) • Resource Guide (Complete) • Program Information • Activity Routines (Explanations and Introductory Examples)
For substitutes	• Previous Student Packets (unfinished work) • Practice 1 – 8 (may be completed independently any time after instruction of those topics in the packet) • Spiral Review • Vocabulary Review • Extra Problems • Carole’s Puzzles and Games
For parents	• Resource Guide (Complete) • Parent Letter (English and Spanish)

MATH BACKGROUND

Ratios are Everywhere

Under every rug there is a ratio.

In mathematics:

- the value of the ratio of the circumference of a circle to its diameter (π)
- the value of the ratio of lengths of corresponding sides of similar triangles
- the value of the ratios of side lengths of right triangles (trigonometric ratios)
- the value of the ratio of the "increase in the *y*-variable" to the "increase in the *x*-variable" (slope of a line)

In science:

- laws of physics, such as the ratio of momentum to velocity of falling objects
- conversion rates, such as feet to meters or minutes to hours
- comparisons, such as nineteen out of twenty glaciers are receding

In daily activities:

- two cups water for every cup oatmeal (recipe)
- a dozen almonds per serving
- thirty miles per hour (a speed limit)
- twenty-seven miles per gallon (fuel consumption)

In pricing:

- cheese at $5 per pound
- farmland at $8,000 per acre

In sports and exercise:

- odds of Boston winning the World Series
- calories burned in fifteen minutes jogging

Whenever we refer to percentages, we are using ratios. The battery life of our electronic device, the sales tax on our pizza, and the discount on sale items are given as a percentage**.**

Ratio, Rate, Unit Rate, and Value

The words “ratio” and “rate” have various shades of meaning in common language. The definitions in school mathematics textbooks vary. The Common Core State Standards for Mathematics (CCSS-M) and Progressions prescribe a formal definition of “ratio,” and at least implicitly a definition of “unit rate.” On the other hand, “rate” is treated as a term in common language. No formal definition of “rate” appears in the documents.

- A ratio is a pair of positive numbers in a specific order. The ratio of a to b is denoted by $a : b$ (read “a to b,” or “a for every b”).

  Examples of ratios: $3 : 2$, $\frac{4}{5} : 2$, $3.14 : 10$.

  These are NOT ratios: $0 : 0$, $2 : -3$.

- Unit rate associated with a ratio: Suppose $a : b$ is a ratio, and $b \neq 0$. The unit rate associated to $a : b$ is the number $a \div b$, which may have units attached to it. If a and b have units attached to them, say “a-units” and “b-units,” the appropriate unit of measure for the unit rate is “a-units per b-unit.”

  Example: The ratio “400 miles every 8 hours” has unit rate “50 miles per hour.” There is a convenient calculation device that leads to the unit for the unit rate:

  $$\frac{400 \text{ miles}}{8 \text{ hours}} = \frac{400}{8}\,\frac{\text{miles}}{\text{hours}} = 50\,\frac{\text{miles}}{\text{hours}} = 50 \text{ miles per hour.}$$

- Value of a ratio: The value of a ratio $a : b$, $b \neq 0$, is the quotient number $a \div b$.

  Example: The value of the ratio $6 : 3$ is $6 \div 3 = 2$. The value of the ratio $7 : 2$ is 3.5.

  The value of the ratio “400 miles every 8 hours” is $\frac{400}{8} = 50$.

Both terms “value” and “unit rate” are based on the same numerical value, the quotient number $a \div b$. The difference between the terms is that *all* ratios $a : b$ with $b \neq 0$ have a value, whereas we generally talk about unit rates only for ratios that have units attached to them. In the latter case, the unit rate is equal to the value of the ratio with “something per something” attached.

Geometric Interpretation of Equivalent Ratios

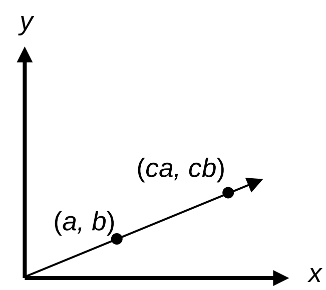
Two ratios are <u>equivalent</u> if each number in one ratio is a multiple of the corresponding number in the other ratio by the same positive number. Thus the ratio $a : b$ is equivalent to the ratio $ca : cb$ for all numbers $c > 0$.

When $b \neq 0$, the <u>value of a ratio</u> $a : b$ is the quotient number $a \div b$. We extend the definition of value to ratios $a : b$ with $b = 0$ by declaring that the value of the ratio $a : 0$ is $+\infty$. This is analogous to thinking of a vertical line in the plane as having slope $+\infty$. Though $+\infty$ is not a number, it is a perfectly legitimate value for a function.

Now that we have extended the definition of value of a ratio to cover the case when $b = 0$, we can give a simple geometrical characterization of equivalent ratios in terms of rays in the plane.

Each ratio $a : b$ determines a point (a, b) in the first quadrant of the coordinate plane. This correspondence $a : b \rightarrow (a, b)$ maps ratios to the first quadrant, including the positive x-axis and y-axis but omitting the origin. Ratios $0 : b$ with value 0 are mapped to points $(0, b)$ on the positive x-axis, and ratios $a : 0$ with value $+\infty$ are mapped to points $(a, 0)$ on the positive y-axis. Under this correspondence, the ratios $ca : cb$ equivalent to $a : b$ correspond to the points (ca, cb) on the ray through (a, b) emanating from the origin. In fact, if we assign a slope of $+\infty$ to a vertical line, then the following statements are valid for all ratios:

- The ratios equivalent to $a : b$ correspond to the ray (half-line) issuing from the origin through (a, b).
- The slope of the ray through (a, b) is the value of the ratio $a : b$.
- Two ratios are equivalent if, and only if, they have the same value.



Equivalent Ratios and Proportional Relationships

Two positive variables x and y are in a <u>proportional relationship</u> if the values of y are the same constant multiple of the values of x, that is, $y = cx$ for some constant c. The constant c is the <u>constant of proportionality</u>. The graph of the pairs of values (x,y) lie on the ray of slope c emanating from the origin. If x and y are in a proportional relationship, then the ratios $y : x$ of the values of y to the corresponding values of x all have the same value c, since $c = \frac{y}{x}$. Thus the ratios $y : x$ are all equivalent.

Conversely, if the ratios $y : x$ of the values of y to the corresponding values of x are all equivalent, and c is the common value of the ratios, then x and y are in a proportional relationship , and c is the constant of proportionality.

Reasoning and Proof: Why Cross-Multiplication Works for Equations of the Form $\frac{a}{b} = \frac{c}{d}$

Equations in the form $\frac{a}{b} = \frac{c}{d}$ are commonly referred to as proportions.

Prove: If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ (assuming $b \neq 0$, $d \neq 0$).

Statement:	Reason:
$\frac{a}{b} = \frac{c}{d}$	given
$a \bullet \frac{1}{b} = c \bullet \frac{1}{d}$	definition of division
$(b \bullet d) \bullet \left(a \bullet \frac{1}{b}\right) = (b \bullet d) \bullet \left(c \bullet \frac{1}{d}\right)$	multiplication property of equality
$(a \bullet d) \bullet \left(b \bullet \frac{1}{b}\right) = (b \bullet c) \bullet \left(d \bullet \frac{1}{d}\right)$	commutative and associative properties of multiplication
$a \bullet d \bullet 1 = b \bullet c \bullet 1$	multiplicative inverse property
$a \bullet d = b \bullet c$	multiplicative identity property

How Much Detail Is Needed in a Proof?

In the justification of why cross-multiplication works, steps involving the associative property were omitted so that the essential reasons (definition of division, multiplicative inverses, and multiplicative identity) for the procedure would be more transparent.

Soccer referees are instructed by FIFA Law not to blow the whistle for every piddling foul, as it disrupts the flow of the game. By the same token, mathematicians do not include piddling details in proofs, as they disrupt the flow of the proof and conceal the main arguments.

TEACHING TIPS

Applying Standards for Mathematical Practice (SMP)		
Here is an abbreviated version of the SMPs and some ways they are applied in this packet.		
SMP1	**Make sense of problems and persevere in solving them.** • Understand a problem and look for entry points. • Consider simpler or analogous problems. • Monitor progress and alter solution course as needed. • Make connections between multiple representations. • Check answers with a different method.	[3.1] For **Twinkie the Dog**, students make sense of the problem and adjust predictions with more information. [3.3] **Jenna's Cornbread Recipe** involves ratios of fractions. Solution strategies that connect to whole number procedures will likely make more sense to students.
SMP2	**Reason abstractly and quantitatively.** • Use numbers and quantities flexibly in computations. • Attend to the meaning of quantities. • Decontextualize a problem using symbols, manipulate them, and then interpret based on the context.	[3.3] In the **Double Number Lines and Equations** slide deck, a series of "If – Then – Example – Generalize" statements begin with a context (cost of baseballs) on a double number line. The lesson guides students to decontextualize so that properties of equations of the form $\frac{a}{b}=\frac{c}{d}$ (aka "proportions") can be established. Then students generalize the properties.
SMP3	**Construct viable arguments and critique the reasoning of others.** • Use assumptions, definitions, established results, examples, and counter examples to analyze an argument and discuss its merits or flaws. • Make and test conjectures based on evidence. • Analyze situations by breaking them into cases. • Understand and analyze the approaches of others.	[3.0, 3.3] Students critique student work in **Length and Area Patterns** and **Practice 4**. [3.1, 3.2] Students explain why particular situations are (or are not) proportional relationships.

Applying Standards for Mathematical Practice (Continued)		
SMP4	**Model with mathematics.** • Attach meaningful mathematics to everyday problems and questions of interest. • Make reasonable assumptions and approximations to simplify a situation. • Identify quantities, use mathematical tools (such as multiple representations, formulas, equations) to analyze relationships. • Interpret results and draw conclusions in the context of the situation.	[3.1, 3.2] Students use multiple representations (tables, tape diagrams, double number lines, graphs, and equations) to model situations as they solve problems and draw conclusions, especially whether a situation represents a proportional relationship. [3.1] In **An Introduction to Proportional Relationships**, **Getting Started**, the better buy for bagels is based on cost. But students may want to consider other factors such as quality or travel time to the store. Considering these factors is a part of the mathematical modeling process and important to decision-making.
SMP5	**Use appropriate tools strategically.** • Select and use tools strategically (and flexibly) to visualize, explore, and compare information. • Use technological tools and resources to solve problems and deepen understanding.	[3.1, 3.2, 3.3] Throughout the lessons, students decide whether or not arithmetic should be performed using mental math, pencil and paper, or a calculator.
SMP6	**Attend to precision.** • Calculate accurately and efficiently. • Explain thinking using mathematical vocabulary. • Use symbols appropriately • Specify units of measure.	[3.1, 3.2] Students scale graphs appropriately and specify units on axes as they graph points that may represent proportional relationships. [Review] **Match and Compare Sort** requires precision in deciphering related vocabulary terms.
SMP7	**Look for and make use of structure.** • Recognize the structure of a symbolic representation and generalize it. • See complicated objects as composed of chunks of simpler objects.	[3.3] For **Double Number Lines and Equations**, the “If – Then – Example – Generalize” slides give students the opportunity to identify the structure within an equation and generalize it.
SMP8	**Look for and make use of repeated reasoning.** • Identify repeated calculations and patterns. • Generalize procedures based on repeated patterns or calculations. • Find shortcuts based on repeated patterns or calculations.	[3.3] For **Double Number Lines and Equations**, students observe that the cross multiplication property (if $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$) is a shortcut for solving this type of equation, which is often referred to as a “proportion,” and continue to use it when helpful.

<table>
<tr><th colspan="3">Strategies to Support Diverse Populations</th></tr>
<tr><td colspan="3">Classrooms typically include students with different learning styles and needs. Here are some specific ways that MathLinks supports special populations. Strategies essential to the academic success of English learners are noted with a star (*). See General Program Information for more details.</td></tr>
<tr><th></th><th>General Examples</th><th>MathLinks Examples</th></tr>
<tr><td>Know your Learner</td><td>✓ Understand student attributes that support or interfere with learning

✓ Determine preferred learning and interaction styles

✓ Assess student knowledge of prerequisite mathematics content

✓ Check for understanding continuously

✓ Provide differentiation opportunities for intervention or enrichment to reach more learners

✓ Encourage students to write about their attitudes and feelings towards math

✓ Use contexts that link to students' cultures*</td><td>Built into the MathLinks Design:
SP: Getting Started, Spiral Review, Monitor Your Progress, Packet Reflection
TE: References to Journals
PR: Extra Problems, Essential Skills, Nonroutine Problems, Projects
OR: Skill Boosters, Assessment Options

[Anytime] Many students struggle with rational number operations. Ask students to write a short letter to you about their successes or challenges. Respond to each letter with a sincere “thank you for sharing,” and a short sentence to encourage effort.

[3.0, 3.1, 3.2] Watch for and address common misconceptions that students may have about proportional reasoning, such as additive vs. multiplicative thinking.

[3.0, 3.1, 3.2, 3.3] Students learn many approaches for solving proportional reasoning problems. Encourage them to rely on representations that make sense and to try others to further build understanding.</td></tr>
<tr><td>Increase Academic Language through Mathematics</td><td>✓ Provide opportunities for students to read, write, or speak about their mathematical learning

✓ Explain the academic vocabulary needed to access mathematical ideas, providing both examples and non-examples

✓ Use strategically organized groups that attend to language needs*

✓ Use rich mathematical contexts and sophisticated language to help ELs progress in their linguistic development*

✓ Use cognates and root words (when appropriate) to link new math terms to students' background knowledge*</td><td>Built into the MathLinks Design:
SP: Word Bank, Vocabulary Review, Student Resources
TE: Grouping suggestions, References to Journals, Suggested problems for the MathLinks Rubric
PR: Math Talks
OR: Critique student work on Slide Decks

[3.1, 3.2, 3.3] For Twinkie the Dog, and anywhere in this packet, encourage students to dig into the real-life examples and complex tasks that apply the proportional ideas they are learning.

[Review] Poster Problems provide the opportunity to review math content and practice language skills.

[Review] For Match and Compare Sort, students use mathematical vocabulary in a safe environment.</td></tr>
</table>

Components cited: Student Packet (SP), Teacher Edition (TE), Packet Resources (PR), Other Resource (OR)

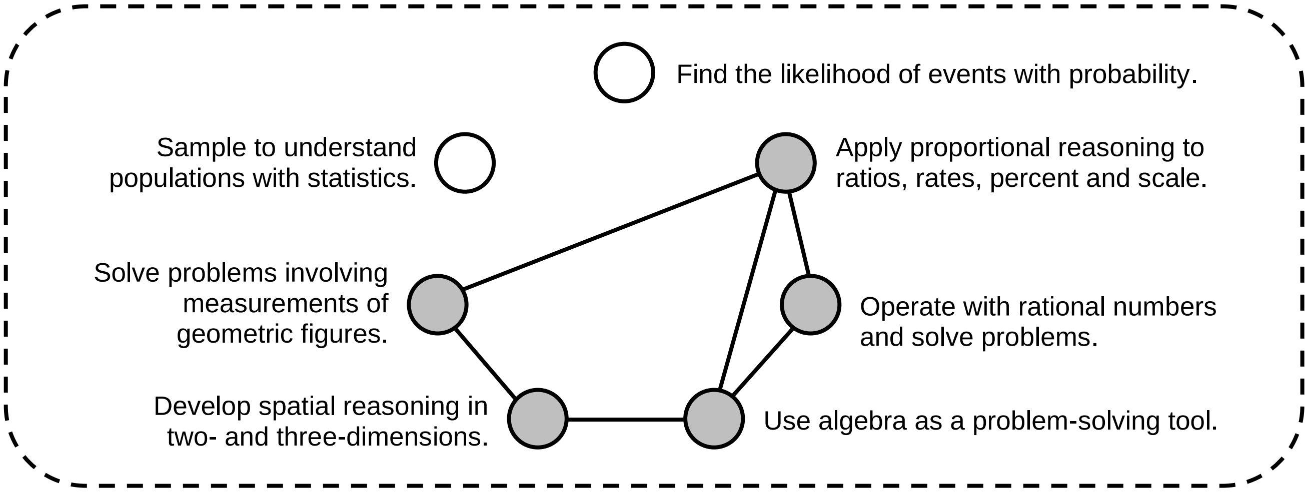
Strategies to Support Diverse Populations (Continued)		
	General Examples	***MathLinks* Examples**
Increase Comprehensible Input	✓ Link concepts to past learning ✓ Make concepts meaningful through hands-on activities, visuals, demonstrations, and color-coding ✓ Use a think-aloud strategy to model appropriate thinking processes and academic language use ✓ Use graphic organizers to help students record information and data, see patterns, and generalize them ✓ Use multiple representations (pictures, numbers, symbols, words, contexts) of math ideas to create meaning and make connections ✓ Strategically sequence and scaffold to make mathematics accessible ✓ Simplify written instructions, rephrase explanations, and use verbal and visual clues*	Built into the *MathLinks* Design: SP: Structured workspace TE: Slide Deck Alternatives, Reproducibles, Materials OR: Slide Decks [3.0] Students review tables, graphs, and double number lines prior to extending their understanding of proportional relationships. [3.1, 3.2, 3.3] Proportional reasoning is developmental. Encourage counting strategies (additive) as well as proportional strategies (multiplicative) for students who are not yet thinking proportionally. Review the concepts of part and whole (whole : part and part : part) examples using manipulatives (e.g., 3 blue blocks : 7 red blocks). Review representations like tape diagrams and double number line diagrams to help with proportional reasoning. Check frequently for understanding by having students draw or restate solutions in their own words.
Promote Student Interaction	✓ Use flexible group configurations that support content objectives ✓ Use strategies and activities that promote teacher/student and student/student interactions (e.g., think-pair-share, Poster Problems) ✓ Encourage elaborate responses through questioning ✓ Allow processing time and appropriate wait time, recognizing the importance of the different requirements for speaking, reading, and writing in a new language* ✓ Allow alternative methods to express mathematical ideas (e.g., visuals, students' first language)*	Built into the *MathLinks* Design: SP: Lesson and Review activities TE: References for Journals, Suggested problems for the *MathLinks* Rubric PR: Math Talks, various games and puzzles OR: Slide Decks, Activity Routines [Review] **Poster Problems**, **Matching Activity**, and **Match and Compare Sort** give students opportunities to exchange ideas and support each other. Be sure to place English learners in groups where they will feel safe to do this.

Components cited: Student Packet (SP), Teacher Edition (TE), Packet Resources (PR), Other Resources (OR)

Big Ideas and Connections

The Center for Mathematics and Teaching is dedicated to igniting and nurturing passion for mathematics in middle school students. We see the classroom as a place of joy and wonder, collaboration and purpose, perseverance and empowerment. We want all students to succeed in mathematics, as they explore its beauty in patterns, concepts, connections, and applications.

MathLinks: Grade 7 is organized around seven big ideas. This graphic provides a snapshot of the ideas in Packet 3 and their connections to each other.



These ideas build on past work and prepare students for the future. Some of these include:

Prior Work	What's Ahead
• Read a protractor (Grade 4) • Computational fluency with decimals and fractions (Grade 6) • Ratio concepts and reasoning (Grade 6) • Percent of a quantity (Grade 6) • Measurement and conversion of measurement units (Grades 4, 5) • Length and area (Grades 3, 4, 5, 6)	• Understand connections between proportional relationships, lines, and linear equations (Grades 8+) • Use functions to model relationships between quantities (Grades 8+) • Solve problems using equations (Grades 8+) • Dilations and similarity (Grades 8+) • Represent vector quantities (HS)

Students May Wonder…

- Real life ratios and rates: Ask students to think of real-world examples that have different units of measurement. For example, temperature can be measured using degrees Fahrenheit or degrees Celsius. Distance may be measured in miles or kilometers. Ask students to identify rates in their everyday lives. Encourage creative units of measurement like “math problems per minute” or “high-fives per day.”
- ***What is the most expensive item in the grocery store?*** Encourage students to explore unit rates in a real life setting and share what they learned.

Algebra in *MathLinks*: Grade 7

Algebra topics primarily appear in the CCSS-M Expressions and Equations and Ratios and Proportional Relationships domains. These areas are the focus of four packets in *MathLinks*: Grade 7, and they extend work introduced in 6th grade.

