Name ____________________________ Period __________ Date _____________

PACKET 3
STUDENT PACKET

Math Links GRADE 7

PROPORTIONAL RELATIONSHIPS

Parent (or Guardian) signature ___________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

expression	equation input-output rule
proportional constant of proportionality proportional relationship	
ratio equivalent ratio	unit price unit rate

LENGTH AND AREA PATTERNS

Follow your teacher's directions for (1) – (7).

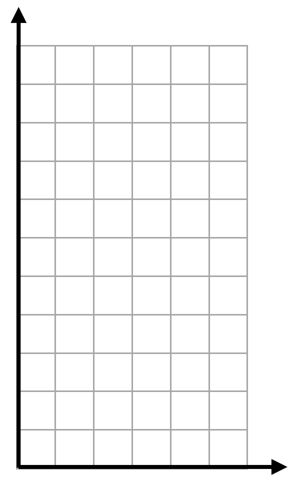
(1)

		Step 1		Step 2		Step 3		Step 4		Step 5
Pattern 1										
Pattern 2										

(2) – (3)

AREA (*A*)			
Step 1			
Step 2			
Step 3			
Step 4			
Step 5			
Step *n*			

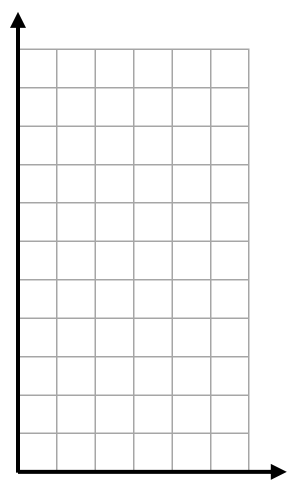
(4)



(5) – (6)

PERIMETER (*P*)			
Step 1			
Step 2			
Step 3			
Step 4			
Step 5			
Step *n*			

(7)



8. Record the meanings of <u>ratio</u>, <u>equivalent ratios</u>, and <u>expression</u> in **My Word Bank**

AN INTRODUCTION TO PROPORTIONAL RELATIONSHIPS

We will use tables and graphs to explore unit rates and unit prices. We will learn what it means for quantities to be in a proportional relationship, and identify the constant of proportionality (unit rate) in tables and graphs.

[7.NS.3, 7.RP.1, 7.RP.2ab, 7.G.1; SMP1, 3, 4, 5, 6]

GETTING STARTED

Shmear 'N Things
4 bagels for $3.00

Hole-y Bread
5 bagels for $4.00

1. Complete the tables below. Assume each shop will sell you any number of bagels at the rates shown above.

Shmear 'N Things	
# of bagels (x)	cost in $ (y)
4	
8	
12	
16	
20	

Hole-y Bread	
# of bagels (x)	cost in $ (y)
5	
10	
15	
20	
25	

2. Fill in the table below using the data tables above.

Shmear 'N Things	$\frac{\text{cost in dollars}}{\text{\# of bagels}}$	$\frac{3}{4}$	$\frac{\square}{8}$			
	Simplify	$\frac{3}{4}$	$\frac{\square}{4}$			
	Unit price (in dollars per bagel)	0.75				
Hole-y Bread	$\frac{\text{cost in dollars}}{\text{\# of bagels}}$	$\frac{4}{5}$	$\frac{\square}{10}$			
	Simplify	$\frac{4}{5}$				
	Unit price (in dollars per bagel)					

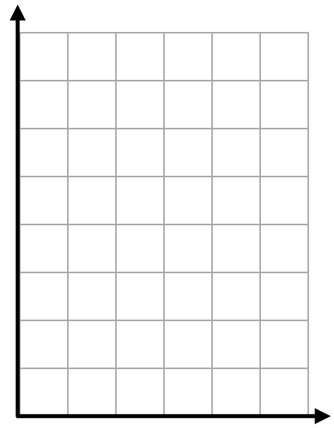
3. Which shop has the better buy? Explain.

PROPORTIONAL RELATIONSHIPS

Follow your teacher's directions for (1) – (4).

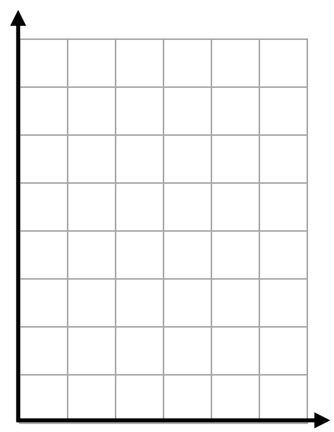
(1a) **Dion's Pillow Project**

(1b)



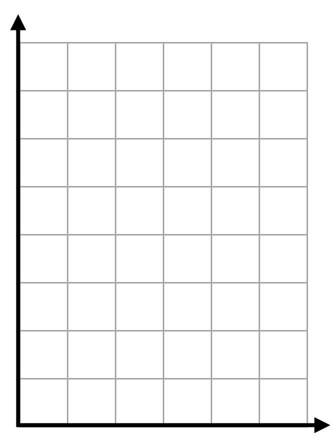
(2) **Ayla's Community Service**

(*x*)	**(*y*)**	
20	60	
50	150	
42	126	
60	180	



(3) **Mateo's Party Rentals**

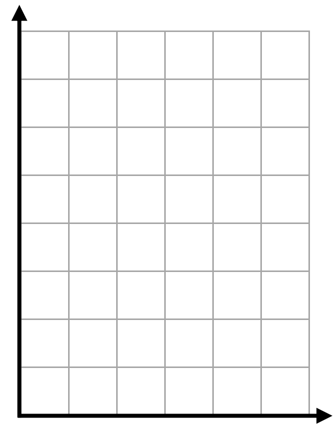
(*x*)	**(*y*)**	
1	20	
2	25	
3	30	
4	35	



PROPORTIONAL RELATIONSHIPS

Continued

(4) **Kim's House Plants**			
	(*x*)	**(*y*)**	
bedroom	100	2	
kitchen	125	5	
den	150	6	
patio	250	10	



5. Choose the ordered pair in each table for problems (1) – (3) that has the smallest *x*-value. Double both the *x*-value and the *y*-value and write them below.

	Ordered pair with least (*x*, *y*) values	**Ordered pair with doubled *x*-value and *y*-value**	**Would this point lie on the line of the existing graph?**	**Is the unit rate the same as other entries in the table?**
Problem 1	(____, ____)	(____, ____)		
Problem 2	(____, ____)	(____, ____)		
Problem 3	(____, ____)	(____, ____)		

6. Which situations from problems (1) – (4) describe proportional relationships? Explain.

7. Record the meanings of <u>equation</u>, <u>unit rate</u>, <u>unit price</u>, <u>proportional (relationship)</u>, and <u>constant of proportionality</u> in **My Word Bank**.

PRACTICE 1

1. Go back to the opening problem. First copy the patterns. Then copy the area and perimeter ratio columns in the table below. Finally, fill in the unit rate columns in the table below.

	Step 1	Step 2	Step 3	Step 4
Pattern 1				
Pattern 2				

Compare Areas and Perimeters – Pattern 1 : Pattern 2				
step #	$A1 : A2$	unit rate $\frac{A2}{A1}$	$P1 : P2$	unit rate $\frac{P2}{P1}$
1				
2				
3				
4				

2. Do the area ratios and perimeter ratios appear to be proportional relationships? Explain.

3. What if each square was NOT a unit square, but rather had a side length equal to $\frac{1}{2}$ unit of length? Fill in the table below for this situation.

Compare Areas and Perimeters – Pattern 1 : Pattern 2				
step #	$A1 : A2$	unit rate $\frac{A2}{A1}$	$P1 : P2$	unit rate $\frac{P2}{P1}$
1				
2				
3				
4				

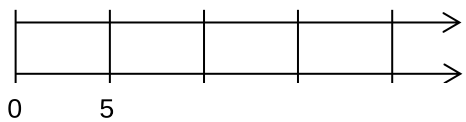
4. Do the area ratios and perimeter ratios appear to be in a proportional relationship? Explain.

TWINKIE THE DOG

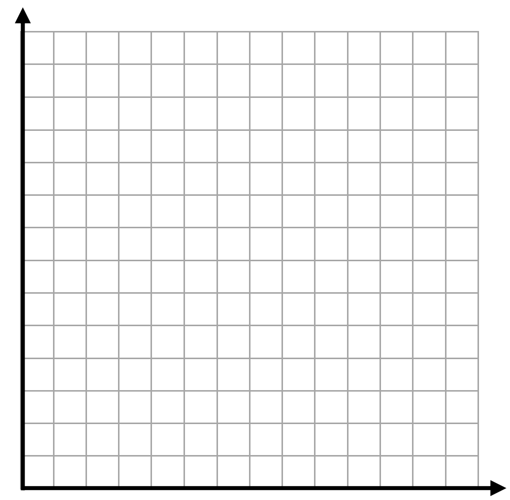
Follow your teacher's directions for (1) – (6).

(1) Twinkie, the Jack Russell Terrier, pops balloons.
Predict how long it will take for her to pop them all.

(2)



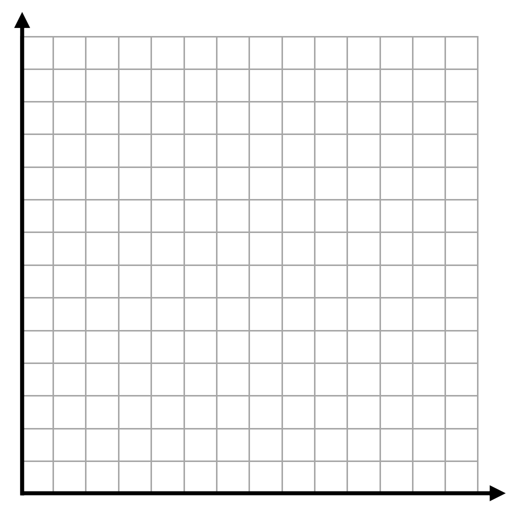
(3)



(4)

(5)

# of sec elapsed (x)	# of balloons (y)	unit rate $\frac{\text{\# of balloons}}{\text{\# of seconds}}$
0	0	



(6)

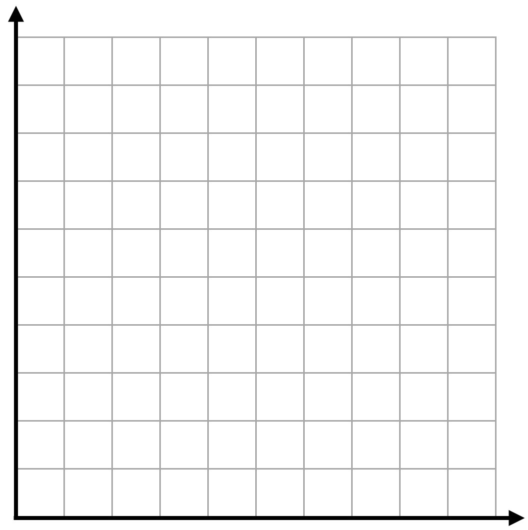
PRACTICE 2

The Enchanted Hill amusement park offers different ticket price packages.

1. Find unit prices for the different packages. Then graph the relationship between cost and number of tickets. Be sure to scale, title, and label your graph appropriately.

Ticket To Ride

number of tickets (x)	cost in $ (y)	$\frac{\text{cost (\\$)}}{\text{ticket}}$
1	3	
5	15	
10	20	
15	25	
20	28	



2. Does the ticket pricing represent a proportional relationship? Explain.

3. Which ticket option offers the best price in cost per ticket? Which would you choose? Explain.

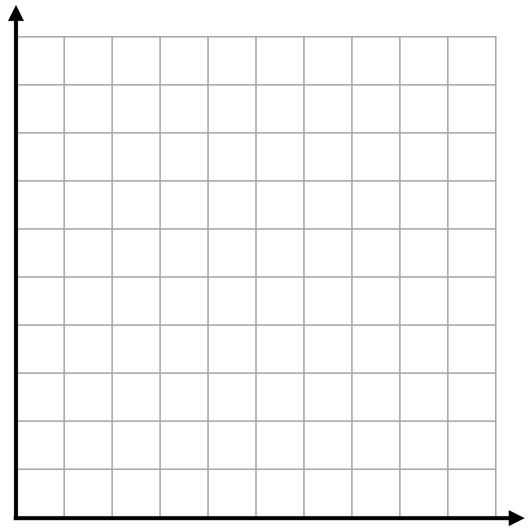
PRACTICE 2

Continued

4. Complete the table. Then graph the relationship between cost and number of tickets. Be sure to scale, title, and label your graph appropriately.

R n' R Tickets

number of tickets (*x*)	**cost in $ (*y*)**	$\frac{\textbf{cost (\$)}}{\textbf{ticket}}$
4	10	
16		2.50
2	5	
	12.50	2.50

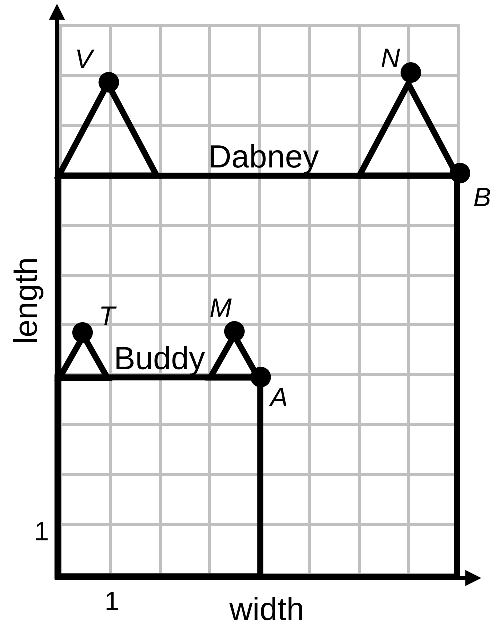


5. Does this represent a proportional relationship? Explain.

6. Which basketball purchasing option offers the best buy? Which would you choose? Explain.

BUDDY, DABNEY, AND KILROY ARE BACK!

Recall Buddy and Dabney from a previous lesson. Here are the backs of their heads.