- In Packet 2, **Percent and Scale** (this packet), students analyze and solve problems involving numerical and algebraic expressions and involving percents.

- In Packet 3, **Proportional Relationships**, students connect different representations (i.e., visual contexts, tables, graphs, equations, word descriptions) as they solve problems involving proportional relationships. Special attention is paid to whether two quantities are in a proportional relationship by analyzing tables, graphs, and equations. Students continue to develop flexibility when working with variables, expressions, and equations. Double number lines facilitate the learning of how to solve proportions (i.e., equations in the form $\frac{x}{a} = \frac{b}{c}$).

- In Packet 6, **Expressions**, students use a visual context to write numerical and algebraic expressions, paving the way to greater flexibility working with variables and expressions. Equations of the $y = mx + b$ are explored without formally addressing function, slope, and vertical intercept, which is done in 8th grade.

  The counter manipulative used for developing integer operations in Packets 4 and 5 (**Rational Number Addition and Subtraction** and **Rational Number Multiplication and Division**) is extended to include cups to represent an unknown in an equation. This model gives students a tool for exploring and rewriting more difficult expressions.

- In Packet 7, **Solving Equations and Inequalities,** students extend the use of substitution to solve equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Cups and counters help to facilitate the learning of the procedures. Students also learn to solve inequalities with negative coefficients and open/closed boundary points.

Additionally, in Packets 8, 9, and 10 (**Plane and Solid Figures**; **Length, Area, and Volume**; **and Sampling**), students apply their knowledge of proportional relationships and algebra to solve problems in other domains.

"Simplify" and the Common Core State Standards

A quick word search of the Common Core State Standards in Mathematics will locate the word "simplify" on page 7 only:

> "Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation…"

In other words, no standard explicitly asks students to "Simplify $\frac{4}{6}$, calling for the "answer of $\frac{2}{3}$."

According to Bill McCallum, one of the authors of the CCSS-M document, this is intentional. The writers of the Standards want students to be flexible thinkers who use appropriate forms of numbers and expressions in given situations. In these lessons, students write expressions and rational numbers in multiple forms {e.g., $2 - 3(1 - 6) = 2 - 3(-5) = 2 - (-15) = 17$ or $4\frac{1}{2} = \frac{9}{2} = \frac{-18}{-4} = 4.5$}, and we give guidance to the teacher to discuss equivalence and conventions.

In this program, we still find it useful to give the instruction "simplify," and we expect students will write numbers and expressions in some conventionally accepted simpler form.

Proportional Reasoning Representations

Throughout this packet students will encounter, or choose to use, representations for solving proportional reasoning problems from 6^{th} grade or earlier in this course. A few are presented below as a refresher.

“Tables of equivalent ratios” or **“ratio tables.”** Technically, these tables do not have ratios as entries. But numbers in the tables may be written as ratios or thought of as ratios.

Entries in the columns of the table to the right represent equivalent ratios of stars to circles.

# of Stars	2	4	6
# of Circles	3	6	9

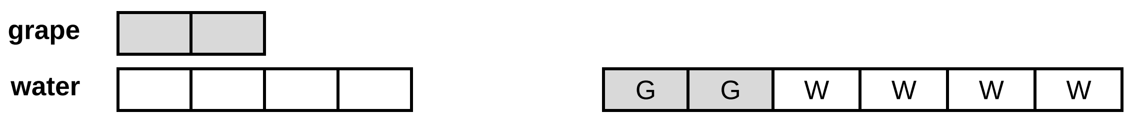
Example: 2 stars : 3 circles is equivalent to 4 stars : 6 circles
4 stars : 6 stars is equivalent to 6 circles : 9 circles

Tables may also have variables as column heads. In this case, the rows would have entries that indicate equivalent ratios.

Tables may have more than two rows or columns, corresponding to more than two variables. In this case, any two columns or rows determine number pairs that form equivalent ratios.

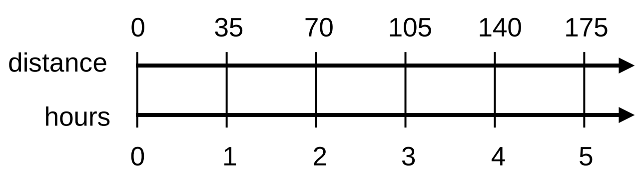
A **tape diagram** is a visual model consisting of strips divided into rectangular segments whose areas represent relative sizes of quantities. Tape diagrams are typically used when quantities have the same units.

Example: Below are two tape diagrams that show that the ratio of grape concentrate to water is 2 : 4.



A **double number line diagram** is a graphical representation of two variables, in which the corresponding values are placed on two parallel number lines for easy comparison. Double number lines are often used to compare two quantities that have different units.

Example: The double number line below shows corresponding ratios if a car goes 70 miles for every 2 hours.



We intentionally do not show negative numbers on our double number lines that correspond to ratios. According to the Progressions for the Common Core State Standards in Mathematics document for proportional reasoning, a <u>ratio</u> is a pair of non-negative numbers $A : B$, which are not both zero. Therefore, all double number lines in this document show only rays emanating from 0.

Algebra in *MathLinks*: Grade 7

Algebra topics primarily appear in the CCSS-M Expressions and Equations and Ratios and Proportional Relationships domains. These areas are the focus of four packets in *MathLinks*: Grade 7, and they extend work introduced in 6th grade.

- In Packet 2, **Percent and Scale**, students analyzed and solved problems involving numerical and algebraic expressions and involving percents.

- In Packet 3, **Proportional Relationships** (this packet), students connect different representations (i.e., visual contexts, tables, graphs, equations, word descriptions) as they solve problems involving proportional relationships. Special attention is paid to whether two quantities are in a proportional relationship by analyzing tables, graphs, and equations. Students continue to develop flexibility when working with variables, expressions, and equations. Double number lines facilitate the learning of how to solve proportions (i.e., equations of the form $\frac{x}{a} = \frac{b}{c}$).

- In Packet 6, **Expressions**, students use a visual context to write numerical and algebraic expressions, paving the way to greater flexibility working with variables and expressions. Equations of the form $y = mx$ are contrasted with those in the form $y = mx + b$ without formally addressing function, slope, and vertical intercept, which is done in 8th grade.

  The manipulative used for developing integer operations in Packets 4 and 5 (**Rational Number Addition and Subtraction** and **Rational Number Multiplication and Division**) is extended to include "cups and counters." This model gives students a tool for exploring and rewriting more difficult expressions.

- In Packet 7, **Equations and Inequalities,** students extend the use of mental math to solve equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. The Cups and Counters model helps to facilitate the learning of the procedures. Students also learn to solve inequalities with negative coefficients and open/closed boundary points.

Additionally, in Packets 8, 9, and 10 (**Plane and Solid Figures**; **Length, Area, and Volume**; **and Sampling**), students apply their knowledge of proportional relationships and algebra to solve problems in other domains.

REPRODUCIBLES

R3-1 MATCHING ACTIVITY: NUTS!

Note: Each column below has four equivalent representations. Cut into 16 cards for students to match. (Note: The error in the "Mixed Nuts" table is intentional.)

TRAIL MIX 2 pounds for $12.00		**CHOCO NUTS** 4 pounds for $10.00		**MIXED NUTS** 3 pounds for $9.00		**FRUIT 'N NUTS** $\frac{1}{2}$ pound for $1.75	
# of lbs	**price in $**	**# of lbs**	**price in $**	**# of lbs**	**price in $**	**# of lbs**	**price in $**
2	12	2	5	2	6	2	7
4	24	4	10	4	16	4	14
0.5	3	0.5	1.25	0.5	1.5	0.5	1.75
1	6	1	2.5	1	3	1	3.5
Unit Rate $6 per pound		**Unit Rate** $2.50 per pound		**Unit Rate** $3.00 per pound		**Unit Rate** $3.50 per pound	
Equation Let x = # of lbs and y = price in $ $y = 6x$		**Equation** Let x = # of lbs and y = price in $ $y = 2.5x$		**Equation** Let x = # of lbs and y = price in $ $y = 3x$		**Equation** Let x = # of lbs and y = price in $ $y = 3.5x$	

R3-2 MATCH AND COMPARE SORT CARDS: PROPORTIONAL RELATIONSHIPS

I △ INDEPENDENT VARIABLE	I ○ DEPENDENT VARIABLE
II △ UNIT RATE	II ○ UNIT PRICE
III △ PROPORTIONAL RELATIONSHIP	III ○ CONSTANT OF PROPORTIONALITY
IV △ INPUT-OUTPUT RULE	IV ○ EQUATION
A △ ✓ the graph of one of these is a straight line through the origin ✓ the values of all ordered pairs are some constant multiple of the values of any other, like (2, 5), (4, 10), and (8, 20)	A ○ ✓ a statement that asserts that two expressions are equal ✓ example: 20 = 15 + 5
B △ ✓ an equation that establishes a specific output value for each input value ✓ example: $y = 2.5x$	B ○ ✓ in a proportional relationship described by the equation $y = 3x$, it is 3 ✓ The unit rate in a proportional relationship
C △ ✓ the value of a ratio ✓ example: 45 miles per hour	C ○ ✓ a variable whose value is determined by the values of the independent variable ✓ typically, the output
D △ ✓ a variable whose value may be specified ✓ typically, the input	D ○ ✓ the price for one unit of measure ✓ example: $1.10 per orange

Name ____________________________ Period __________ Date _____________

PACKET 3
ANSWER KEY

MathLinks GRADE 7

PROPORTIONAL RELATIONSHIPS

Materials Grouping Reproducibles Slide Deck Journal Idea

Parent (or Guardian) signature ______________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

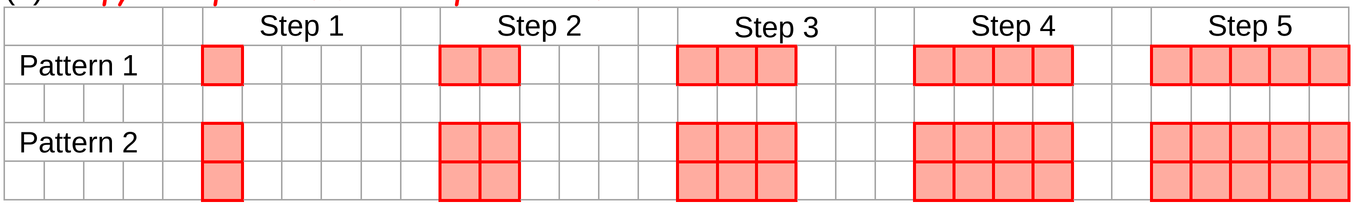
expression	equation input-output rule
proportional constant of proportionality proportional relationship	
ratio equivalent ratio	unit price unit rate

LENGTH AND AREA PATTERNS

[SMP3]

Follow your teacher's directions for (1) – (7).

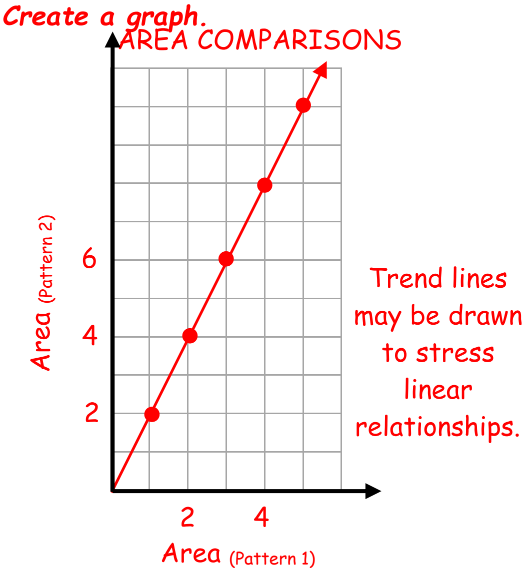
(1) *Copy each pattern. Draw Steps 4 and 5.*



(2) – (3) *Complete the table.*

AREA (A)			
	Pattern 1 ($A1$)	Pattern 2 ($A2$)	Ratio $A1 : A2$
Step 1	1	2	1 : 2
Step 2	2	4	2 : 4 or 1 : 2
Step 3	3	6	3 : 6 or 1 : 2
Step 4	4	8	4 : 8 or 1 : 2
Step 5	5	10	5 : 10 or 1 : 2
Step n	n	$2n$	

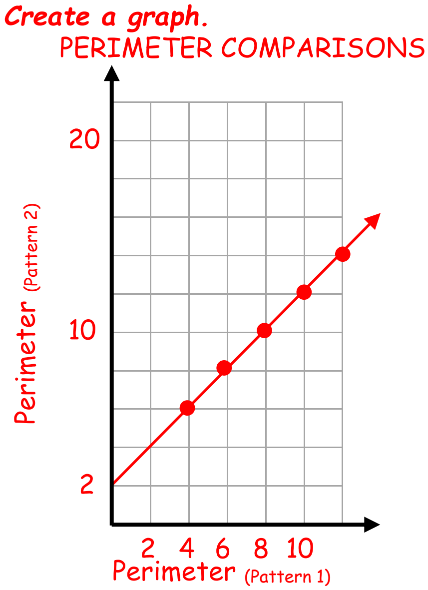
(4) *Create a graph.*



(5) – (6) *Complete the table.*

PERIMETER (P)			
	Pattern 1 ($P1$)	Pattern 2 ($P2$)	$P1 : P2$
Step 1	4	6	4 : 6 or 2 : 3
Step 2	6	8	6 : 8 or 3 : 4
Step 3	8	10	8 : 10 or 4 : 5
Step 4	10	12	10 : 12 or 5 : 6
Step 5	12	14	12 : 14 or 6 : 7
Step n	$2n + 2$	$2n + 4$	

(7) *Create a graph.*



8. Record the meanings of <u>ratio</u>, <u>equivalent ratios</u>, and <u>expression</u> in **My Word Bank**

LESSON NOTES S3.0: LENGTH AND AREA PATTERNS

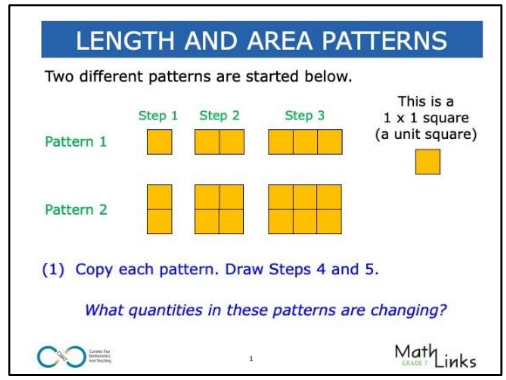
On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Students focus on growing geometric patterns for a review of multiple representations (tables, ratios, graphs, and algebraic rules). Toward the end of Lesson 1, we will revisit this exploration to further examine proportional relationships. This work builds towards a more formal study of linear functions in Grade 8.

- Slide 1: Show the growing patterns and ask students to verbalize them. ***What is a description of pattern 1?*** Start with 1 square tile, then add 1 more for each step. ***Pattern 2?*** Start with 2 square tiles, then add 2 more for each step.

  For (1), ask students to copy the patterns and extend them. Provide square tiles for those who need to build first.

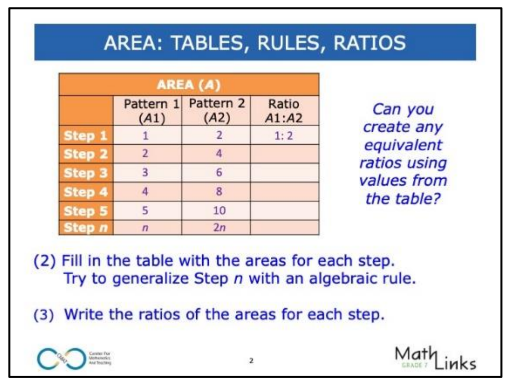
  What is changing in these patterns? What quantities could we record in a table? Dignify students' ideas by recording them. We will keep track of changing areas and perimeters.



- Slide 2: Ask students to copy the table headings. The notations $A1$ and $A2$ refer to areas of patterns 1 and 2 respectively. The third column will be used later. Pose (2) and provide time for students to finish the table.

  Do you see ways to create equivalent ratios with values in this table? Use the discussion to assess student familiarity with <u>ratios</u> and <u>equivalent ratios</u>. Ratios of values between *steps* are equivalent (e.g., values in lines 2 and 5 all have ratios of 2 : 5). Ratios of areas between *patterns* are equivalent ($A1$: $A2$ are all 1 : 2).

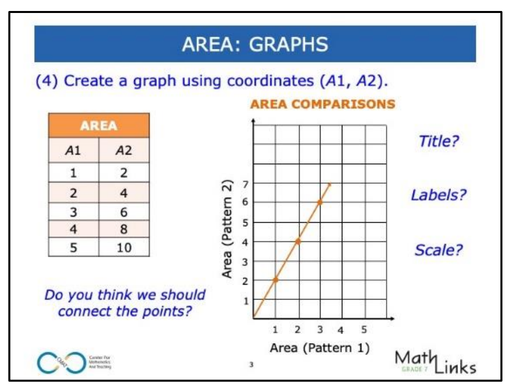
  Pose (3) and ask students to complete the table.



- Slide 3: For (4), students graph ordered pairs based on the ratios of $A1$: $A2$. Check labeling and scales.

  Should we "connect the dots" for these graphs? Since only whole number values for these variables makes sense, this pattern is graphed with discrete points. However, it is permissible to draw a "trend line" if we want to show an overall shape of a graph.

  What are some features of this graph? The points appear to lie along a straight line. For every 1 unit increase in $A1$, there is a 2 unit increase in $A2$.



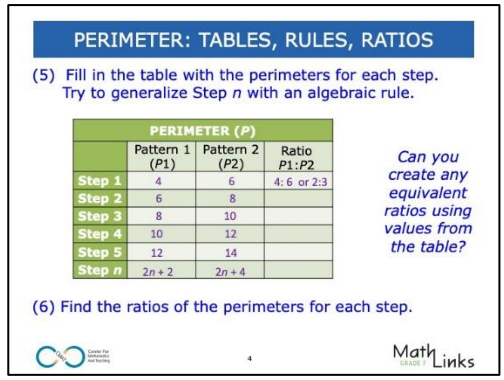
LESSON NOTES S3.0: LENGTH AND AREA PATTERNS

Continued

- Slide 4: For (5) and (6), follow a sequence of questions similar to the area pattern on slide 2.

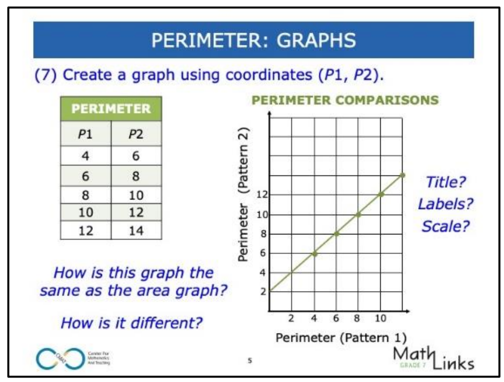
  How did you determine the general rules? Encourage students to describe the rule in terms of the step number. Talia's top number line increases by a constant rate, but the bottom number line does not.

  Can you create equivalent ratios from values in this table? No. There are no equivalent ratios within the perimeter patterns (e.g., values in lines 2 and 5 all have different ratios), nor are their equivalent ratios between patterns (e.g., $P1 : P2$ are not the same in different rows).



- Slide 5: For (7), students graph ordered pairs based on the ratios of $P1 : P2$. Remind students to label and scale graphs.

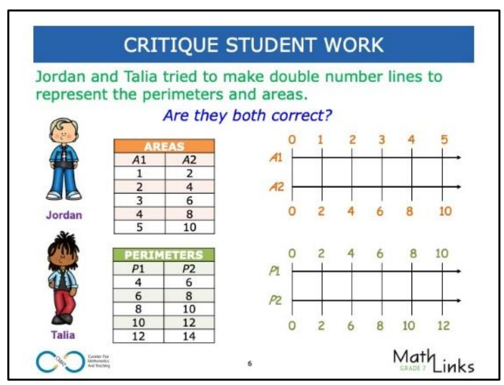
  Compare the two graphs. How are they the same? How are they different? Both are straight lines, and show that as the one measure increases, so does the other. Both increase at constant (but different) rates. The area graph goes through the origin, while the perimeter graph does not. (This fact will become important as we explore features of proportional relationships in this packet.)