1. Find the ratio of Buddy’s width to Dabney’s width. _____

2. Find the ratio of Buddy’s length to Dabney’s length. _____

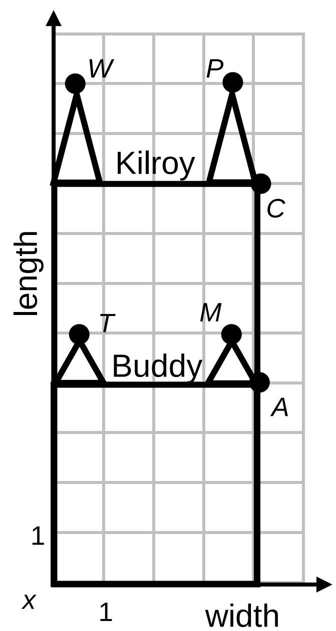
3. What is the multiplier (scale factor) that creates Dabney’s head from Buddy’s head? _____

4. Draw rays through the following corresponding points on their heads:
   - Ray *AB* (through top right of head)
   - Ray *MN* (through top of right ear)
   - Ray *TV* (through top of left ear)

   Would these rays extend back through the origin? ____

   What does this tell you about the relationship between the ordered pairs of Dabney’s coordinates and Buddy’s coordinates?

Now compare the heads of Buddy and Kilroy.



5. Find the ratio of Buddy’s width to Kilroy’s width. _____

6. Find the ratio of Buddy’s length to Kilroy’s length. _____

7. Why is there no multiplier (scale factor) that creates Kilroy’s head from Buddy’s head?

8. Draw rays through the corresponding points on their heads:
   - Line segment *AC* (through top right of head)
   - Line segment *MP* (through top of right ear)
   - Line segment *TW* (through top of left ear)

   Would these rays extend back through the origin? ____

   What does this tell you about the relationship between the ordered pairs of Buddy’s coordinates and Kilroy’s coordinates?

9. Which pair of friends have proportional faces? ___________________

DIGGING DEEPER INTO PROPORTIONAL RELATIONSHIPS

We will use tables, double number lines, graphs, and equations to explore what it means for a relationship between quantities to be proportional. We will pay special attention to the meaning of specific ordered pairs of quantities represented in the different representations.

[7.NS.3, 7.EE.3, 7.RP.1, 7.RP.2abcd; SMP3, 4, 5, 6]

GETTING STARTED

Complete each table and fill in the blanks.

1a.

x	1	2	3	4	5	6	10	15	
y	4	8	12						80

b. Rate of change: for every increase of x by 1, y increases by _____.

c. Input-output rule (words): Multiply an x-value by _____ to get the corresponding y-value.

d. Input-output rule (equation): y = _____ ; the coefficient of x is _____.

e. If $x = 100$, then y = _____.

f. If $y = 100$, then x = _____.

2a.

x	1	2	3	4	5	6	8	11	
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$				$6\frac{1}{2}$

b. Rate of change: for every increase of x by 1, y increases by _____.

c. Input-output rule (words): Multiply an x-value by _____ to get the corresponding y-value

d. Input-output rule (equation): y = _____; the coefficient of x is _____.

e. If $x = 100$, then y = _____.

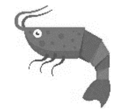
f. If $y = 100$, then x = _____.

3. Record the meaning of input-output rule in **My Word Bank**.

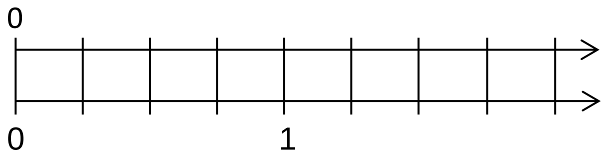
CAP'N SHERMAN'S SHRIMP SHOP

Follow your teacher's directions.

A customer bought _____ pounds of shrimp for _____.



(1)



(2) Use the double number line to find the cost for…

a.

b.

(3) Use the double number line to find the amount of shrimp you can purchase for…

a.

b.

(4)
(7)

0		
	4.90	
1		
	14.70	
2		
3		
	39.20	
5		
	98.00	

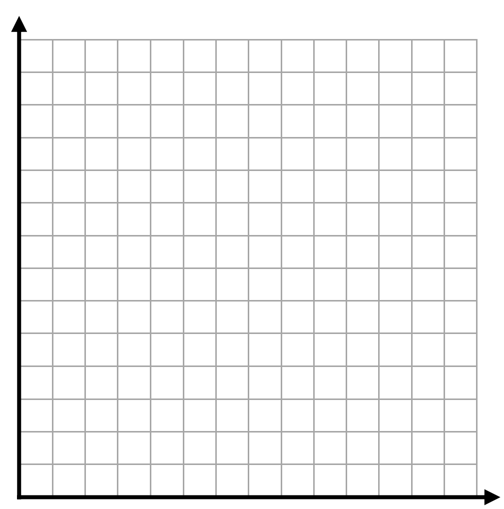
(5)

(6)

(8)

(10)

(9)



PRACTICE 3

Fruity-Fizzy-Water (FFW) is made using 5 cups of soda water for every 2 cups of fruit juice.

1. Fill in the table for different mixtures of FFW. Show work as needed.

2. Complete the paragraph:

   To keep the same flavor, a 1 cup increase in soda water requires an increase of _____ cups of juice. The unit rate of cups of juice per 1 cup soda water is _____. An equation that relates the amounts of juice to soda water is __________. One ordered pair is (1, _____). Within the context of FFW, this represents

   ____

   ____

cups of soda water (*x*)	**cups of fruit juice (*y*)**
0	
1	
2	
3	
4	
5	
6	
x	

   Another ordered pair is (0, _____). Within the context of FFW, this represents

   ____ .

Show work as needed for problems 3 – 5.

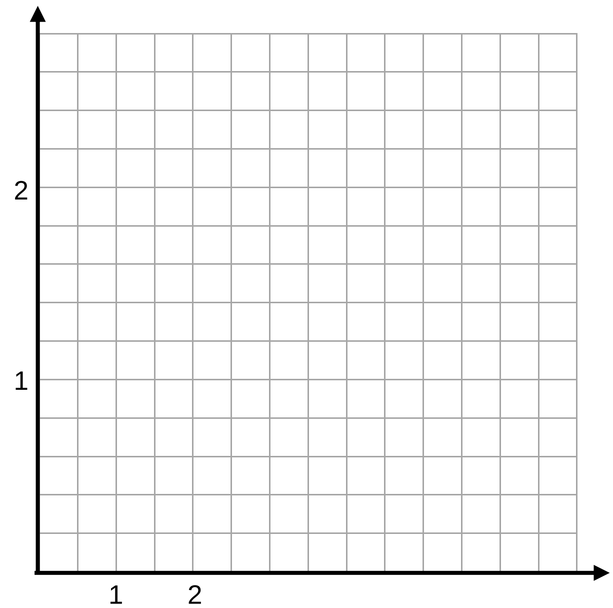
3. How many cups of juice are needed to make the exact same flavor of FFW if 40 cups of soda water are used?

4. How many cups of soda water are needed to make the exact same flavor of FFW if 40 cups juice of are used?

5. How many cups of FFW can be made with using 10 cups of juice?

PRACTICE 3
Continued

6. Make a graph to represent cups of soda water and juice.



7. Draw the following right triangles on the diagram and complete the table.

	Vertices of right triangles	Length of vertical leg (change in y)	Length of horizontal leg (change in x)	$\frac{\text{change in } y}{\text{change in } x}$
Triangle A	$(0, 0)$, $(0, \frac{2}{5})$, $(1, \frac{2}{5})$			
Triangle B	$(1, \frac{2}{5})$, $(1, 1\frac{1}{5})$, $(3, 1\frac{1}{5})$			
Triangle C	$(3, 1\frac{1}{5})$, $(3, 2\frac{2}{5})$, $(6, 2\frac{2}{5})$			

8. What is the meaning of the ratio of the lengths of the legs (last column in the table) in the context of the problem?

9. Write a few reasons that explain why the data in the tables and on this graph represent a proportional relationship.

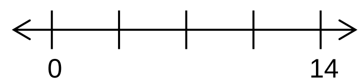
EQUATIONS AND PROBLEMS

We will write and solve equations created using equivalent rates, commonly referred to as "proportions." We will solve proportional reasoning problems using multiple strategies, including equations.