- Slide 6: To deepen understanding of the appropriate use of double number lines, students critique the reasoning of Jordan and Talia.

  Who made correct double number lines? Jordan only. ***Why?*** Double number lines must have scales with equal intervals and show equivalent ratios. Talia's top number line increases by a constant rate, but the bottom number line does not. Furthermore, the $P1$ to $P2$ ratios for tick marks are not equivalent.

  Students will learn in the first lesson that the area relationship represents a proportional relationship, while the perimeter relationship does not. Only proportional relationships can be displayed on double number lines.



SLIDE DECK ALTERNATIVE S3.0: LENGTH AND AREA PATTERNS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

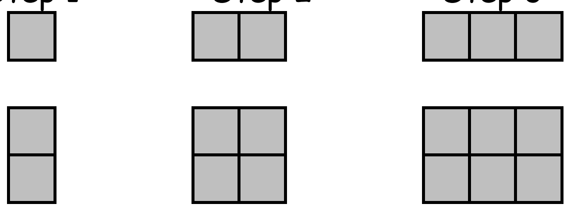
Slides 1 – 3

This is a 1 × 1 square (unit square). Two different patterns are started below.

Step 1 Step 2 Step 3 Step 4 Step 5

Pattern 1

Pattern 2



(1) Copy each pattern. Draw steps 4 and 5.

What quantities in these patterns are changing?

(2) - (3) Fill in the table with areas for each step and the ratios of areas in the last column. Try to generalize Step *n* with an algebraic rule.

AREA (A)			
Step	Pattern 1 (A1)	Pattern 2 (A2)	Ratio A1 : A2
1	1	2	1 : 2
2			
3			
4			
5			
n			

Discuss equivalent ratios that you see using values from the table.

(4) Create a graph using ordered pairs (A1, A2).

Title? Labels? Scale?

Do you think we should connect the points?

SLIDE DECK ALTERNATIVE S3.0: LENGTH AND AREA PATTERNS

Continued

Slides 4 – 5

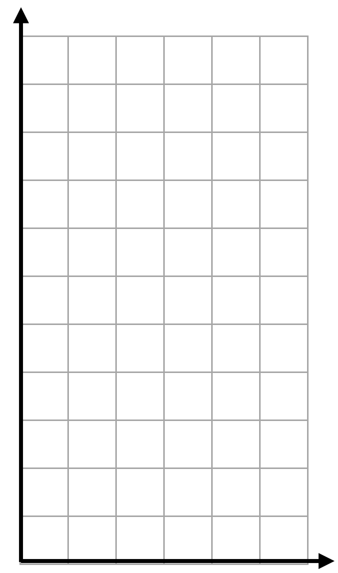
(5) - (6) Fill in the table with perimeters for each step and the ratios of perimeters in the last column. Try to generalize Step *n* with an algebraic rule.

PERIMETER (P)			
Step	Pattern 1 ($P1$)	Pattern 2 (P 2)	Ratio P 1 : P 2
1	4	6	4 : 6 = 2 : 3
2			
3			
4			
5			
n			

Discuss equivalent ratios that you see using values from the table.

(7) Create a graph using ordered pairs (P 1, P 2).

Title? Labels? Scale?



How is this graph the same as the area graph? How is it different?

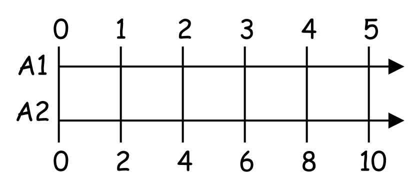
Slide 6

What did Jordan do correctly on the double number lines that Talia did not?

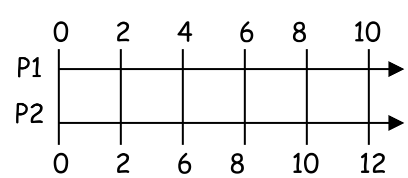
Jordan's table	
Areas	
A1	A2
1	2
2	4
3	6
4	8
5	10

Talia's table	
Perimeters	
P1	P2
4	6
6	8
8	10
10	12
12	14

Jordan



Talia



AN INTRODUCTION TO PROPORTIONAL RELATIONSHIPS

We will use tables and graphs to explore unit rates and unit prices. We will learn what it means for quantities to be in a proportional relationship, and identify the constant of proportionality (unit rate) in tables and graphs.

[7.NS.3, 7.RP.1, 7.RP.2ab, 7.G.1; SMP1, 3, 4, 5, 6]

GETTING STARTED

[SMP4]

Shmear 'N Things
4 bagels for $3.00

Hole-y Bread
5 bagels for $4.00

1. Complete the tables below. Assume each shop will sell you any number of bagels at the rates shown above.

Shmear 'N Things	
# of bagels (x)	cost in $ (y)
4	3
8	6
12	9
16	12
20	15

Hole-y Bread	
# of bagels (x)	cost in $ (y)
5	4
10	8
15	12
20	16
25	20

2. Fill in the table below using the data tables above.

Shmear 'N Things	$\frac{\text{cost in dollars}}{\text{\# of bagels}}$	$\frac{3}{4}$	$\frac{\boxed{6}}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{15}{20}$
	Simplify	$\frac{3}{4}$	$\frac{\boxed{3}}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
	Unit price (in dollars per bagel)	0.75	0.75	0.75	0.75	0.75
Hole-y Bread	$\frac{\text{cost in dollars}}{\text{\# of bagels}}$	$\frac{4}{5}$	$\frac{\boxed{8}}{10}$	$\frac{12}{15}$	$\frac{16}{20}$	$\frac{20}{25}$
	Simplify	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$
	Unit price (in dollars per bagel)	0.8	0.8	0.8	0.8	0.8

3. Which shop has the better buy? Explain.
Based on cost, Shmear 'N Things is the better buy, because $0.75/bagel < $0.80/bagel. But other factors may come into play, such as taste of bagel or driving distance to the store.

PROPORTIONAL RELATIONSHIPS

[SMP5,6]

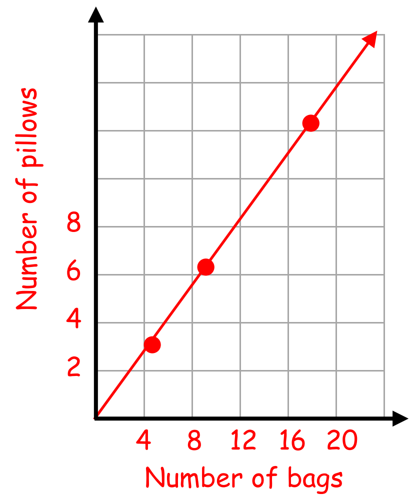
Follow your teacher's directions for (1) – (4).

(1a) Dion's Pillow Project *Table*

# of bags (x)	# of pillows (y)	Unit rate $\frac{\#\text{ of pillows}}{\#\text{ of bags}}$
24	16	$\frac{16}{24} = \frac{2}{3}$
18	12	$\frac{12}{18} = \frac{2}{3}$
9	6	$\frac{6}{9} = \frac{2}{3}$
4.5	3	$\frac{3}{4.5} = \frac{2}{3}$

(1b) *Graph*

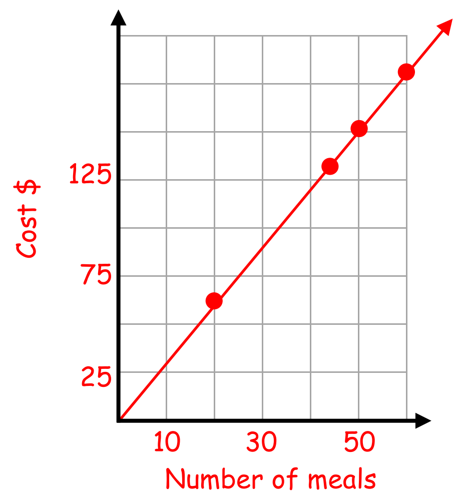
DION'S PILLOW PROJECT



(2) Ayla's Community Service

# of meals (x)	Cost in \$ (y)	Unit rate $\frac{\text{cost in \$}}{\#\text{ of meals}}$
20	60	$\frac{60}{20} = 3$
50	150	$\frac{150}{50} = 3$
42	126	$\frac{126}{42} = 3$
60	180	$\frac{180}{60} = 3$

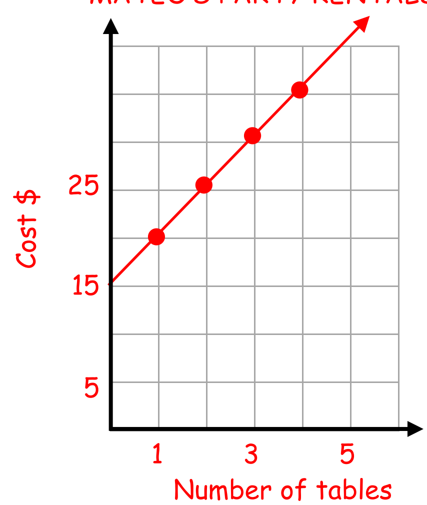
AYLA'S COMMUNITY SERVICE



(3) Mateo's Party Rentals

# of tables (x)	Cost in \$ (y)	Unit rate $\frac{\text{cost in \$}}{\#\text{ of tables}}$
1	20	$\frac{20}{1} = 20$
2	25	$\frac{25}{2} = 12.5$
3	30	$\frac{30}{3} = 10$
4	35	$\frac{35}{4} = 8.75$

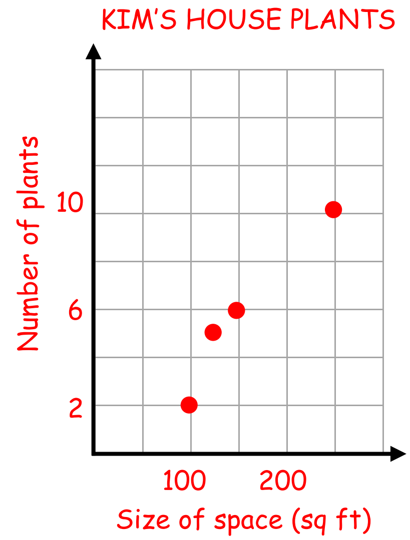
MATEO'S PARTY RENTALS



PROPORTIONAL RELATIONSHIPS

Continued

(4) **Kim's House Plants**			
Part of the house	Size of the space in sq ft (*x*)	# of plants in the space (*y*)	Unit rate $\frac{\text{\# of plants}}{\text{sq ft}}$
bedroom	100	2	$\frac{2}{100} = 0.02$
kitchen	125	5	$\frac{5}{125} = 0.04$
den	150	6	$\frac{6}{150} = 0.04$
patio	250	10	$\frac{10}{250} = 0.04$



5. Choose the ordered pair in each table for problems (1) – (3) that has the smallest *x*-value. Double both the *x*-value and the *y*-value and write them below.

	Ordered pair with least (*x*, *y*) values	Ordered pair with doubled *x*-value and *y*-value	Would this point lie on the line of the existing graph?	Is the unit rate the same as other entries in the table?
Problem 1	(4.5 , 3)	(9 , 6)	yes	yes
Problem 2	(20 , 60)	(40 , 120)	yes	yes
Problem 3	(1 , 20)	(2 , 40)	no	no

6. Which situations from problems (1) – (4) describe proportional relationships? Explain.
The situations in problems (1) – (2) are proportional relationships because ratios are constant multiples of one another, each entry in the respective tables are equivalent unit rates, and both graphs are lines through the origin.

7. Record the meanings of <u>equation</u>, <u>unit rate</u>, <u>unit price</u>, <u>proportional (relationship)</u>, and <u>constant of proportionality</u> in **My Word Bank**.

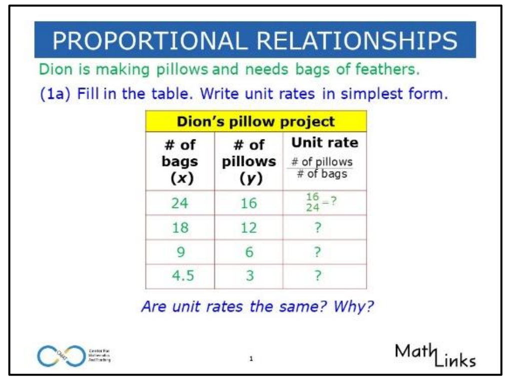
LESSON NOTES S3.1a: PROPORTIONAL RELATIONSHIPS

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Given four different situations, students complete tables, find unit rates, and create graphs in order to look for elements that determine a proportional relationship. They learn that in a proportional relationship, all ratios must be constant multiples of one another, all entries that could be in a given table must have equivalent unit rates, and all possible graphed points must be in a line (ray) that goes through the origin.

- Slide 1: Use this example to review the meaning of <u>unit rate</u>. For (1a), students copy the given information into the table and find the unit rate (number of pillows per bag) for the first entry. Reveal each entry one-by-one, allowing time for students to calculate the unit rate.

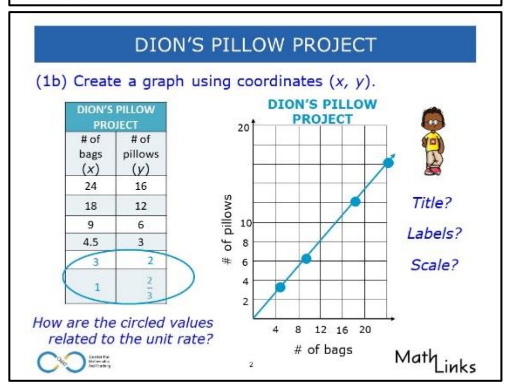
  How do the unit rates relate? They are all equal to $\frac{2}{3}$, because the ratio of # of bags to # of pillows is the same.



- Slide 2: For (1b), students create a graph from the data, attending to appropriate labels and scaling. Encourage students to extend a trend line back toward the origin to see if it goes through the origin or not.

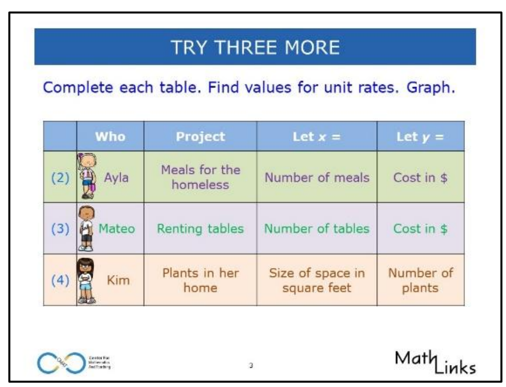
  Does it make sense to connect the graphed points? It depends whether factional values are relevant for the given variables. Fractions are appropriate for the number of bags. Whole numbers make sense for the number of pillows. Discuss the value of a trend line to show the linear pattern.

  Reveal the two circled values. ***How are they related to the unit rate?*** The unit rate is $\frac{2}{3}$. The first of these entries can be interpreted as 2 pillows for every 3 bags. The next shows the unit rate directly.



- Slide 3: Students continue with (2) – (4). Allow time for individual and collaborative work. Remind students to draw lines back toward the origin.

  A special unit rate is a <u>unit price</u>. For which situations did you compute unit prices? Ayla and Mateo in (2) and (3).



LESSON NOTES S3.1a: PROPORTIONAL RELATIONSHIPS

Continued

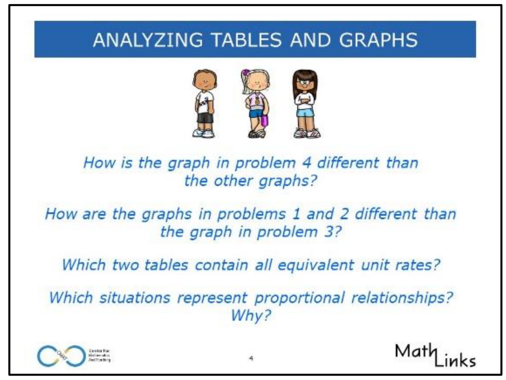
- Slide 4: Discuss important characteristics of a proportional relationship using the examples created in (1) – (4).

  How is the graph in problem 4 different than the other graphs? The points do not fall on a straight line like the other three graphs.

  How are the graphs in problems 1 – 2 different from the graph in problem 3? The lines graphed in problems (1) and (2) would go through the origin if drawn back toward the y-axis. The line graphed for problem 3 would intersect with the y-axis above the origin.

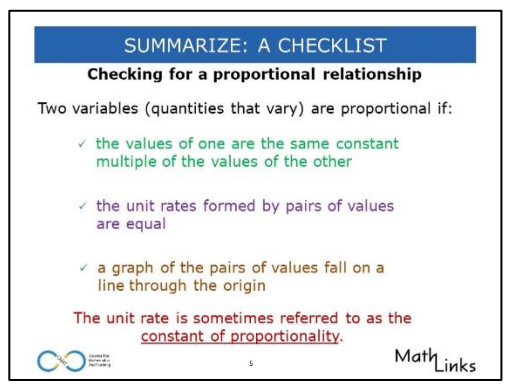
  Which two tables contain all equivalent unit rates? The unit rates in (1) all equal $\frac{2}{3}$. All unit rates in (2) equal 3.

  Ask students to look up proportional relationship in **Student Resources** and discuss the features illustrated so far. ***Which situations represent a proportional relationship?*** (1) and (2) ***Why?*** Each value of one is a constant multiple of the corresponding value of the other. The unit rates formed by pairs of values are equal. Their graph falls on a line (ray) through the origin. The equation feature of a proportional relationship will be explored in an upcoming lesson.



- Slide 5: Use the slide to further summarize features of a proportional relationship.

  What is the constant of proportionality in (1) and (2)? It is the same as the unit rate. For (1), it is $\frac{2}{3}$. For (2), it is 3.



SLIDE DECK ALTERNATIVE S3.1a: PROPORTIONAL RELATIONSHIPS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1 – 2

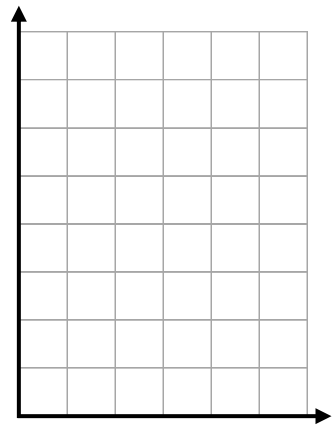
Dion is making pillows and needs bags of feathers.

(1a) Fill in the table.
Write unit rates in simplest form.

DION'S PILLOW PROJECT		
# of bags (x)	# of pillows (y)	Unit rate $\frac{\text{\# of pillows}}{\text{\# of bags}}$
24	16	
18	12	
9	6	
4.5	3	

Are the unit rates the same? Why?

(1b) Create a graph using coordinates (x, y). Don't forget title, labels, scale.



Slide 3

Finish each table. Find values for unit rates. Graph.

	Who	**Project**	**Let x =**	**Let y =**
(2)	Ayla	Meals for the homeless	Number of meals	Cost in $
(3)	Mateo	Renting tables	Number of tables	Cost in $
(4)	Kim	Plants in her home	Size of space in square feet	Number of plants

SLIDE DECK ALTERNATIVE S3.1a: PROPORTIONAL RELATIONSHIPS

Continued

Slide 4

How is the graph in problem (4) different than the other graphs?

How are the graphs in problems (1) and (2) different than the graph in problem (3)?

Which two tables contain all equivalent unit rates?

Which situations represent proportional relationships? Why?

Slide 5

Features of a Proportional Relationship

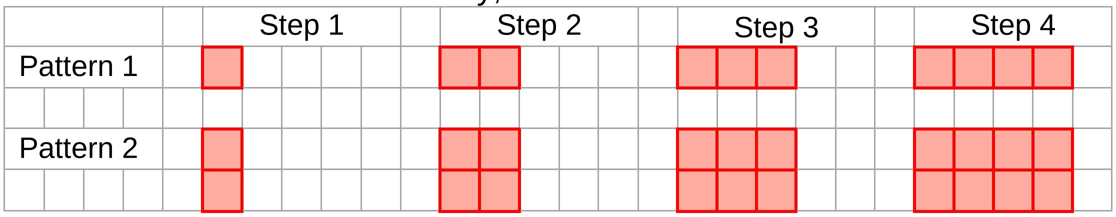
Two variables (quantities that vary) are proportional if:

- ✓ the values of one are the same constant multiple of the values of the other
- ✓ the unit rates formed by pairs of values are equal
- ✓ a graph of the pairs of values fall on a line through the origin

The unit rate is sometimes referred to as the <u>constant of proportionality</u>.

PRACTICE 1

1. Go back to the opening problem. First copy the patterns. Then copy the area and perimeter ratio columns in the table below. Finally, fill in the unit rate columns in the table below.



Compare Areas and Perimeters – Pattern 1 : Pattern 2				
step #	$A1 : A2$	unit rate $\frac{A2}{A1}$	$P1 : P2$	unit rate $\frac{P2}{P1}$
1	1 : 2	2	4: 6	$\frac{3}{2}$
2	2 : 4	2	6 : 8	$\frac{4}{3}$
3	3 : 6	2	8 : 10	$\frac{5}{4}$
4	4 : 8	2	10 : 12	$\frac{6}{5}$

2. Do the area ratios and perimeter ratios appear to be proportional relationships? Explain.
Yes for the area ratios. They are all equivalent. No for the perimeter ratios. They are not equivalent. This is why we cannot draw a double number line for the perimeter relationship.

3. What if each square was NOT a unit square, but rather had a side length equal to $\frac{1}{2}$ unit of length? Fill in the table below for this situation.

Compare Areas and Perimeters – Pattern 1 : Pattern 2				
step #	$A1 : A2$	unit rate $\frac{A2}{A1}$	$P1 : P2$	unit rate $\frac{P2}{P1}$
1	$\frac{1}{4} : \frac{1}{2}$	2	2 : 3	$\frac{3}{2}$
2	$\frac{1}{2} : 1$	2	3 : 4	$\frac{4}{3}$
3	$\frac{3}{4} : \frac{3}{2}$	2	4 : 5	$\frac{5}{4}$
4	1 : 2	2	5 : 6	$\frac{6}{5}$

4. Do the area ratios and perimeter ratios appear to be in a proportional relationship? Explain.
Yes for the area ratios. They all have equivalent values. No for the perimeter ratios. They do not all have equivalent values.

TWINKIE THE DOG

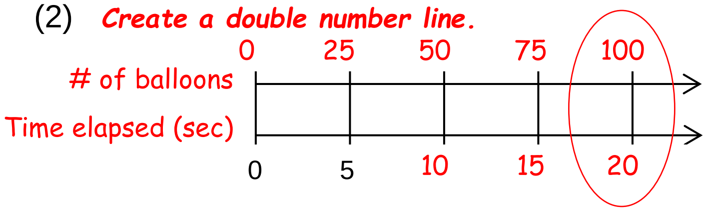
[SMP1,5,6]

Follow your teacher's directions for (1) – (6).

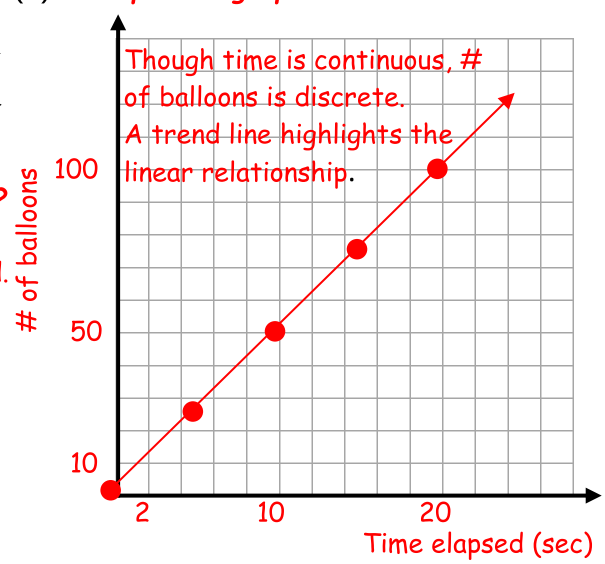
(1) Twinkie, the Jack Russell Terrier, pops balloons.

Predict how long it will take for her to pop them all.

(2) ***Create a double number line.***



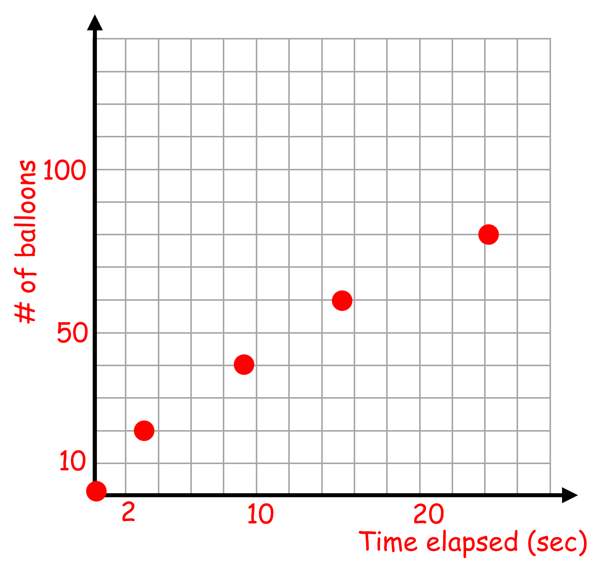
(3) ***Complete a graph.***



(4) ***Do you think Twinkie will break Cally's record?***
Answers will vary. Since Cally's record is 41.67 seconds, Twinkie is on track to break the record. But once some balloons are popped, the remaining balloons may be more spread out. Twinkie may be unable to keep up the pace and take more than 20 seconds to pop all of the balloons.

(5) ***Make a table and a graph from video data.***
Points may vary. Approximate data from video:

# of sec elapsed (x)	# of balloons (y)	unit rate $\frac{\text{\# of balloons}}{\text{\# of seconds}}$
0	0	
3	20	6.7
9	40	4.4
15	60	4
24	80	3.3



(6) ***Does Twinkie's record pace represent a proportional relationship?***
This is not a proportional relationship. Neither variable is a constant multiple of the other. The unit rates are different for different pairs of values. The graph goes though the origin, but it is not a straight line. Twinkie popped fewer balloons per second (slowed down) as she progressed.

LESSON NOTES S3.1b: TWINKIE THE DOG*

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Twinkie's balloon-popping exploits require students to first ponder (from the first part of a video) whether she can continue at a constant rate, and then use multiple representations to show what happens if she does. Then after watching the conclusion of the video, we find that Twinkie's balloon popping, in contrast, does not represent a proportional relationship.

- Slide 1: Click the picture to play the 5-second video. Record students' responses to the questions to recognize their ideas.

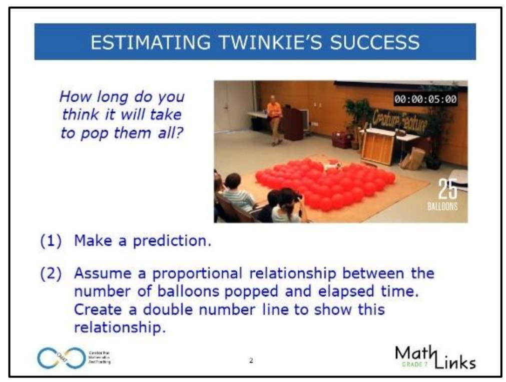
  What do we know? There are 100 balloons. Twinkie pops 25 balloons in the first 5 sec so there are 75 balloons left.

  What do you wonder? Some possibilities: Can she pop them all? How long it will take? Will she quit? Will she speed up or slow down? Will she get distracted by the kids in the room?

- Slide 2: ***How long will it take to pop them all?*** Encourage students to first guess a lower limit and upper limit for their estimates. Share reasoning and representations.

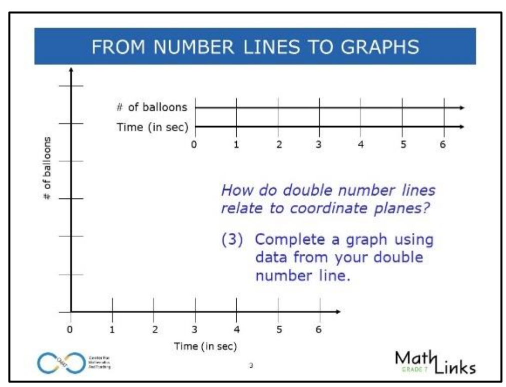
  If Twinkie could pop 25 balloons every 5 seconds, then how many balloons would she pop in 10 seconds? 15 seconds? 20 seconds? This would illustrate a proportional relationship because the values of the ratios $\left(\frac{\#\text{ of balloons}}{\text{time (sec)}}\right)$ are equivalent.

  For (1), students commit their predictions to writing.
  For (2), students create a double number line.



- Slide 3: Illustrate the relationship between a double number line and the axes of a coordinate plane. For (3), students complete a graph based on their data.

  Does this graph seem reasonable? At this point students may think that a proportional model will hold up, though some may question whether the dog can maintain a constant rate. Allow students to revise estimates.



*If desired, go to http://www.101qs.com/3933 for the lesson "World Record Dog" designed by Dan Meyer.

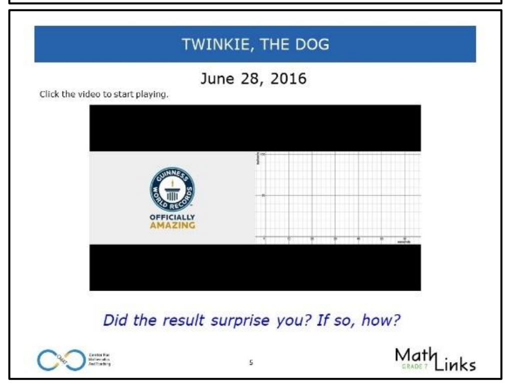
LESSON NOTES S3.1b: TWINKIE THE DOG

Continued

- Slide 4: Discuss Cally's world record. For (4), students explain whether they think Twinkie will break Cally's record. Encourage mathematical answers to the question, not just simple guesses. Allow opportunities for students to revise any estimates that may have been made.

- Slide 5: Play Part 2 of the video to reveal Twinkie's record-breaking accomplishment. Encourage students to explain why results did (or did not) match their predictions.

  Replay the video several times (if needed), and pause it along the way to collect some data for time from the start, x, and # of balloons popped, y. Discuss the graph as desired, but students will create their own soon.

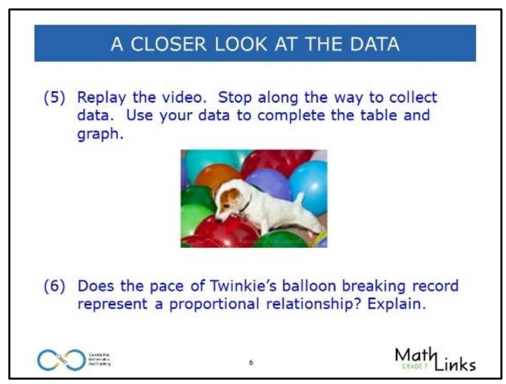


- Slide 6: For (5), students copy the data collected, complete the table, find unit rates, and complete the graph.

  What do the unit rates and the graph tell us about whether Twinkie is slowing down, speeding up, or staying at the same rate? She is slowing down since at each interval (though not "equally spaced" intervals) there are fewer balloons popped per second. This is seen in the graph. It increases more steeply (at a greater rate) at first, and increases more flatly (at a lesser rate) as time progresses.

  Pose (6) to see if students recognize that in fact, Twinkie's pace does NOT represent a proportional relationship. Make sure they explain their reasons: 1. the ratios extracted from the table do not represent constant multiples of one another; 2. the unit rates are not equal; 3. Even though the graph goes through the origin, it's not a line (ray).

  Why does not make sense to create a double number line to represent Twinkie's balloon popping? Double number lines are used to represent proportional situations.



SLIDE DECK SALTERNATIVE 3.1b: TWINKIE THE DOG

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option. Use the video at http://www.101qs.com/3933.

Slides 1 – 4

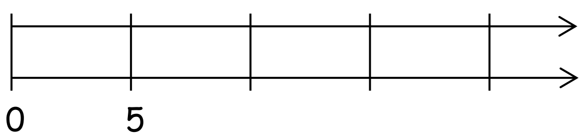
Watch a 5-second video where a Jack Russell Terrier, Twinkie, pops balloons.

What do we know?

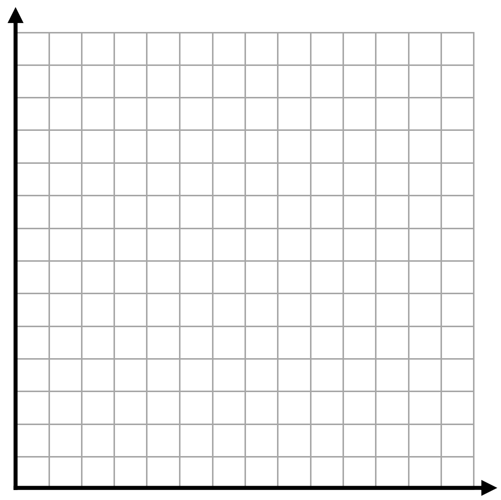
What do you wonder?

(1) Predict how long it will take for her to pop them all.

(2) Assume a proportional relationship between the number of balloons popped and time elapsed. Create a double number line to show this relationship.



(3) Complete a graph using data from your double number line.



(4) Another Jack Russell Terrier, Cally, popped 100 balloons in 41.67 seconds on "Britain's Got Talent" on May 15, 2015. Do you think Twinkie will break Cally's record? Explain.

Slides 5 – 6

Watch the entire video 40-second video.

Did the result surprise you? If so, how?

(5) Replay the video. Stop along the way to collect data. Use your data to complete the table and graph.

(6) Does the pace of Twinkie's balloon-breaking record represent a proportional relationship? Explain.

PRACTICE 2

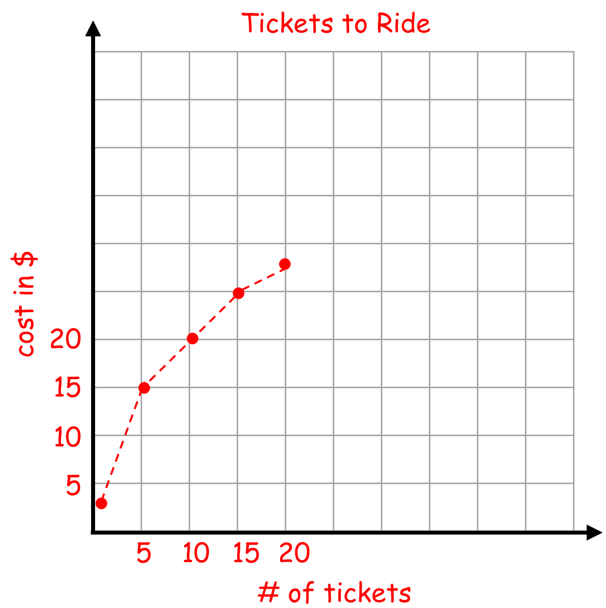
(Using the *MathLinks* Rubric) See Activity Routines in the Teacher Portal for directions. [SMP3, 4, 5, 6]

The Enchanted Hill amusement park offers different ticket price packages.

1. Find unit prices for the different packages. Then graph the relationship between cost and number of tickets. Be sure to scale, title, and label your graph appropriately.

Ticket To Ride

number of tickets (x)	cost in \$ (y)	cost (\$) / ticket
1	3	3
5	15	3
10	20	2
15	25	1.67
20	28	1.40



The trend line is not straight because this is not a linear relationship and not a proportional relationship.

2. Does the ticket pricing represent a proportional relationship? Explain.
   No. Entries are not constant multiples of one another. Unit prices are not equal for corresponding values of variables. The points, when graphed, do not fall on a straight line through the origin.

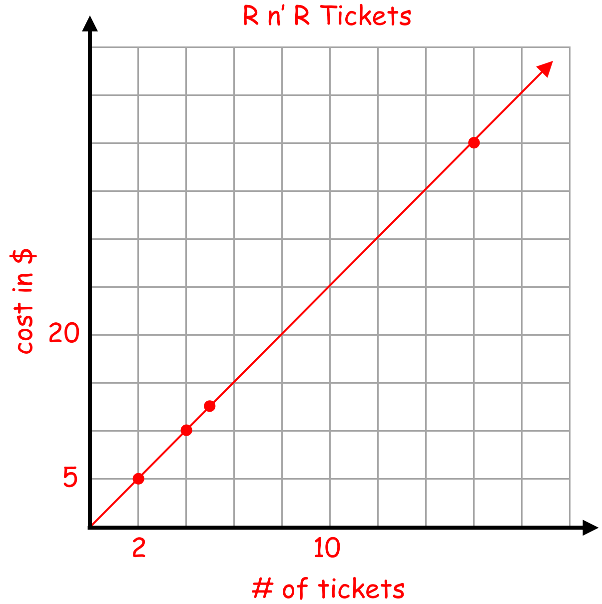
3. Which ticket option offers the best price in cost per ticket? Which would you choose? Explain.
   The best deal is 20 tickets for \$28. However, it is not good to buy more tickets than needed.

PRACTICE 2
Continued

4. Complete the table. Then graph the relationship between cost and number of tickets. Be sure to scale, title, and label your graph appropriately.

R n' R Tickets

number of tickets (x)	cost in \$ (y)	cost (\$) / ticket
4	10	2.50
16	40	2.50
2	5	2.50
5	12.50	2.50



The trend line is not straight because this is not a linear relationship and not a proportional relationship.

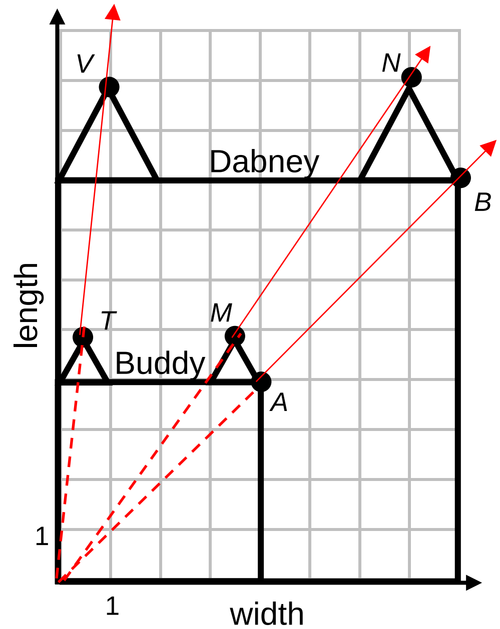
5. Does this represent a proportional relationship? Explain.
Yes. Entries are constant multiples of one another. Unit prices are equal. Points on the graph fall on a straight line through the origin.

6. Which basketball purchasing option offers the best buy? Which would you choose? Explain.
There is no difference in unit price for the various options because this represents a proportional relationship. However, it is probably best to choose the number of tickets you expect to use. That way you won' have to wait in line to buy tickets or pay for more than needed.

BUDDY, DABNEY, AND KILROY ARE BACK!

[SMP3]

Recall Buddy and Dabney from a previous lesson. Here are the backs of their heads.



1. Find the ratio of Buddy's width to Dabney's width. 4 : 8

2. Find the ratio of Buddy's length to Dabney's length. 4 : 8

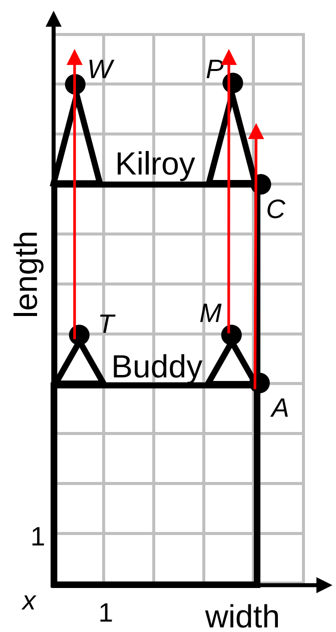
3. What is the multiplier (scale factor) that creates Dabney's head from Buddy's head? 2

4. Draw rays through the following corresponding points on their heads:
   - Ray *AB* (through top right of head)
   - Ray *MN* (through top of right ear)
   - Ray *TV* (through top of left ear)

   Would these rays extend back through the origin? yes

   What does this tell you about the relationship between the ordered pairs of Dabney's coordinates and Buddy's coordinates?
   These ordered pairs lie in a straight line through the origin. They are in a proportional relationship.