[7.RP.1, 7.RP.2bc,7.NS.3, 7.EE.3; SMP1, 2, 3, 5, 7, 8]

GETTING STARTED

1. What number times 4 is equal to 14?	2. Label each tick mark on the number line.



Solve each equation using any method.

3. $\frac{56}{m} = 8$	4. $5 = \frac{k}{9}$	5. $\frac{1}{3}h = 11$
6. $\frac{6}{5} = \frac{36}{p}$	7. $\frac{27}{d} = \frac{3}{7}$	8. $\frac{3}{4} = \frac{v}{14}$

9. Circle all of the true equations below.
   Notice that they are variations of the true equation: $\frac{1}{2} = \frac{4}{8}$.

   a. $\frac{2}{1} = \frac{8}{4}$ b. $\frac{1}{4} = \frac{2}{8}$ c. $\frac{4}{1} = \frac{8}{2}$ d. $\frac{1}{8} = \frac{4}{2}$

   Choose an incorrect equation above and explain why it is NOT true.

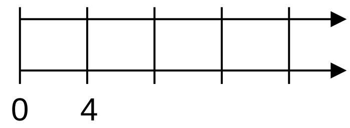
10. Explain what is incorrect about each statement.

a. JB is 10 and Ang is 15. When JB is 20, Ang will be 30.	b. It takes 3 people 4 hours to paint a room, so it will take 6 people 8 hours to paint the room.

DOUBLE NUMBER LINES AND EQUATIONS

Follow your teacher's directions.

(1) Four baseballs cost $_____.



(2)	(3)
(4)	(5) (6)

PRACTICE 4

1. Some students explored the equation $\frac{3}{5} = \frac{6}{10}$ and rewrote it in a few different ways.

   a. Circle the three true equations.

Abner:	Nick:	Buck:	Winton:
$\frac{3}{6} = \frac{5}{10}$	$\frac{6}{3} = \frac{5}{10}$	$\frac{5}{3} = \frac{10}{6}$	$3 \bullet 10 = 6 \bullet 5$

   b. For the equation that is not true, explain to that student why it is not true and a way to revise the work.

2. Rewrite the equation $\frac{2}{7} = \frac{6}{21}$ in three other ways to create true equations.

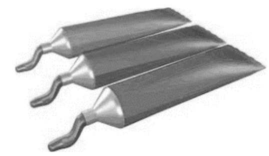
Solve each equation using any method.

3. $\frac{2}{5} = \frac{x}{20}$	4. $\frac{x}{17} = \frac{3}{17}$	5. $\frac{55}{x} = \frac{5}{2.1}$
6. $\frac{2.5}{5} = \frac{x}{12}$	7. $\frac{20}{7} = \frac{6}{x}$	8. $\frac{2}{x} = \frac{3}{13}$

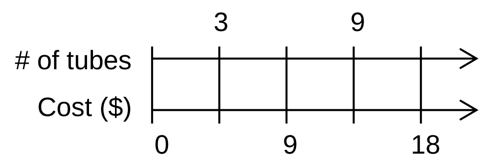
9. Explain how you solved the equation in problem 8 above.

PRACTICE 5

Fact statements for problems 1 – 5:
- 3 tubes of artist paint cost $4.50.
- You can buy any number of paint tubes at that rate.



1. Fill in the missing numbers for tick marks on the double number line to the right. Then use it to help with writing and solving equations below.



2. How many tubes can you buy for $12?	3. What is the cost of 50 tubes of paint?
4. How many tubes of paint can you buy for $42?	5. What is the unit price for a tube of paint?

6. While waiting for the bus, you notice that 3 trucks drive by for every 10 cars.

   a. At this rate, about how many trucks would you see if 56 cars drove by?

   b. If you saw 13 trucks drive by, about how many total vehicles drove by during that time?

JENNA'S CORNBREAD RECIPE

Granny and Auntie both love the cornbread Jenna brought to the family dinner, so Jenna says, "Here's what I did. I started by using $1\frac{1}{2}$ cups of milk, $2\frac{1}{2}$ cups of cornmeal, $1\frac{1}{4}$ cups of flour, and…" "Wait!" Granny says. "I just want to make it for myself, not for a party!" Auntie agrees. Jenna says, "You both know a lot about ratios. I'll give you the rest and you figure it out!"

Granny and Auntie want their cornbread to taste the same as Jenna's. Analyze the cornbread recipe representations below. Let M and C represent parts milk and cornmeal, respectively.

1. Finish the tape diagram below using some of Jenna's initial quantities.

M	M	M	C	C	C	C	C
$1\frac{1}{2}$ cups			$2\frac{1}{2}$ cups				

2. Granny intends to use only 1 cup of milk. Finish the tape diagram below to represent the quantities Granny will need.

M	M	M	C	C	C	C	C

1 cup milk requires ______ cups cornmeal.

3. Auntie plans to use 1 cup of cornmeal. Finish the tape diagram below to represent the quantities Auntie will need.

M	M	M	C	C	C	C	C

1 cup cornmeal requires ______ cups milk.

4. Compute. Describe what this represents in the context of this situation.

$$\frac{1\frac{1}{2}}{2\frac{1}{2}}$$

5. How many cups of milk are needed for $\frac{3}{4}$ cups of cornmeal?

PRACTICE 6

A cornbread recipe used $1\frac{1}{2}$ cups of milk, $2\frac{1}{2}$ cups of cornmeal, and $1\frac{1}{4}$ cups of flour.

Write and solve equations that represent these statements. If any exact measure resulting from your calculations seem unreasonable, offer a close, more reasonable estimate.

1. How many cups of milk are needed for 1 cup of flour?	2. How many cups of cornmeal are needed for 1 cup of flour?
3. How many cups of flour are needed for $\frac{2}{3}$ cups of cornmeal?	4. How many cups of flour are needed for $2\frac{1}{2}$ cups of milk?

PRACTICE 7: EXTEND YOUR THINKING

Solve using any method.

COMMUNITY GARDEN Student volunteers from a local high school are turning a vacant lot into a community garden. A community beautification planner estimates the time it will take **1 person** to complete each of the following tasks. (Assume that everyone works at about the same rate):

- 8 hours to prepare the soil
- 40 hours to plant the flowers
- 18 hours to build a fence
- 14 hours to paint the fence

1. How many hours will it take for 2 people to prepare the soil together?

2. How many hours will it take for 4 people to plant the flowers together?

3. If 5 people are going to work together to plant the flowers, and they work 4 hours per day, how many days will be needed to complete the job?

4. Eight people are going to work together to build and paint the fence. If they want to complete the job in two days, and to work the same number of hours on the first day as the second day, how many hours does each person need to work each day?

PAINTING You want to paint your bedroom with your favorite shade of purple. Making this shade requires $\frac{1}{2}$ quart blue paint for every $\frac{1}{3}$ quart red paint.

5. If you want to mix blue and red paint in the same ratio to make 5 gallons of your favorite purple paint, how many quarts of blue paint and how many quarts of red paint will you need?

PRACTICE 8: EXTEND YOUR THINKING

Solve using any method.