Now compare the heads of Buddy and Kilroy.



5. Find the ratio of Buddy's width to Kilroy's width. 4 : 4

6. Find the ratio of Buddy's length to Kilroy's length. 4 : 8

7. Why is there no multiplier (scale factor) that creates Kilroy's head from Buddy's head?
   The ratios of widths and lengths are different so there is no one multiplier.

8. Draw rays through the corresponding points on their heads:
   - Line segment *AC* (through top right of head)
   - Line segment *MP* (through top of right ear)
   - Line segment *TW* (through top of left ear)

   Would these rays extend back through the origin? No

   What does this tell you about the relationship between the ordered pairs of Buddy's coordinates and Kilroy's coordinates?
   These points do not lie in a straight line through the origin. They are not in a proportional relationship.

9. Which pair of friends have proportional faces? Buddy and Dabney

1+

Monitor Your Progress

DIGGING DEEPER INTO PROPORTIONAL RELATIONSHIPS

We will use tables, double number lines, graphs, and equations to explore what it means for a relationship between quantities to be proportional. We will pay special attention to the meaning of specific ordered pairs of quantities represented in the different representations.

[7.NS.3, 7.EE.3, 7.RP.1, 7.RP.2abcd; SMP3, 4, 5, 6]

GETTING STARTED

Complete each table and fill in the blanks.

1a.

x	1	2	3	4	5	6	10	15	20
y	4	8	12	16	20	24	40	60	80

b. Rate of change: for every increase of x by 1, y increases by 4.

c. Input-output rule (words): Multiply an x-value by 4 to get the corresponding y-value.

d. Input-output rule (equation): y = $4x$; the coefficient of x is 4.

e. If $x = 100$, then y = 400.

f. If $y = 100$, then x = 25.

2a.

x	1	2	3	4	5	6	8	11	13
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	4	$5\frac{1}{2}$	$6\frac{1}{2}$

b. Rate of change: for every increase of x by 1, y increases by $\frac{1}{2}$.

c. Input-output rule (words): Multiply an x-value by $\frac{1}{2}$ to get the corresponding y-value

d. Input-output rule (equation): y = $\frac{1}{2}x$; the coefficient of x is $\frac{1}{2}$.

e. If $x = 100$, then y = 50.

f. If $y = 100$, then x = 200.

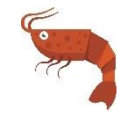
3. Record the meaning of input-output rule in **My Word Bank**.

CAP'N SHERMAN'S SHRIMP SHOP

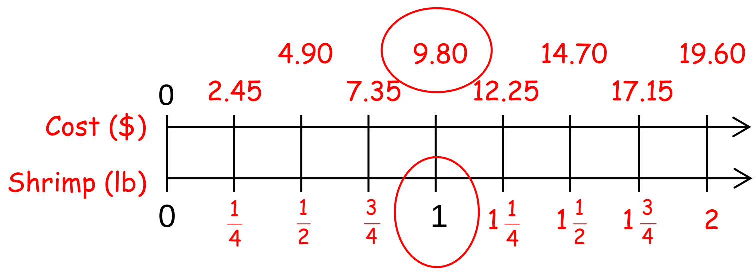
[SMP3,4,5,6]

Follow your teacher's directions.

A customer bought $\frac{3}{4}$ pounds of shrimp for $7.35.



(1) *Copy the fact statement above. Make a double number line. Circle the unit price per pound.*



(2) Use the double number line to find the cost for…

a. 2 pounds of shrimp → $19.60

b. 1.5 lb of shrimp → $14.70

(3) Use the double number line to find the amount of shrimp you can purchase for…

a. $2.45 → $\frac{1}{4}$ pound

b. $17.15 → $1\frac{3}{4}$ pounds

(4) *Complete the first two columns.*

(7) *Find the unit prices. Use division.*

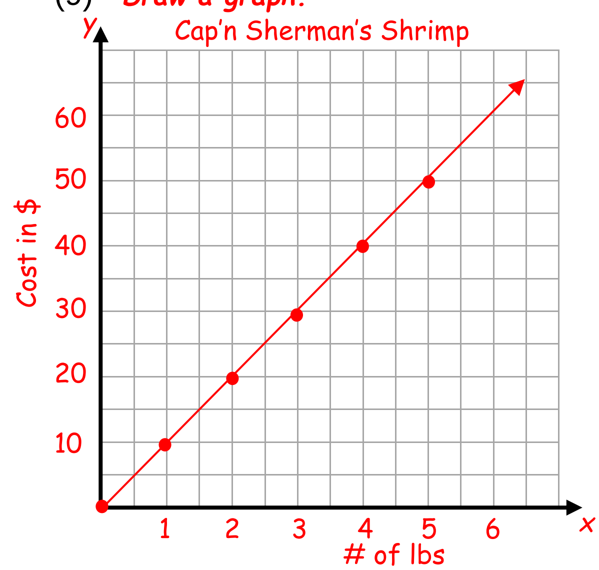
lbs shrimp (x)	Cost in $ (y)	Unit price $\left(\frac{y}{x}\right)$
0	0	
$\frac{1}{2}$	4.90	9.8
1	9.80	9.8
$1\frac{1}{2}$	14.70	9.8
2	19.60	9.8
3	29.40	9.8
4	39.20	9.8
5	49.00	9.8
10	98.00	9.8
x	$9.8x$	9.8

(5) *Explain the meaning of (0, 0).*
Buying 0 pounds of shrimp costs $0.

(6) *Explain the meaning of (1, 9.8).*
Buying 1 pound of shrimp costs $9.80.
This is the unit price (cost/lb).

(8) *Find an input-output rule (last row in the table).*
$y = 9.8x$

(9) *Draw a graph.*



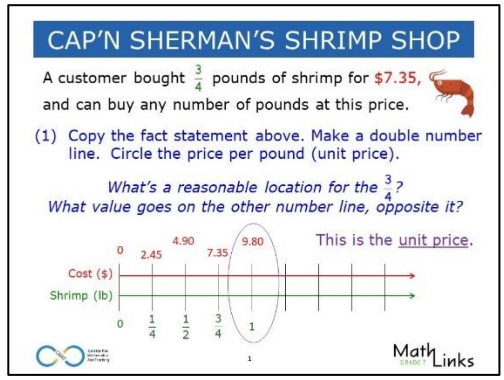
(10) *What is the constant of proportionality?* 9.8

LESSON NOTES S3.2: CAP'N SHERMAN'S SHRIMP SHOP

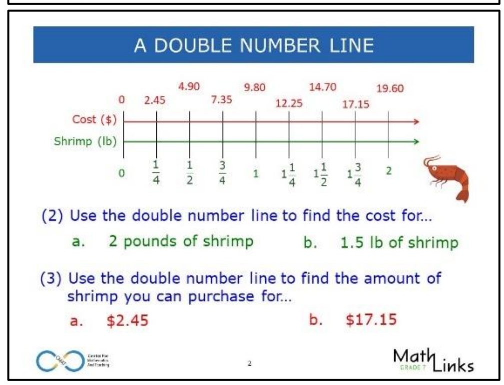
On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

At Cap'n Sherman's Shrimp Shop, a proportional relationship involving fractions and decimals is explored through multiple representations. Students interpret different (x, y) coordinates in context, specifically (0, 0) and (1, r) where r is the unit rate. Students write an equation of the form $y = kx$ for this proportional situation. They connect unit rate, r, to the constant of proportionality, k.

- Slide 1: For (1), students fill in the missing numbers in the fact statement and create a double number line showing cost vs. pounds of shrimp. Discuss how the unit rate in this case is a unit price, since it represents the price per pound of shrimp.



- Slide 2: Reveal (2) and (3) one at a time as students use their double number line to find the missing quantities. Share and discuss as needed before moving on.

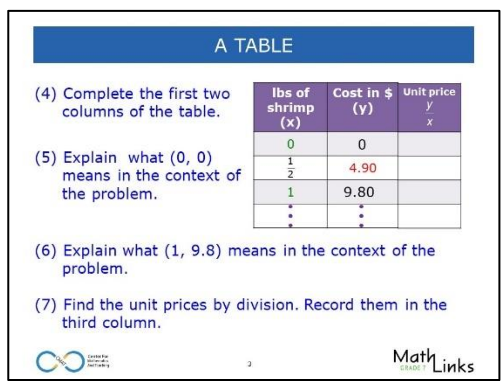


- Slide 3: For (4), allow time for students to use their double number line data to create a table.

  For (5) and (6), students explain the meaning of the ordered pairs (0, 0) and (1, 9.80) in the context of the Shrimp Shop. Again, share and discuss as needed.

  For (7), students fill in a third column in the table with unit price. Consider having students use calculators and share their work.

  What do you notice? All of the unit prices are equal. This value corresponds to the cost when the number of pounds is equal to 1.



LESSON NOTES S3.2: CAP'N SHERMAN'S SHRIMP SHOP

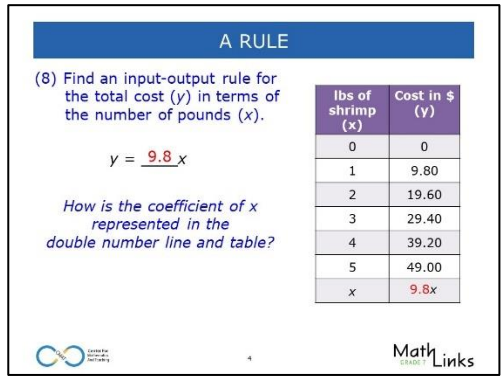
Continued

- Slide 4: For (8), students find an input-output rule in the form $y = kx$. Ask students to record it in the last row of the table.

  At this point, pause so students see how the coefficient in the equation reveals itself in different representations.

  Where does this coefficient show up in the double number line? It is the price for one pound of shrimp (unit price).

  Where do you find it in the table? Again, it is the cost for 1 pound. It is also the value of the ratio of cost/pound in the third column.



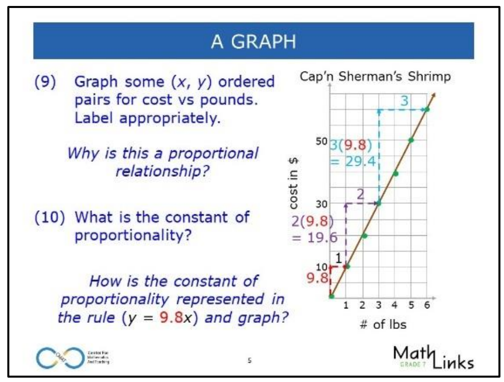
- Slide 5: For (9), students make a graph of the data from the table. Be sure they label and scale appropriately.

  How do we know this is a proportional relationship? Any ordered pair of values in the table is a constant multiplier of any other ordered pair. The unit rates formed by ordered pairs of values are equal. The graph of these ordered pairs fall on a line through the origin.

  Students complete (10). Review the term constant of proportionality as needed.

  How can we identify the constant of proportionality from the equation? It is the coefficient of x.

  How can we identify the constant of proportionality from the graph? This may be an eye opener for students. Reveal the triangles that show for every increase of 9.8 vertically, there is an increase of 1 horizontally. This is shown with the legs of three right triangles. Note that this is an informal preview of the important 8th grade concept of slope of a line.

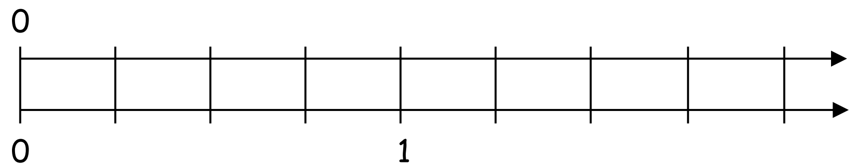


SLIDE DECK ALTERNATIVE S3.2: CAP'N SHERMAN'S SHRIMP SHOP

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1 – 3

A customer bought $\frac{3}{4}$ pounds of shrimp for \$7.35,
and can buy any number of pounds at this price.



What's a reasonable location for the $\frac{3}{4}$ *?*
What value goes on the other number line, opposite it?

(1) Copy the fact statement above. Make a double number line. Circle the price per pound (unit price).

(2) Use the double number line. Find the cost for:

a. 2 pounds of shrimp

b. 1.5 pounds of shrimp

(3) Use the double number line. Find the amount of shrimp you can purchase for:

a. \$2.45

b. \$17.15

(4) Complete the first two columns of the table.

(5) Explain what (0,0) means in the context of the problem.

lbs shrimp (x)	Cost in \$ (y)	Unit price $\left(\frac{y}{x}\right)$

(6) Explain what (1, 9.8) means in the context of the problem.

(7) Find the unit prices by division. Record them in the third column.

SLIDE DECK ALTERNATIVE S3.2: CAP'N SHERMAN'S SHRIMP SHOP

Continued

Slide 4

(8) Find an input-output rule for the total cost (y) in terms of the number of pounds (x).

How is the coefficient of x represented in the double number line and table?

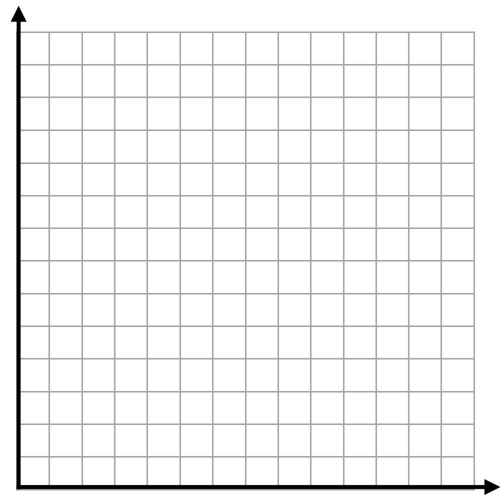
Slide 5

(9) Graph some (x, y) ordered pairs for cost vs. pounds. Label appropriately.

Why is this a proportional relationship?

(10) What is the constant of proportionality?

How is the constant of proportionality represented in the rule (y = 9.8x) and the graph?



PRACTICE 3

[SMP3]

Fruity-Fizzy-Water (FFW) is made using 5 cups of soda water for every 2 cups of fruit juice.

Note: it's probably easiest to start with the $x = 5$ entry and work from there.

1. Fill in the table for different mixtures of FFW. Show work as needed.

cups of soda water (x)	cups of fruit juice (y)
0	0
1	$2 \div 5 = \frac{2}{5}$
2	$2 \bullet \frac{2}{5} = \frac{4}{5}$
3	$3 \bullet \frac{2}{5} = \frac{6}{5}$ or $1\frac{1}{5}$
4	$4 \bullet \frac{2}{5} = \frac{8}{5}$ or $1\frac{3}{5}$
5	2
6	$6 \bullet \frac{2}{5} = \frac{12}{5}$ or $2\frac{2}{5}$
x	$\frac{2}{5}x$

2. Complete the paragraph:

   To keep the same flavor, a 1 cup increase in soda water requires an increase of $\frac{2}{5}$ cups of juice. The unit rate of cups of juice per 1 cup soda water is $\frac{2}{5}$. An equation that relates the amounts of juice to soda water is $y = \frac{2}{5}x$. One ordered pair is (1, $\frac{2}{5}$). Within the context of FFW, this represents

   the unit rate, or the amount of fruit juice needed for each cup of soda water.

   Another ordered pair is (0, 0). Within the context of FFW, this represents

   Answers will vary. One possible answer: 0 cups of soda are needed for every 0 cups of juice.

Show work as needed for problems 3 – 5.

3. How many cups of juice are needed to make the exact same flavor of FFW if 40 cups of soda water are used?
   16 cups of juice

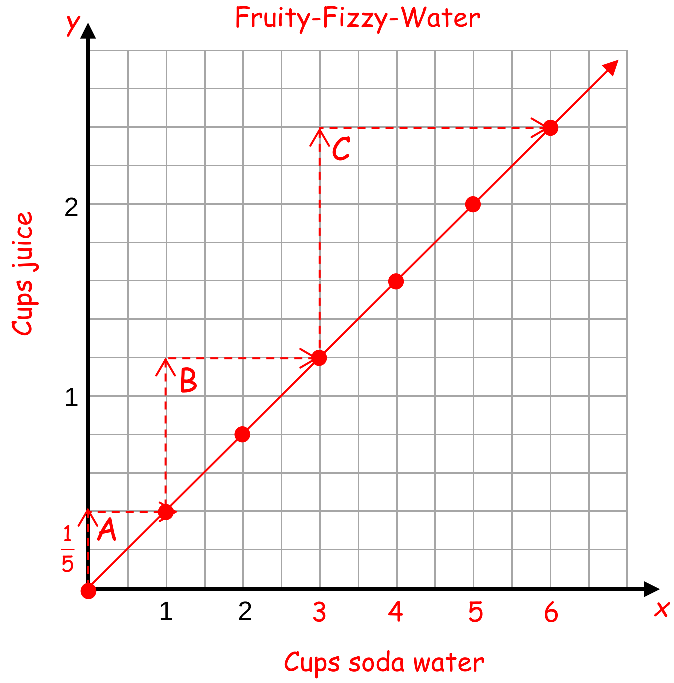
4. How many cups of soda water are needed to make the exact same flavor of FFW if 40 cups juice of are used?
   100 cups of soda water

5. How many cups of FFW can be made with using 10 cups of juice?
   35 cups FFW (10 cups juice, 25 cups soda water)

PRACTICE 3

Continued

6. Make a graph to represent cups of soda water and juice.



7. Draw the following right triangles on the diagram and complete the table.

	Vertices of right triangles	Length of vertical leg (change in y)	Length of horizontal leg (change in x)	$\frac{\text{change in } y}{\text{change in } x}$
Triangle A	$(0, 0)$, $(0, \frac{2}{5})$, $(1, \frac{2}{5})$	$\frac{2}{5}$	1	$\frac{2}{5}$
Triangle B	$(1, \frac{2}{5})$, $(1, 1\frac{1}{5})$, $(3, 1\frac{1}{5})$	$\frac{4}{5}$	2	$\frac{4}{10} = \frac{2}{5}$
Triangle C	$(3, 1\frac{1}{5})$, $(3, 2\frac{2}{5})$, $(6, 2\frac{2}{5})$	$\frac{6}{5}$	3	$\frac{6}{15} = \frac{2}{5}$

8. What is the meaning of the ratio of the lengths of the legs (last column in the table) in the context of the problem?
   It is another way to see the unit rate within the graph.

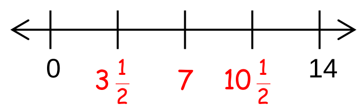
9. Write a few reasons that explain why the data in the tables and on this graph represent a proportional relationship.
   Each ordered pair is a constant multiple of the others; the values of the ratios are equivalent (equal unit rates); the graph is a line (ray) through the origin.

EQUATIONS AND PROBLEMS

We will write and solve equations created using equivalent rates, commonly referred to as "proportions." We will solve proportional reasoning problems using multiple strategies, including equations.

[7.RP.1, 7.RP.2bc,7.NS.3, 7.EE.3; SMP1, 2, 3, 5, 7, 8]

GETTING STARTED

1. What number times 4 is equal to 14? $3\frac{1}{2}$	2. Label each tick mark on the number line. 

Solve each equation using any method.

3. $\frac{56}{m} = 8$ 7	4. $5 = \frac{k}{9}$ 45	5. $\frac{1}{3}h = 11$ 33
6. $\frac{6}{5} = \frac{36}{p}$ 30	7. $\frac{27}{d} = \frac{3}{7}$ 63	8. $\frac{3}{4} = \frac{v}{14}$ $10\frac{1}{2}$

9. Circle all of the true equations below.
   Notice that they are variations of the true equation: $\frac{1}{2} = \frac{4}{8}$.

   a. $\frac{2}{1} = \frac{8}{4}$ b. $\frac{1}{4} = \frac{2}{8}$ c. $\frac{4}{1} = \frac{8}{2}$ d. $\frac{1}{8} = \frac{4}{2}$

   Choose an incorrect equation above and explain why it is NOT true.
   Part d states that $\frac{1}{8}$ = 2, which is a false statement.

10. Explain what is incorrect about each statement.

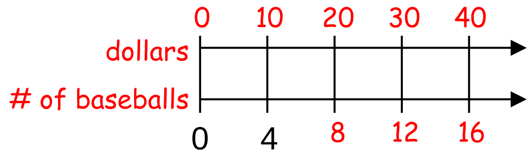
a. JB is 10 and Ang is 15. When JB is 20, Ang will be 30. Ang will always be 5 years older. When JB is 20, Ang will be 25.	b. It takes 3 people 4 hours to paint a room, so it will take 6 people 8 hours to paint the room. It should take twice as many people half the time to finish, so 6 people should take 2 hours.

DOUBLE NUMBER LINES AND EQUATIONS

[SMP2, 5, 7, 8]

Follow your teacher's directions.

(1) Four baseballs cost $\underline{\ 10\ }$.



Explanations and examples may vary. Some possible answers:

(2) ***Copy the statement. Generalize. Illustrate with another example.***

a. If $\frac{20}{8} = \frac{30}{12}$, then $\frac{8}{20} = \frac{12}{30}$

b. Ratios of dollars to baseballs are being compared in the "if" part. Rates of baseballs to dollars are being compared in the "then" part. In other words, if a statement equating two fractions is true, then we can "flip" both fractions and the new equation is still true.

c. If $\frac{10}{4} = \frac{40}{16}$, then $\frac{4}{10} = \frac{16}{40}$

(3) ***Copy the statement. Generalize. Illustrate with another example.***

a. If $\frac{20}{8} = \frac{30}{12}$, then $\frac{20}{30} = \frac{8}{12}$

b. Different units (dollars to baseballs) are being compared in the "if" part. The same units (dollars to dollars and baseballs to baseballs) are being compared in the "then" part. In other words, equivalent ratios may be created by comparing same units. If a statement equating two fractions is true, fractions created from corresponding numerators and denominators will be equivalent.

c. If $\frac{10}{4} = \frac{40}{16}$, then $\frac{10}{40} = \frac{4}{16}$

(4) ***Copy the statement. Generalize. Illustrate with another example.***

a. If $\frac{20}{8} = \frac{30}{12}$, then $20 \bullet 12 = 30 \bullet 8$

b. If a statement equating two fractions is true, then the product of the "diagonals" is equal. We call this the "cross multiplication property."

c. If $\frac{10}{4} = \frac{40}{16}$, then $10 \bullet 16 = 40 \bullet 4$

(5) ***How much will 9 baseballs cost if 4 baseballs cost $10? Use the double number line to set up the equation. Use the cross-multiplication property to solve it.***

$$\frac{10}{4} = \frac{x}{9}$$
$$4x = 90$$
$$x = 22.5$$

9 baseballs cost $22.50.

(6) ***Use a ratio other than 10 : 4. Write a different equation and solve for x.***

Ratio choices will vary. This one compares within units.

$$\frac{x}{20} = \frac{9}{8}$$
$$8x = 180$$
$$x = 22.5$$

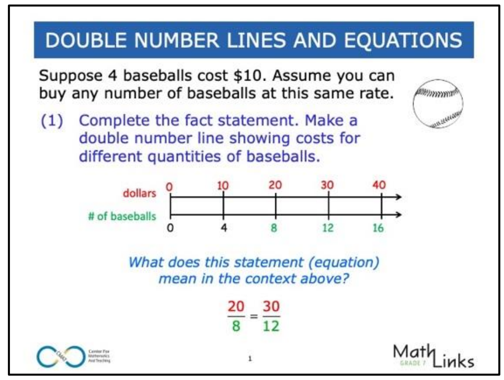
LESSON NOTES S3.3: DOUBLE NUMBER LINES AND EQUATIONS

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

Students use double number lines to explore and generalize some legal manipulations for equations formed by equal rates. They observe that equivalent fractions maintain equivalence when the reciprocals of both are taken. They also see that equations formed by comparing corresponding units maintains equality. Finally, they observe that a well-known shortcut, often referred to as "the cross-multiplication property," holds for equations of the form $\frac{a}{b} = \frac{c}{d}$, commonly referred to as "proportions."

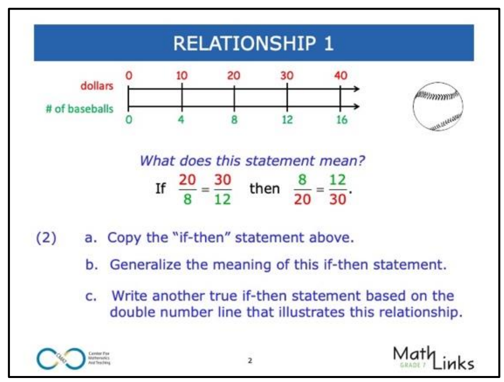
- Slide 1: For (1), pose the baseball-related ratio context. Ask students to copy the fact statement and create a double number line. Then reveal the finished double number line and ask students to interpret the equation $\frac{20}{8} = \frac{30}{12}$ in the context of the situation.

  What does this equivalent fraction statement mean in this context? $20 for 8 baseballs is the same rate as $30 for 12 baseballs; $\frac{20}{8} = 2.5$ and $\frac{30}{12} = 2.5$ (both are equivalent to $2.50 per ball).



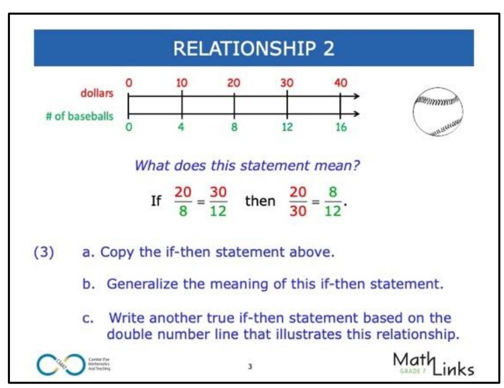
- Slide 2: Discuss the "if-then" statement. ***What does this statement mean?*** If $20 for 8 baseballs is the same rate as $30 for 12 baseballs, then 8 baseballs for $20 is the same rate as 12 baseballs for $30. This is the same statement, but in a different, consistent order.

  For (2), students copy the "if-then" statement, and try to explain and generalize it in their own words. They create another example based on the double number line. Use the answer key if needed to promote discussion.



- Slide 3: Discuss the "if-then" statement. ***What does this statement mean?*** This statement illustrates that we can also consistently compare dollars to dollars and baseballs to baseballs.

  Pose (3) and discuss in a similar manner as before.

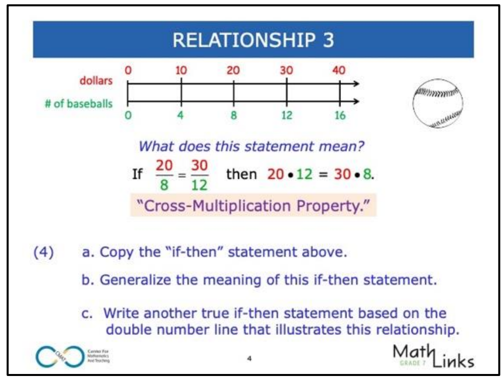


LESSON NOTES S3.3: DOUBLE NUMBER LINES AND EQUATIONS

Continued

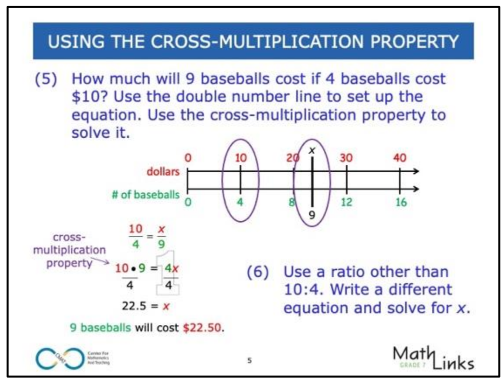
- Slide 4: Discuss this “if-then” statement. ***What does this statement mean?*** This statement illustrates that the “cross products” in the equation are equal. We refer to this as the well-known “cross multiplication property,” and make it plausible on slides 5 and 6.

  Pose (4) and discuss in a similar manner as before.



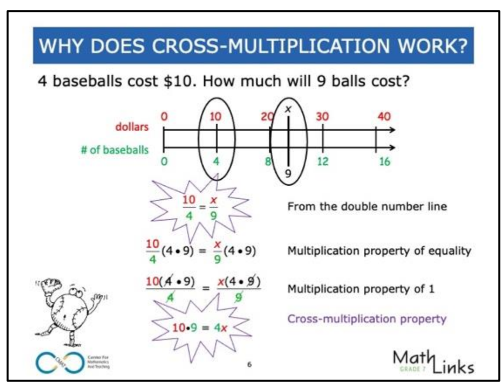
- Slide 5: For (5), students create an equation and solve it. The double number line is an excellent tool to help students set up proportional reasoning equations correctly.

  For (6), students are asked to validate the previous answer by using different ratios to write and solve a different, but related equation. Since there are many possible options, ask students to share their equations.



- Slide 6: The cross-multiplication property is a valid equation-solving procedure that bypasses several steps for solving proportion equation.

  Guide students slowly through the sequence of steps so they see why the property works to solve equations of the form $\frac{a}{b} = \frac{c}{d}$. It is important that students know that this shortcut is based on fundamental mathematical principles. It is not simply a trick or senseless procedure.

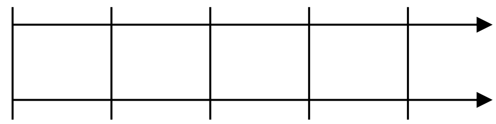


SLIDE DECK ALTERNATIVE S3.3:
DOUBLE NUMBER LINES AND EQUATIONS

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1 – 4

(1) 4 baseballs cost $10. You can buy any number of baseballs at this rate. Make a double number line showing costs for different quantities of baseballs.



What does $\frac{20}{8} = \frac{30}{12}$ *mean in this context?*

For (2) - (4):

a. Copy the "if-then" statement.

b. Generalize the meaning of this statement.

c. Write another true if-then statement based on the double number line that illustrates this relationship.

For (2): *Interpret this statement*: If $\frac{20}{8} = \frac{30}{12}$, *then* $\frac{8}{20} = \frac{12}{30}$

For (3): *Interpret this statement*: If $\frac{20}{8} = \frac{30}{12}$, *then* $\frac{20}{30} = \frac{8}{12}$

For (4): *Interpret this statement*: If $\frac{20}{8} = \frac{30}{12}$, *then* $20 \bullet 12 = 30 \bullet 8$

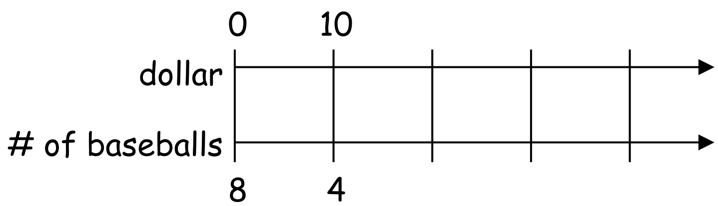
The statement in problem 4 is referred to as the "cross-multiplication property."

SLIDE DECK ALTERNATIVE S3.3: DOUBLE NUMBER LINES AND EQUATIONS

Continued

Slides 5 – 6

(5) How much will 9 baseballs cost if 4 baseballs cost $10? Use the double number line to set up the equation and include the ratio 10 : 4. Use the cross-multiplication property to solve for the unknown.



(6) Using a ratio other than 10 : 4, write another equation and solve it for the unknown.

Why does the cross-multiplication property work?

PRACTICE 4

[SMP3, 5]

1. Some students explored the equation $\frac{3}{5} = \frac{6}{10}$ and rewrote it in a few different ways.

   a. Circle the three true equations.

Abner:	Nick:	Buck:	Winton:
$\frac{3}{6} = \frac{5}{10}$	$\frac{6}{3} = \frac{5}{10}$	$\frac{5}{3} = \frac{10}{6}$	$3 \bullet 10 = 6 \bullet 5$

   b. For the equation that is not true, explain to that student why it is not true and a way to revise the work.

   Nick is stating that $2 = \frac{1}{2}$. If he uses the cross-multiplication property, he will see that $6 \bullet 10 \neq 5 \bullet 3$. If he wants to take the original equation and create a fraction by comparing the numerators $(6 \rightarrow 3)$, then the equivalent fraction will compare denominators in the same order $(10 \rightarrow 5)$. A correct equation $\left(\frac{6}{3} = \frac{10}{5}\right)$ is the inverse of Abner's equation.

2. Rewrite the equation $\frac{2}{7} = \frac{6}{21}$ in three other ways to create true equations.

   Some possible answers:

$\frac{7}{2} = \frac{21}{6}$	$\frac{7}{21} = \frac{2}{6}$	$2 \bullet 21 = 6 \bullet 7$

Solve each equation using any method.

3. $\frac{2}{5} = \frac{x}{20}$ $x = 8$	4. $\frac{x}{17} = \frac{3}{17}$ $x = 3$	5. $\frac{55}{x} = \frac{5}{2.1}$ $x = 23.1$
6. $\frac{2.5}{5} = \frac{x}{12}$ $x = 6$	7. $\frac{20}{7} = \frac{6}{x}$ $x = 2.1$	8. $\frac{2}{x} = \frac{3}{13}$ $x = 8\frac{2}{3} = 8.\overline{6}$ or about 8.7

9. Explain how you solved the equation in problem 8 above.

   Answers will vary. Some possible answers:

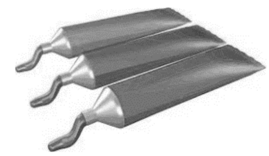
   (1) Since $2 \cdot 1\frac{1}{2} = 3$, then $x \cdot 1\frac{1}{2} = 13 \rightarrow x = 13 \div 1\frac{1}{2} = 8\frac{2}{3}$

   (2) Cross multiplication proerty: $2 \bullet 13 = 3x \rightarrow 26 = 3x \rightarrow 8\frac{2}{3} = x$

PRACTICE 5

Fact statements for problems 1 – 5:

- 3 tubes of artist paint cost $4.50.
- You can buy any number of paint tubes at that rate.



1. Fill in the missing numbers for tick marks on the double number line to the right. Then use it to help with writing and solving equations below.

# of tubes	0	3	6	9	12
Cost ($)	0	4.50	9	13.50	18

Equations may vary, but solutions will not.

2. How many tubes can you buy for $12?

$$\begin{array}{r} \text{\# tubes} \rightarrow \\ \$ \rightarrow \end{array} \frac{6}{9} = \frac{x}{12}$$

$9x = 72$
$x = 8$
8 tubes for $12

3. What is the cost of 50 tubes of paint?

$$\begin{array}{r} \text{\# tubes} \rightarrow \\ \$ \rightarrow \end{array} \frac{6}{9} = \frac{50}{x}$$

$6x = 450$
$x = 75$
$75 for 50 tubes

4. How many tubes of paint can you buy for $42?

$$\begin{array}{r} \text{\# tubes} \rightarrow \\ \$ \rightarrow \end{array} \frac{6}{9} = \frac{x}{42}$$

$9x = 252$
$x = 28$
28 tubes for $42

5. What is the unit price for a tube of paint?

$$\begin{array}{r} \text{\# tubes} \rightarrow \\ \$ \rightarrow \end{array} \frac{6}{9} = \frac{1}{x}$$

$6x = 9$
$x = 1.5$
$1.50 for 1 tube

6. While waiting for the bus, you notice that 3 trucks drive by for every 10 cars.

   a. At this rate, about how many trucks would you see if 56 cars drove by?
   about 16 or 17 trucks

   b. If you saw 13 trucks drive by, about how many total vehicles drove by during that time?
   about 43 or 44

JENNA'S CORNBREAD RECIPE

[SMP1]

Granny and Auntie both love the cornbread Jenna brought to the family dinner, so Jenna says, "Here's what I did. I started by using $1\frac{1}{2}$ cups of milk, $2\frac{1}{2}$ cups of cornmeal, $1\frac{1}{4}$ cups of flour, and…" "Wait!" Granny says. "I just want to make it for myself, not for a party!" Auntie agrees. Jenna says, "You both know a lot about ratios. I'll give you the rest and you figure it out!"

Granny and Auntie want their cornbread to taste the same as Jenna's. Analyze the cornbread recipe representations below. Let M and C represent parts milk and cornmeal, respectively.

1. Finish the tape diagram below using some of Jenna's initial quantities.

M	M	M	C	C	C	C	C
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$1\frac{1}{2}$ cups			$2\frac{1}{2}$ cups				

2. Granny intends to use only 1 cup of milk. Finish the tape diagram below to represent the quantities Granny will need.

M	M	M	C	C	C	C	C
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
1 cup			$1\frac{2}{3}$ cup				

1 cup milk requires __$1\frac{2}{3}$__ cups cornmeal.

3. Auntie plans to use 1 cup of cornmeal. Finish the tape diagram below to represent the quantities Auntie will need.

M	M	M	C	C	C	C	C
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
$\frac{3}{5}$ cup			1 cup				

1 cup cornmeal requires __$\frac{3}{5}$__ cups milk.

4. Compute. Describe what this represents in the context of this situation.

$$\frac{1\frac{1}{2}}{2\frac{1}{2}}$$

$= 1\frac{1}{2} \div 2\frac{1}{2} = \frac{3}{2} \div \frac{5}{2} = \frac{3}{5}$

There are $\frac{3}{5}$ cups of milk for each cup of cornmeal. This is the unit rate of parts milk per 1 part cornmeal.

A shorthand for representing the meaning of this fraction can be "$\frac{\text{milk}}{\text{cornmeal}}$."

5. How many cups of milk are needed for $\frac{3}{4}$ cups of cornmeal?

$$\frac{1\frac{1}{2}}{2\frac{1}{2}} = \frac{m}{\frac{3}{4}}$$

$$m = \frac{9}{20}$$

This is about $\frac{1}{2}$ cups milk.

PRACTICE 6

A cornbread recipe used $1\frac{1}{2}$ cups of milk, $2\frac{1}{2}$ cups of cornmeal, and $1\frac{1}{4}$ cups of flour.

Write and solve equations that represent these statements. If any exact measure resulting from your calculations seem unreasonable, offer a close, more reasonable estimate.

Let, *m*, *c*, and *f* represent cups milk, cornmeal, and flour respectively. Equations may vary.

1. How many cups of milk are needed for 1 cup of flour?

$\frac{\text{milk}}{\text{flour}}: \frac{1\frac{1}{2}}{1\frac{1}{4}} = \frac{m}{1}$

$m = \frac{6}{5}$

about $1\frac{1}{4}$ cups milk

2. How many cups of cornmeal are needed for 1 cup of flour?

$\frac{\text{cornmeal}}{\text{flour}}: \frac{2\frac{1}{2}}{1\frac{1}{4}} = \frac{c}{1}$

$c = 2$

2 cups cornmeal

3. How many cups of flour are needed for $\frac{2}{3}$ cups of cornmeal?

$\frac{\text{flour}}{\text{cornmeal}}: \frac{1\frac{1}{4}}{2\frac{1}{2}} = \frac{f}{\frac{2}{3}}$

$f = \frac{1}{3}$

$\frac{1}{3}$ cups flour

4. How many cups of flour are needed for $2\frac{1}{2}$ cups of milk?

$\frac{\text{flour}}{\text{milk}}: \frac{1\frac{1}{4}}{1\frac{1}{2}} = \frac{f}{2\frac{1}{2}}$

$f = \frac{25}{12}$

about 2 cups flour

PRACTICE 7: EXTEND YOUR THINKING

Solve using any method.

COMMUNITY GARDEN Student volunteers from a local high school are turning a vacant lot into a community garden. A community beautification planner estimates the time it will take **1 person** to complete each of the following tasks. (Assume that everyone works at about the same rate):

- 8 hours to prepare the soil
- 40 hours to plant the flowers
- 18 hours to build a fence
- 14 hours to paint the fence

1. How many hours will it take for 2 people to prepare the soil together?

   4 hours

2. How many hours will it take for 4 people to plant the flowers together?

   10 hours

3. If 5 people are going to work together to plant the flowers, and they work 4 hours per day, how many days will be needed to complete the job?

   2 days

4. Eight people are going to work together to build and paint the fence. If they want to complete the job in two days, and to work the same number of hours on the first day as the second day, how many hours does each person need to work each day?

   2 hours each day for 2 days

PAINTING You want to paint your bedroom with your favorite shade of purple. Making this shade requires $\frac{1}{2}$ quart blue paint for every $\frac{1}{3}$ quart red paint.