PRINTING A school has four printers that print pages at different rates. Determine the number of pages per minute for each:	
1. The printer in the main office prints $2\frac{1}{2}$ pages per second.	2. The printer in the attendance office prints 50 pages per $\frac{1}{2}$ minute.
3. The printer in the counselor's office prints 160 pages in 2 minutes.	4. The printer in the faculty lounge prints 1 page every 2 seconds.
Which printer prints the fastest?	
AT THE PICNIC Some friends were challenged to some fun races.	
5. The winning hopping race was at a rate of 3 miles per hour. If the hopping racer finished in 25 minutes, what was the length of the race course?	6. In a crawling race, the winner completed $1\frac{1}{2}$ miles in $\frac{1}{3}$ of an hour. What is this rate in miles per hour?

REVIEW

POSTER PROBLEMS: PROPORTIONAL RELATIONSHIPS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions. Use a calculator as needed.

Poster 1 (or 5)	**Poster 2 (or 6)**	**Poster 3 (or 7)**	**Poster 4 (or 8)**
A watch gains 2 minutes in 6 hours.	Mary read 12 pages in 30 minutes.	Betsy cooks 17 hours in a 2-week period.	Hurricane Katrina dropped 14 inches of rain over a 48-hour period.

A. Copy the fact statement and create a double number line.

B. Write a unit rate from the given fact statement using the given units.

C. Write a different, equivalent unit rate by changing one of the units of measurement as assigned:
   - For 1 (or 5) calculate this rate as minutes per day.
   - For 2 (or 6) calculate this rate as pages per hour.
   - For 3 (or 7) calculate this rate as hours per day.
   - For 4 (or 8) calculate this rate as inches per day.

D. Create a follow up question that can be answered using the double number line or one of the unit rates.

Part 3: Work in partners or groups to check your original poster, and then to answer the question created for part D.

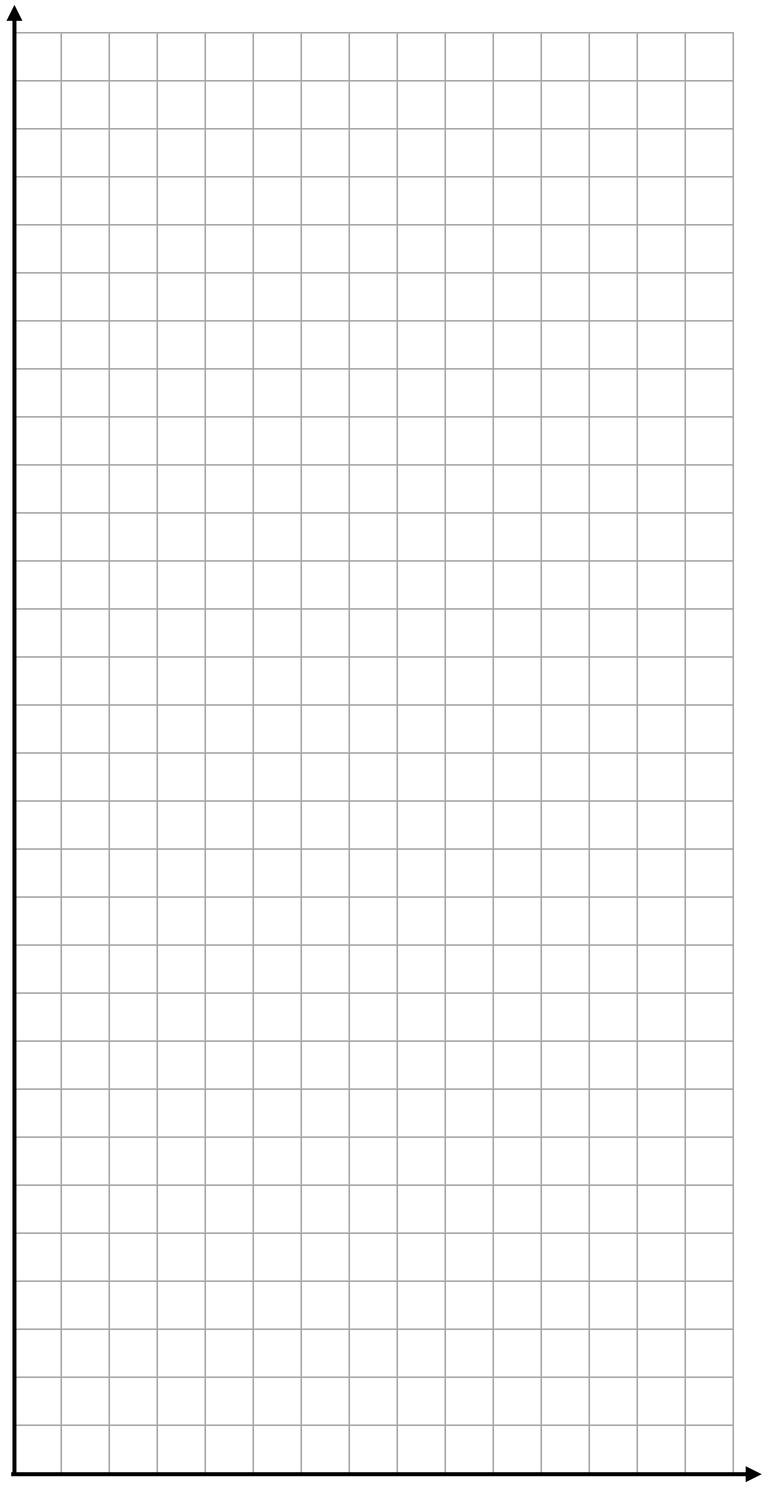
MATCHING ACTIVITY: NUTS

1. Your teacher will give you some cards that represent proportional relationships (one card has an error). Work with a partner to match cards with equivalent representations and find the error.

2. What was the error? How do you know? Fix it on the card.

3. Graph price in dollars vs. number of pounds for each mixture. Label and scale appropriately. Use different colors if possible.

   Do you think the points should be connected? Explain.