5. If you want to mix blue and red paint in the same ratio to make 5 gallons of your favorite purple paint, how many quarts of blue paint and how many quarts of red paint will you need? 12 quarts blue and 8 quarts red

PRACTICE 8: EXTEND YOUR THINKING

Solve using any method.

PRINTING A school has four printers that print pages at different rates. Determine the number of pages per minute for each:	
1. The printer in the main office prints $2\frac{1}{2}$ pages per second. 150 pages per minute	2. The printer in the attendance office prints 50 pages per $\frac{1}{2}$ minute. 100 pages per minute
3. The printer in the counselor's office prints 160 pages in 2 minutes. 80 pages per minute	4. The printer in the faculty lounge prints 1 page every 2 seconds. 30 pages per minute
Which printer prints the fastest? The printer in the main office	
AT THE PICNIC Some friends were challenged to some fun races.	
5. The winning hopping race was at a rate of 3 miles per hour. If the hopping racer finished in 25 minutes, what was the length of the race course? 1.25 mi or $1\frac{1}{4}$ mi	6. In a crawling race, the winner completed $1\frac{1}{2}$ miles in $\frac{1}{3}$ of an hour. What is this rate in miles per hour? $4\frac{1}{2}$ miles per hour

REVIEW

POSTER PROBLEMS: PROPORTIONAL RELATIONSHIPS

See Activity Routines in the Teacher Portal for instructions

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions. Use a calculator as needed.

Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
A watch gains 2 minutes in 6 hours.	Mary read 12 pages in 30 minutes.	Betsy cooks 17 hours in a 2-week period.	Hurricane Katrina dropped 14 inches of rain over a 48-hour period.

A. Copy the fact statement and create a double number line.
   Double number line entries will vary.

B. Write a unit rate from the given fact statement using the given units.
   $\frac{1}{3}$ minutes/hour; about 0.4 pages/minute; 8.5 hours/week; about 0.3 inches/hour

C. Write a different, equivalent unit rate by changing one of the units of measurement as assigned:
   - For 1 (or 5) calculate this rate as minutes per day. 8 minutes/day
   - For 2 (or 6) calculate this rate as pages per hour. 24 pages/hour
   - For 3 (or 7) calculate this rate as hours per day. about 1.2 hours/day
   - For 4 (or 8) calculate this rate as inches per day. 7.2 inches/day

D. Create a follow up question that can be answered using the double number line or one of the unit rates. Questions will vary.

Part 3: Work in partners or groups to check your original poster, and then to answer the question created for part D.

Chart paper or board space, markers

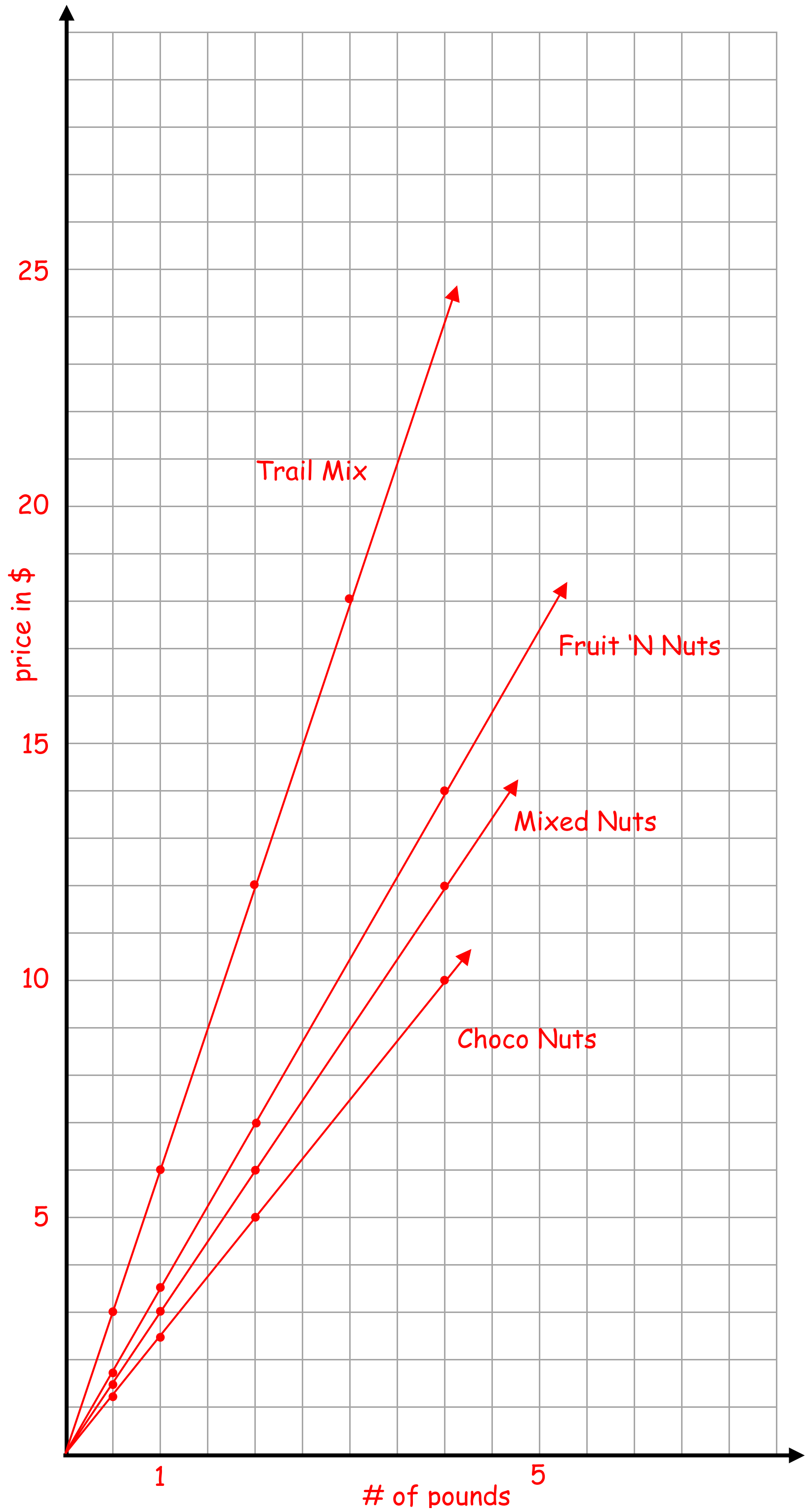
4

Journal Part 3

MATCHING ACTIVITY: NUTS

1. Your teacher will give you some cards that represent proportional relationships (one card has an error). Work with a partner to match cards with equivalent representations and find the error.

2. What was the error? How do you know? Fix it on the card.
   For Mixed Nuts, 4 pounds should cost $12. Since it is supposed to be a proportional relationship, the cost should be $3 per pound.



3. Graph price in dollars vs. number of pounds for each mixture. Label and scale appropriately. Use different colors if possible.

   Do you think the points should be connected? Explain.
   Nuts sold by the pound can be purchased in any quantity, so you can make the case that it makes sense to connect the points. However, we don't purchase in fractions of a penny.

MATCH AND COMPARE SORT: PROPORTIONAL RELATIONSHIPS

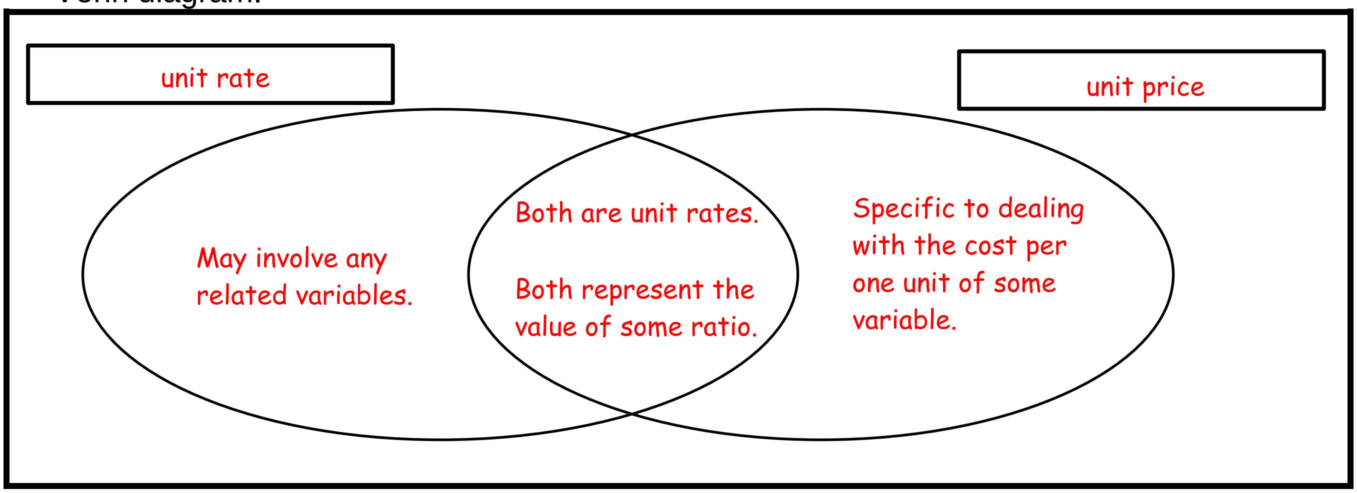
See Activity Routines in the Teacher Portal for instructions.

1. Individually, match your word cards to your description cards, discuss with your partner(s), and record all of your results in the table

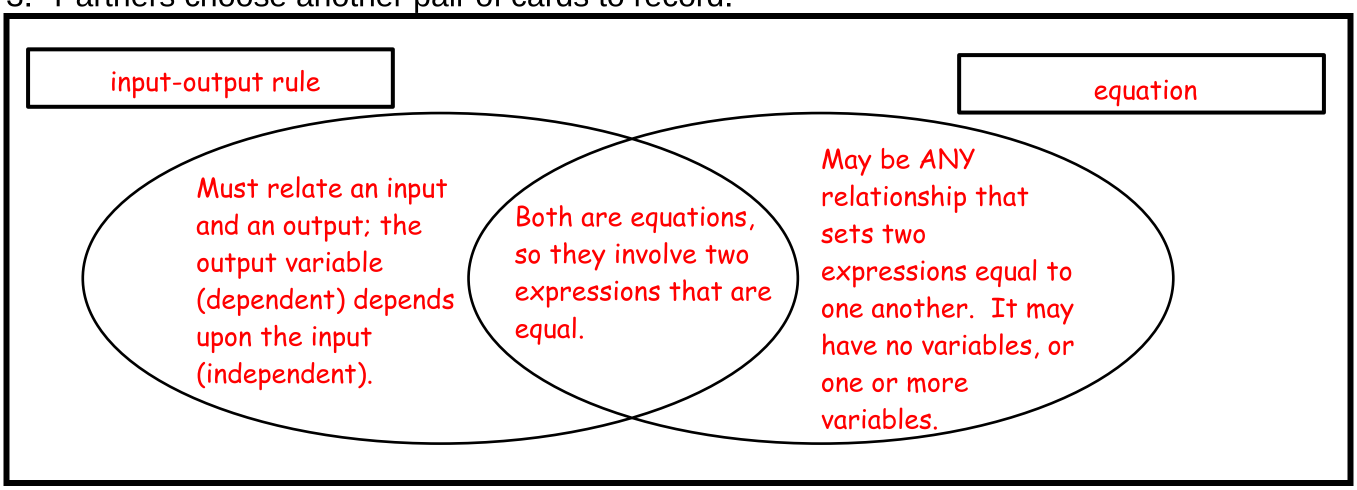
Card set △			Card set ○		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I	independent variable	D	I	dependent variable	C
II	unit rate	C	II	unit price	D
III	proportional relationship	A	III	constant of proportionality	B
IV	input-output rule	B	IV	equation	A

Choice of vocabulary words to compare will vary. Some possible answers:

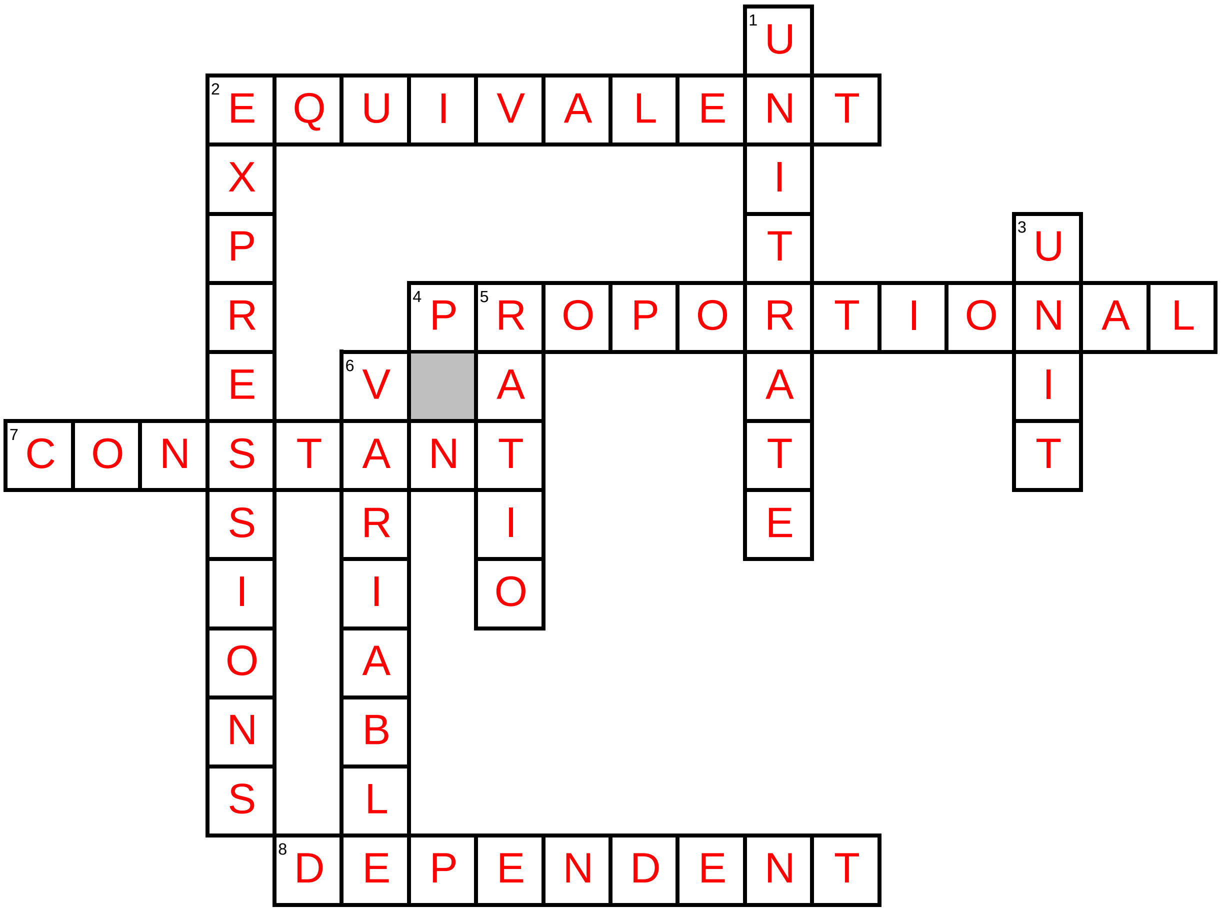
2. With your partner(s), choose one pair of matched word cards and record attributes in the Venn diagram.



3. Partners choose another pair of cards to record.



VOCABULARY REVIEW



Across

2 (See 5 down) For a good sandwich, Damond likes $1\frac{1}{2}$ tsp peanut butter for every 1 tsp jelly. This ratio is ____ to Jayme's.

4 A straight line through the origin describes a ____ relationship.

7 For the input-output rule $y = 3x$, the coefficient of x is 3 and is called the ____ of proportionality.

8 The ____ variable is the variable whose value is determined by the independent variable.

Down

1 the value of a ratio (two words)

2 An equation is a statement with two equivalent ____ (plural).

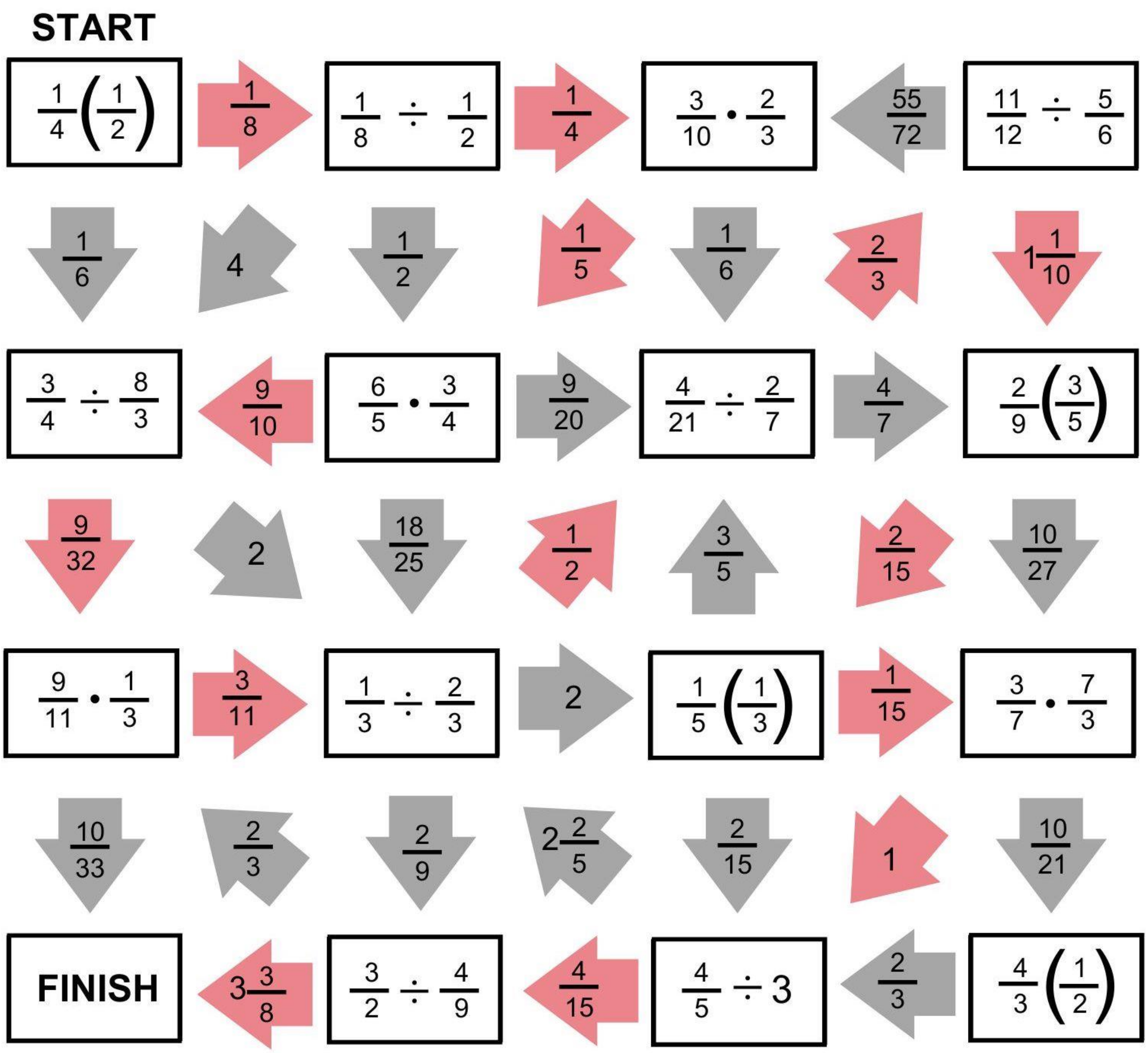
3 A ____ price is a price per one unit of measure.

5 For a good sandwich, Jayme likes her peanut butter and jelly in a ____ of 3 tsp to 2 tsp.

6 In the equation $2x = 10$, x is an unknown. It can also be called a ____.

SPIRAL REVIEW

1. **Math Path Fluency Challenge**: Use what you know about multiplication and division of fractions to find the correct path from Start to Finish. Note all products and quotients are in simplest form. Correct values are indicated in red.