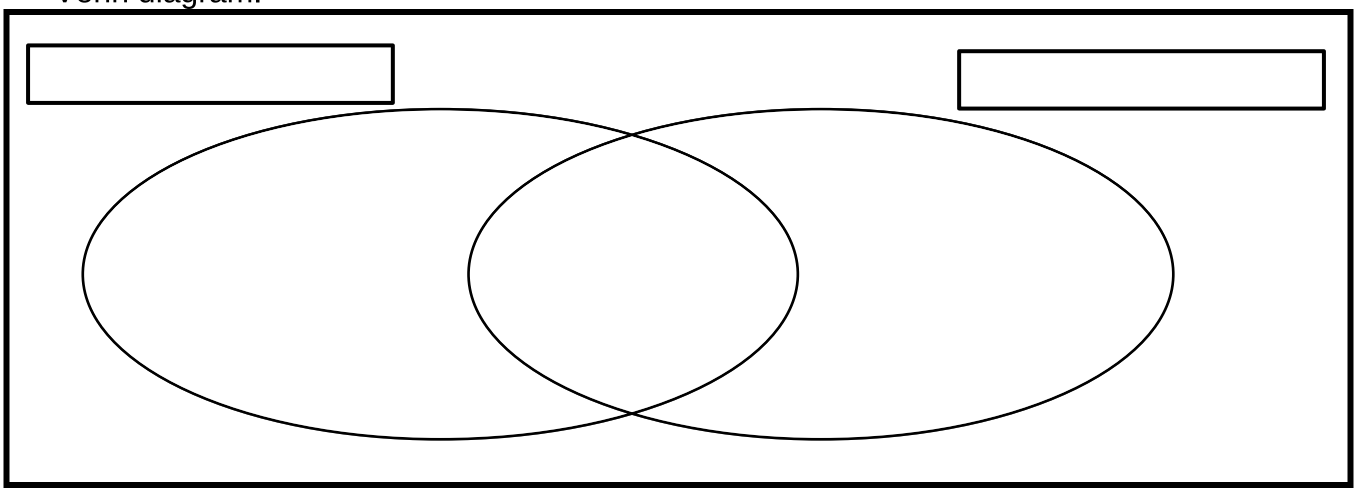


MATCH AND COMPARE SORT: PROPORTIONAL RELATIONSHIPS

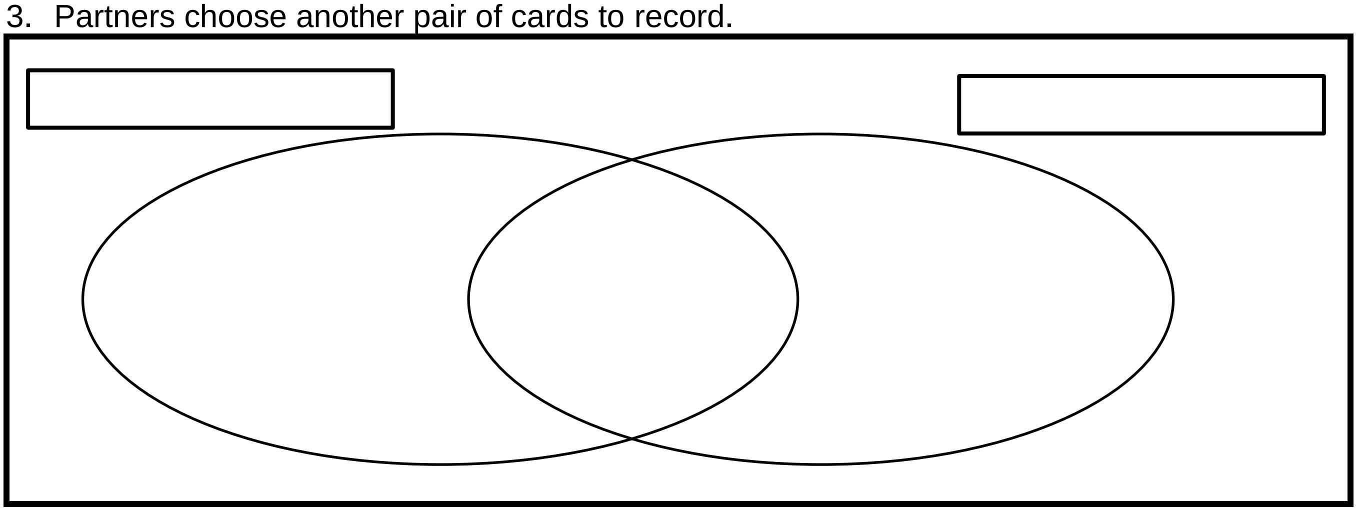
1. Individually, match your word cards to your description cards, discuss with your partner(s), and record all of your results in the table

Card set △			Card set ○		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I			I		
II			II		
III			III		
IV			IV		

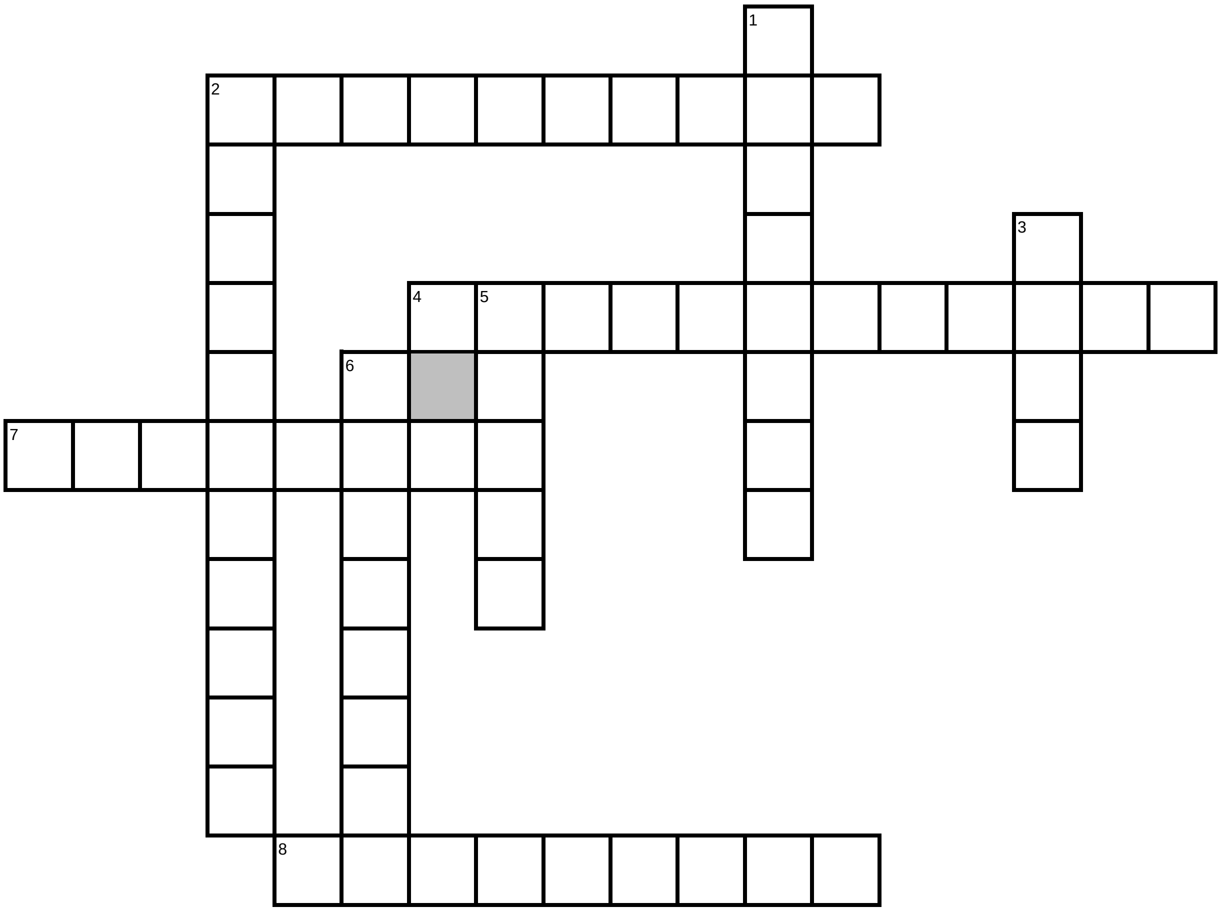
2. With your partner(s), choose one pair of matched word cards and record attributes in the Venn diagram.



3. Partners choose another pair of cards to record.



VOCABULARY REVIEW



Across

2 (See 5 down) For a good sandwich, Damond likes $1\frac{1}{2}$ tsp peanut butter for every 1 tsp jelly. This ratio is ____ to Jayme's.

4 A straight line through the origin describes a ____ relationship.

7 For the input-output rule $y = 3x$, the coefficient of x is 3 and is called the ____ of proportionality.

8 The ____ variable is the variable whose value is determined by the independent variable.

Down

1 the value of a ratio (two words)

2 An equation is a statement with two equivalent ____ (plural).

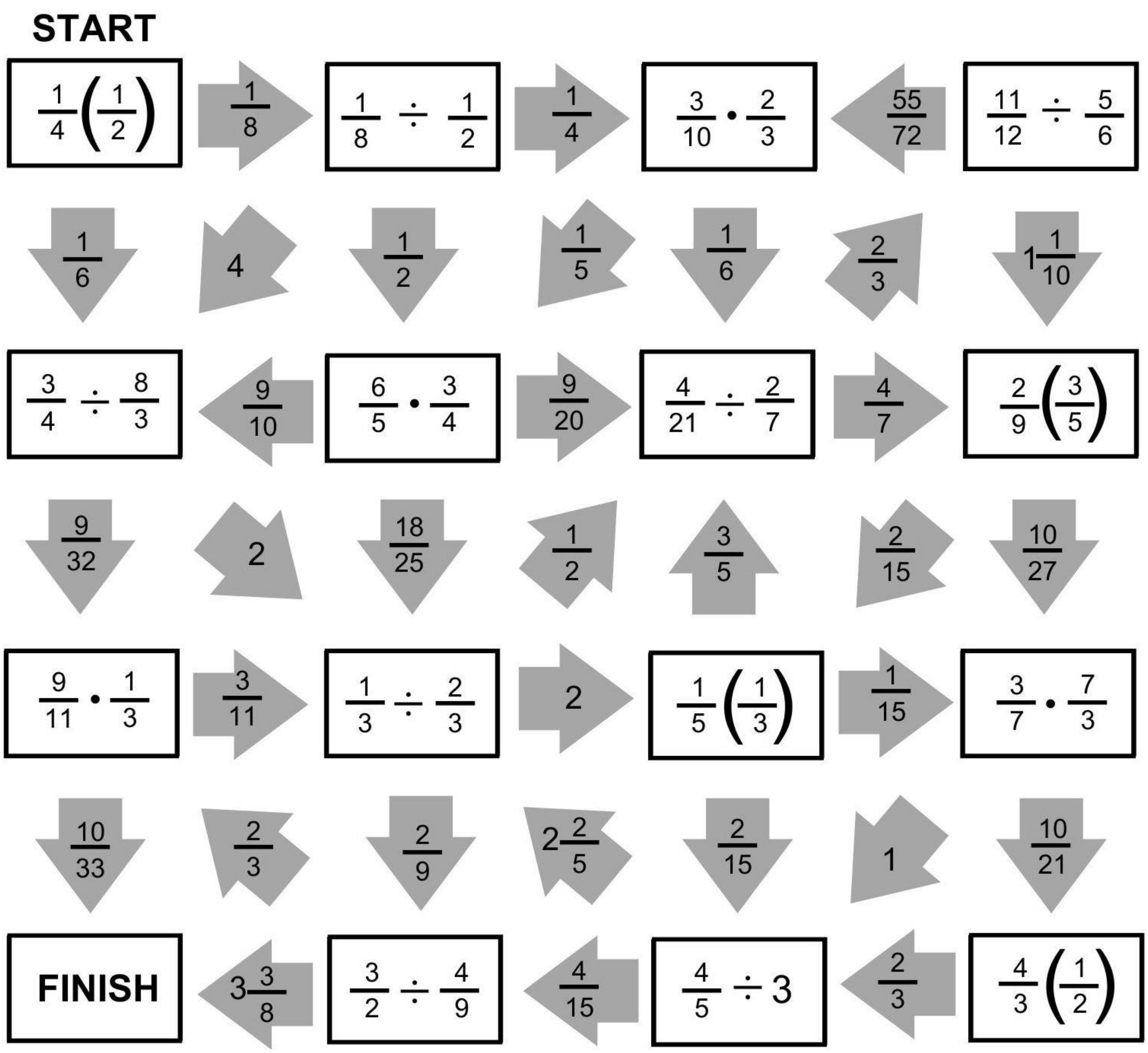
3 A ____ price is a price per one unit of measure.

5 For a good sandwich, Jayme likes her peanut butter and jelly in a ____ of 3 tsp to 2 tsp.

6 In the equation $2x = 10$, x is an unknown. It can also be called a ____.

SPIRAL REVIEW

1. **Math Path Fluency Challenge**: Use what you know about multiplication and division of fractions to find the correct path from Start to Finish. Note all products and quotients are in simplest form.



2. Complete the table:

Fraction			$2\frac{49}{50}$	$\frac{1}{8}$		$\frac{1}{200}$
Decimal		0.35			0.008	
Percent	75%					0.5%

SPIRAL REVIEW

Continued

3. You and a friend go out to lunch. You spend \$6.75 and your friend spends \$8.85.

   a. How much did you spend altogether?

   b. If the sales tax rate is 7.25%, how much tax will be paid?

   c. You leave a \$2.50 tip on your pre-tax total. About what percent was the tip?

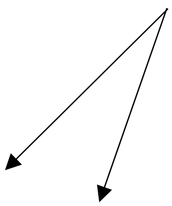
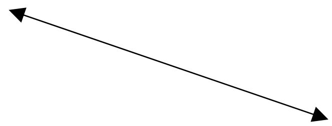
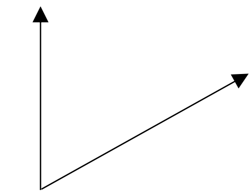
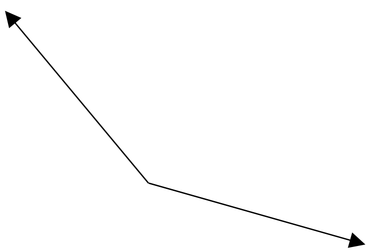
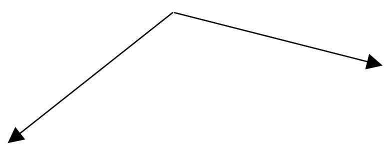
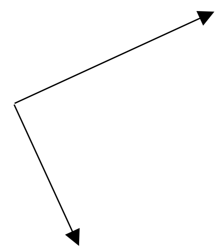
   d. What was the total cost for lunch, including tax and tip?

4. Solve each equation using substitution or mental math.

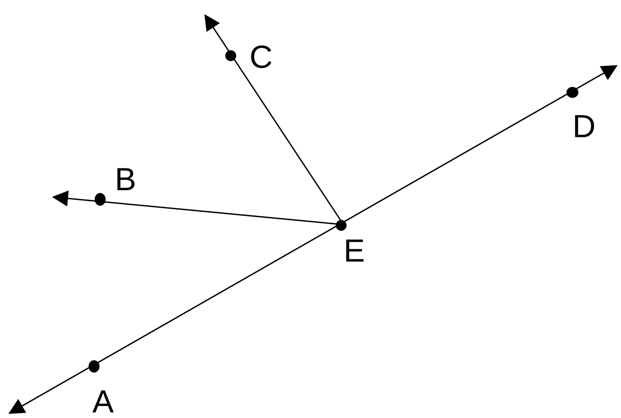
a. $3x = 48$	b. $500 = 270 + y$
c. $240 = 12y$	d. $45 = 67 - s$
e. $\frac{1}{5} + x = 1$	f. $\frac{1}{8}x = 160$

SPIRAL REVIEW
Continued

5. Label the angles as acute, right, obtuse, or straight. Then write a fact about the degree measure of each angle.

a. 	b. 	c. 
d. 	e. 	f. 