2. Complete the table:

Fraction	$\frac{3}{4}$	$\frac{7}{20}$	$2\frac{49}{50}$	$\frac{1}{8}$	$\frac{2}{250}$	$\frac{1}{200}$
Decimal	0.75	0.35	2.98	0.125	0.008	0.005
Percent	75%	35%	298%	12.5%	0.8%	0.5%

SPIRAL REVIEW

Continued

3. You and a friend go out to lunch. You spend \$6.75 and your friend spends \$8.85.

   a. How much did you spend altogether?
   \$15.60 total for two lunches

   b. If the sales tax rate is 7.25%, how much tax will be paid?
   \$1.13 tax

   c. You leave a \$2.50 tip on your pre-tax total. About what percent was the tip?
   About a 16% tip

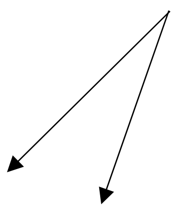
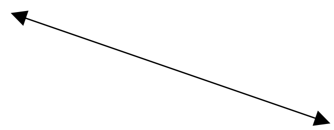
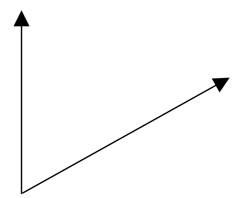
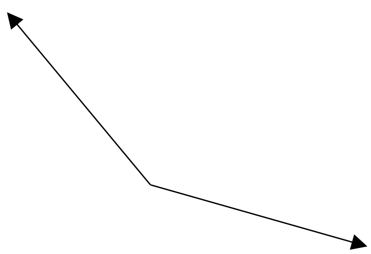
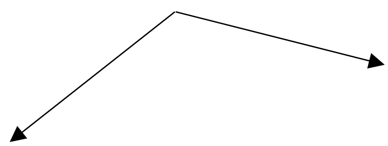
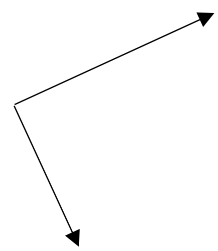
   d. What was the total cost for lunch, including tax and tip?
   \$19.23 total

4. Solve each equation using substitution or mental math.

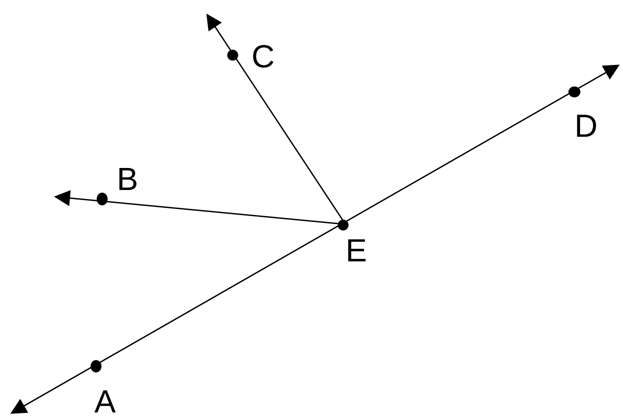
a. $3x = 48$ $x = 16$	b. $500 = 270 + y$ $y = 230$
c. $240 = 12y$ $y = 20$	d. $45 = 67 - s$ $s = 22$
e. $\frac{1}{5} + x = 1$ $x = \frac{4}{5}$	f. $\frac{1}{8}x = 160$ $x = 1{,}280$

SPIRAL REVIEW
Continued

5. Label the angles as acute, right, obtuse, or straight. Then write a fact about the degree measure of each angle. Answers may vary.

a. Acute angle. Less than 90 degrees. 	b. Straight angle. Equals 180 degrees. 	c. Acute angle. Less than 90 degrees. 
d. Obtuse angle. More than 90 degrees. 	e. Obtuse angle. More than 90 degrees. 	f. Right angle. Equals 90 degrees. 

6. Use the picture to the right. Answers may vary.



a. Name an acute angle.
$\angle AEB$, $\angle BEC$

b. Name an obtuse angle.
$\angle BED$

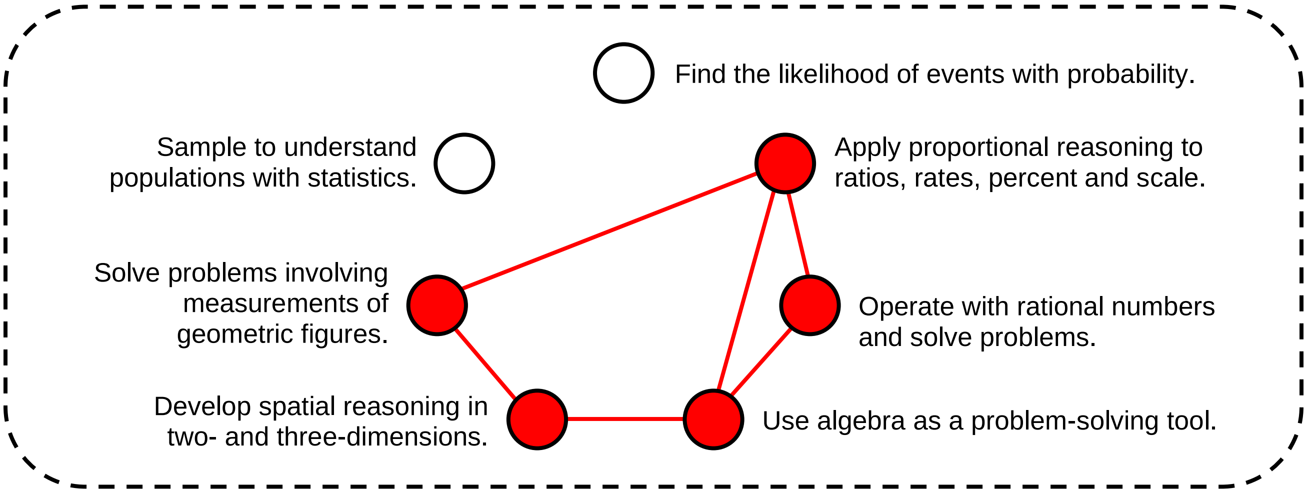
c. Name a right angle.
$\angle CED$

d. Name a straight angle.
$\angle AED$

PACKET REFLECTION

Answers will vary. Some possible answers:

1. **Big Ideas**. Shade all circles that describe big ideas in this packet. Draw lines to show connections that you noticed.



Give an example from this packet of one of the connections above.

Used equations to solve proportional reasoning problems.

2. **Packet Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.

3. **Mathematical Practices.** How did you use mathematical representations to make sense of an everyday problem? [SMP1, 2, 4]

A graph was used to predict if Twinkie the Dog would break a record.
Tape diagrams helped make sense of quantities in **Jenna's Cornbread Recipe**.

4. **Making Connections.** You used tables, graphs, and equations to represent proportional relationships in this packet. Why do you think it is useful to represent proportional relationships in different ways?

Different representations illustrate different aspects of a proportional relationship. For example, the graph of a straight line through the origin indicates a proportional relationship. Sometimes it is easier to understand the relationships using a table or double number line. Sometimes it is more accurate to find a missing quantity with a proportion equation.

STUDENT RESOURCES

Word or Phrase	Definition
complex fraction	A <u>complex fraction</u> is a fraction whose numerator or denominator is a fraction. Two complex fractions are $\frac{\frac{4}{5}}{\frac{1}{2}}$ and $\frac{\frac{1}{5}}{3}$.
constant of proportionality	See <u>proportional</u>.
dependent variable	A <u>dependent variable</u> is a variable whose value is determined by the values of the independent variables. See <u>independent variable</u>.
equation	An <u>equation</u> is a mathematical statement that asserts the equality of two expressions. $18 = 8 + 10$ is an equation that involves only numbers. This is a numerical equation. $18 = x + 10$ is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.
expression	A mathematical <u>expression</u> is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8 + x}{10}$, and $4v – w$.
equivalent ratios	Two ratios are equivalent if each number in one ratio is obtained by multiplying the corresponding numbers in the other ratio by the same positive number. 5 : 3 and 20 : 12 are equivalent ratios because both numbers in the ratio 5 : 3 are multiplied by 4 to get to the ratio 20 : 12.
independent variable	An <u>independent variable</u> is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables. For the equation $y = 3x$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.

<table>
<tr><th>Word or Phrase</th><th>Definition</th></tr>
<tr><td>input-output rule</td><td>An input-output rule for a sequence of values is a rule that establishes explicitly an output value for each given input value.

<table>
<tr><td>input value (x)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>x</td></tr>
<tr><td>output value (y)</td><td>1.5</td><td>3</td><td>4.5</td><td>6</td><td>7.5</td><td>$1.5x$</td></tr>
</table>

In the table above, the input-output rule could be $y = 1.5x$. In other words, to get the output value, multiply the input value by 1.5. If $x = 100$, then $y = 1.5(100) = 150$.</td></tr>
<tr><td>proportional</td><td>Two variables are proportional if the values of one are the same constant multiple of the corresponding values of the other. The variables are said to be in a proportional relationship, and the constant is referred to as the constant of proportionality.

If Wrigley eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.</td></tr>
<tr><td>proportional relationship</td><td>See proportional.</td></tr>
<tr><td>ratio</td><td>A ratio is a pair of positive numbers in a specific order. The ratio of a to b is denoted by $a : b$ (read "a to b," or "a for every b").

The ratio of 3 to 2 is denoted by 3 : 2. The ratio of dogs to cats is 3 to 2. There are 3 cups of water for every 2 cups of juice. The fraction $\frac{3}{2}$ does not represent this ratio, but it does represent the ratio's value (or the unit rate).</td></tr>
<tr><td>unit price</td><td>A unit price is a price for one unit of measure.</td></tr>
<tr><td>unit rate</td><td>The unit rate associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the value $\frac{a}{b}$, to which units may be attached.

The ratio of 40 miles each 5 hours has unit rate of 8 miles per hour.</td></tr>
<tr><td>value of a ratio</td><td>See unit rate.</td></tr>
<tr><td>variable</td><td>A variable is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to quantities that vary in a relationship (as in a formula or an input-output rule). They may refer to unknown quantities in expressions, equations or inequalities. Finally, they may be used to generalize rules of arithmetic.

In the equation $d = rt$, the quantities d, r, and t are variables.
In the equation $2x = 10$, the variable x may be referred to as the unknown.
The equation $a + b = b + a$ generalizes the commutative property of addition for all numbers a and b.</td></tr>
</table>

Testing for a Proportional Relationship

Here are three ways to test if two variables are in a proportional relationship:

- The values of the ratios (unit rates or unit prices) created by data pairs are equivalent.
- An equation in the form $y = kx$ fits all corresponding data pairs.
- Graphed data pairs fall on a line through the origin (0, 0).

Note that this example does **not** represent a proportional relationship. Alexa buys tickets when she goes to the amusement park. This chart shows the costs for different quantities of tickets.

# of tickets	10	20	25	50	100
total cost	\$40	\$60	\$75	\$125	\$200
cost per ticket	\$4	\$3	\$3	\$2.50	\$2

Since the costs per ticket (unit prices) are not the same, ticket purchasing at this amusement park does **not** represent a proportional relationship.

This example **does** represent a proportional relationship. Antonio kept track of the number of miles he traveled each time he filled his tank with gas. Here is some data.

number of miles	100	200	175	300
number of gallons	4	8	7	12
miles per gallon	25	25	25	25

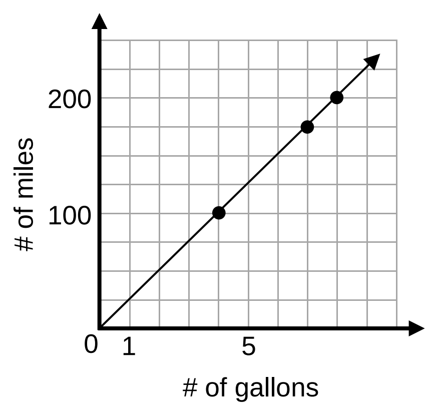
Since the miles per gallon (unit rates) created by the data pairs is the same, this situation represents quantities in a proportional relationship.

Furthermore,

Let x = the number of gallons
Let y = the number of miles

The data fits the equation $y = 25x$ (an equation in the form $y = kx$), which is an equation that represents a proportional relationship.

Finally, if the points for (gallons, miles) are graphed, they will fall on a line through the origin (0,0).



Multiple Representations and Proportional Relationships

Suppose 4 balloons cost $6.00 and each balloon is the same price. Here are some strategies for representing this proportional relationship.

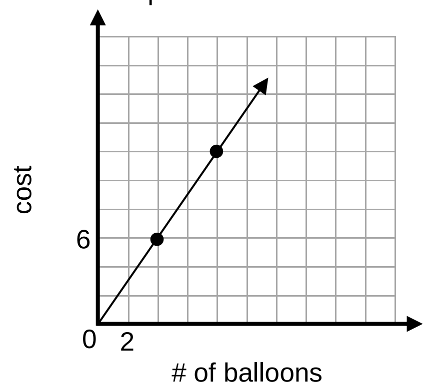
Strategy 1: Tables

Create a table to calculate unit rates. If the unit rates are the same, the variables are in a proportional relationship.

Number of Balloons	Cost	Unit Price
4	$6.00	$1.50
2	$3.00	$1.50
1	$1.50	$1.50
8	$12.00	$1.50

Strategy 2: Graphs

A straight line through the origin indicates quantities in a proportional relationship.



Strategy 3: Equations

An equation of the form $y = kx$ indicates quantities in a proportional relationship. In this case,

y = cost in dollars
x = number of balloons
k = cost per balloon (unit price)

To determine the unit price, create a ratio whose value is: $\dfrac{6 \text{ dollars}}{4 \text{ balloons}} = 1.50 \dfrac{\text{dollars}}{\text{balloons}}$

Therefore, k = $1.50 per balloon, and $y = 1.50x$.

This equation expresses the output as a constant multiple of the input, showing that the relationship is proportional.

Sense-Making Strategies to Solve Proportional Reasoning Problems

How much will 5 pencils cost if 8 pencils cost $4.40?

Strategy 1: Use a "halving" strategy

If 8 pencils cost $4.40, then
4 pencils cost $2.20,
2 pencils cost $1.10, and
1 pencil costs $0.55.

Therefore, 5 pencils cost

$0.55 + $2.20 = $2.75.

Strategy 2: Find unit prices

First, find the cost of one pencil.

$$\frac{\$4.40}{8} = \$0.55$$

Then, multiply by 5 to find the cost of 5 pencils,

($0.55)(5) = $2.75.

Sammie can crawl 12 feet in 3 seconds. At this rate, how far can she crawl in $1\frac{1}{2}$ minutes?

Strategy 1: Make a table

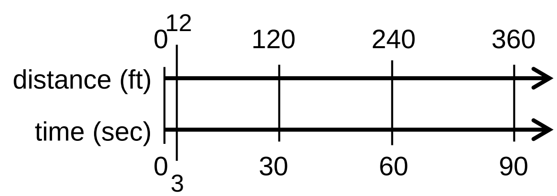
Distance	Time
12 ft	3 seconds
4 ft	1 second
240 ft	60 sec = 1 min
120 ft	30 sec = $\frac{1}{2}$ min
360 ft	90 sec = $1\frac{1}{2}$ min

Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

Strategy 2: Make a Double Number Line

12 feet in 3 seconds is equivalent to
120 feet in 30 seconds

$1\frac{1}{2}$ minutes = 90 seconds.

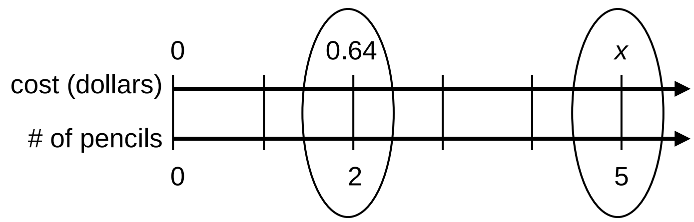


Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

Writing Equations Based on Rates

Here are some ways to set up an equation to solve a rate problem. An equation in the form $\frac{a}{b} = \frac{c}{d}$ is commonly referred to as a "proportion." Double number lines help make sense of this process. (See boxes on the next page for equation solving strategies.)

If 2 pencils cost $0.64, how much will 5 pencils cost?



Strategy 1: Compare rates ("between" two different units)

Create two rates from ratios that compare dollars to pencils. Equate expressions and solve for x.

$$\frac{x}{5} = \frac{0.64}{2}$$

x = 1.60 dollars for 5 pencils.

Note: The equation $\frac{5}{x} = \frac{2}{0.64}$ is another valid "between" equation for this problem.

Strategy 2: Compare like units ('within" the same units)

Create one rate based on corresponding cost ratios and another rate based on the corresponding numbers of pencils ratios. Then, equate expressions and solve for x.

$$\frac{\text{cost}_{\text{case 1}}}{\text{cost}_{\text{case 2}}} = \frac{0.64}{x} \qquad \frac{\text{pencils}_{\text{case 1}}}{\text{pencils}_{\text{case 2}}} = \frac{2}{5}$$

$$\frac{0.64}{x} = \frac{2}{5}$$

x = 1.60 dollars for 5 pencils.

Note: The equation $\frac{x}{0.64} = \frac{5}{2}$ is another valid "within" equation for this problem.

Some Properties Relevant to Solving Equations

Here are some important properties of arithmetic and equality related to solving equations.

- The multiplication property of equality states that equals multiplied by equals are equal. Thus, if $a = b$ and $c = d$, then $ac = bd$.

  Example: If $1 + 2 = 3$ and $5 = 9 - 4$, then $(1 + 2)(5) = 3(9 - 4)$.

- The cross-multiplication property for equations states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ ($b \neq 0$, $d \neq 0$).

  This can be remembered with the diagram: $\frac{a}{b} = \frac{c}{d}$.

  Example: If $\frac{5}{7} = \frac{12}{x}$, then $5 \bullet x = 7 \bullet 12$.
  To see that this property is reasonable, try simple numbers:
  If $\frac{3}{4} = \frac{6}{8}$, then $3 \bullet 8 = 4 \bullet 6$.

Applying Properties to Solve Proportion Equations

Strategy 1: Multiplication Property of Equality

Solve for x:

Multiplication Property of Equality

$$\frac{x}{12} = \frac{3}{8}$$

$$(8 \bullet 12) \bullet \frac{x}{12} = \frac{3}{8} \bullet (8 \bullet 12)$$

$$8x = 36$$

$$x = \frac{36}{8}$$

$$x = 4\frac{1}{2}$$

Strategy 2: Cross-Multiplication Property

Solve for x:

Cross-multiplication property

$$\frac{x}{12} = \frac{3}{8}$$

$$8 \bullet x = (3 \bullet 12)$$

$$8x = 36$$

$$x = \frac{36}{8}$$

$$x = 4\frac{1}{2}$$

Simplifying Complex Fractions
Strategy 1: A complex fraction can be written as a division problem. Example: $\dfrac{\frac{1}{4}}{\frac{3}{8}} = \dfrac{1}{4} \div \dfrac{3}{8} = \dfrac{1}{4} \bullet \dfrac{8}{3} = \dfrac{8}{12} = \dfrac{2}{3}$ Strategy 2: A complex fraction can be multiplied by a form of the “big one” to create a denominator equal to one. Multiply the numerator and denominator each by the reciprocal of the denominator (in this case since the reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$). This process leaves a multiplication problem to compute. Example: $\dfrac{\frac{1}{4}}{\frac{3}{8}} \bullet \dfrac{\frac{8}{3}}{\frac{8}{3}} = \dfrac{\frac{1 \bullet 8}{4 \bullet 3}}{\frac{3 \bullet 8}{8 \bullet 3}} = \dfrac{\frac{8}{12}}{1} = \dfrac{8}{12} = \dfrac{2}{3}$ While Strategy 2 seems to require more steps, this strategy makes more transparent the properties involved in writing the complex fraction in a more usable form.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
7.RP.A	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*
7.RP.2	Recognize and represent proportional relationships between quantities:
a	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c	Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*
d	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
7.EE.B	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
7.G.A	**Draw, construct, and describe geometrical figures and describe the relationships between them.**
7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Woohoo! A blank page!

Do a happy dance! Another blank page!

Last page!