6. Use the picture to the right.



   a. Name an acute angle.

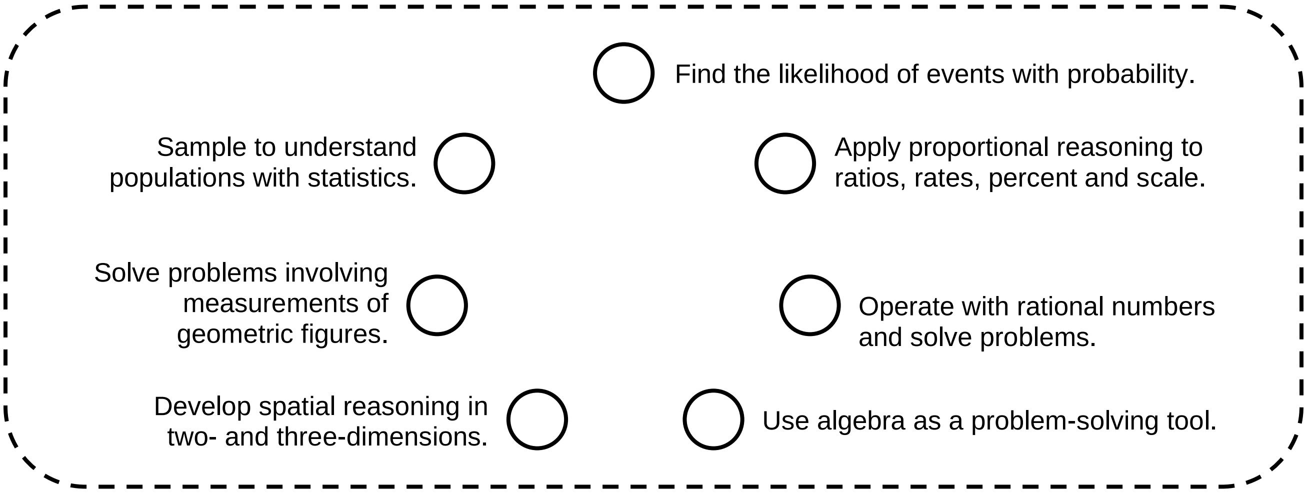
   b. Name an obtuse angle.

   c. Name a right angle.

   d. Name a straight angle.

PACKET REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this packet. Draw lines to show connections that you noticed.



   Give an example from this packet of one of the connections above.

2. **Packet Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before.

3. **Mathematical Practices.** How did you use mathematical representations to make sense of an everyday problem? [SMP1, 2, 4]

4. **Making Connections.** You used tables, graphs, and equations to represent proportional relationships in this packet. Why do you think it is useful to represent proportional relationships in different ways?

STUDENT RESOURCES

Word or Phrase	Definition
complex fraction	A complex fraction is a fraction whose numerator or denominator is a fraction. Two complex fractions are $\frac{\frac{4}{5}}{\frac{1}{2}}$ and $\frac{\frac{1}{5}}{3}$.
constant of proportionality	See proportional.
dependent variable	A dependent variable is a variable whose value is determined by the values of the independent variables. See independent variable.
equation	An equation is a mathematical statement that asserts the equality of two expressions. 18 = 8 + 10 is an equation that involves only numbers. This is a numerical equation. 18 = x + 10 is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.
expression	A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8 + x}{10}$, and $4v – w$.
equivalent ratios	Two ratios are equivalent if each number in one ratio is obtained by multiplying the corresponding numbers in the other ratio by the same positive number. 5 : 3 and 20 : 12 are equivalent ratios because both numbers in the ratio 5 : 3 are multiplied by 4 to get to the ratio 20 : 12.
independent variable	An independent variable is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables. For the equation $y = 3x$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.

Word or Phrase	Definition
input-output rule	An <u>input-output rule</u> for a sequence of values is a rule that establishes explicitly an output value for each given input value. **input value (x)**: 1, 2, 3, 4, 5, x **output value (y)**: 1.5, 3, 4.5, 6, 7.5, $1.5x$ In the table above, the input-output rule could be $y = 1.5x$. In other words, to get the output value, multiply the input value by 1.5. If $x = 100$, then $y = 1.5(100) = 150$.
proportional	Two variables are <u>proportional</u> if the values of one are the same constant multiple of the corresponding values of the other. The variables are said to be in a <u>proportional relationship</u>, and the constant is referred to as the <u>constant of proportionality</u>. If Wrigley eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.
proportional relationship	See <u>proportional</u>.
ratio	A <u>ratio</u> is a pair of positive numbers in a specific order. The ratio of a to b is denoted by $a : b$ (read "*a* to *b*," or "*a* for every *b*"). The ratio of 3 to 2 is denoted by 3 : 2. The ratio of dogs to cats is 3 to 2. There are 3 cups of water for every 2 cups of juice. The fraction $\frac{3}{2}$ does not represent this ratio, but it does represent the ratio's value (or the <u>unit rate</u>).
unit price	A <u>unit price</u> is a price for one unit of measure.
unit rate	The <u>unit rate</u> associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the value $\frac{a}{b}$, to which units may be attached. The ratio of 40 miles each 5 hours has unit rate of 8 miles per hour.
value of a ratio	See <u>unit rate</u>.
variable	A <u>variable</u> is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to quantities that vary in a relationship (as in a formula or an input-output rule). They may refer to unknown quantities in expressions, equations or inequalities. Finally, they may be used to generalize rules of arithmetic. In the equation $d = rt$, the quantities d, r, and t are variables. In the equation $2x = 10$, the variable x may be referred to as the unknown. The equation $a + b = b + a$ generalizes the commutative property of addition for all numbers a and b.

Testing for a Proportional Relationship

Here are three ways to test if two variables are in a proportional relationship:

- The values of the ratios (unit rates or unit prices) created by data pairs are equivalent.
- An equation in the form $y = kx$ fits all corresponding data pairs.
- Graphed data pairs fall on a line through the origin (0, 0).

Note that this example does **not** represent a proportional relationship. Alexa buys tickets when she goes to the amusement park. This chart shows the costs for different quantities of tickets.

# of tickets	10	20	25	50	100
total cost	\$40	\$60	\$75	\$125	\$200
cost per ticket	\$4	\$3	\$3	\$2.50	\$2

Since the costs per ticket (unit prices) are not the same, ticket purchasing at this amusement park does **not** represent a proportional relationship.

This example **does** represent a proportional relationship. Antonio kept track of the number of miles he traveled each time he filled his tank with gas. Here is some data.

number of miles	100	200	175	300
number of gallons	4	8	7	12
miles per gallon	25	25	25	25

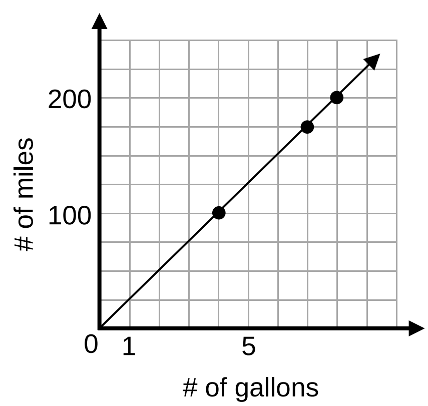
Since the miles per gallon (unit rates) created by the data pairs is the same, this situation represents quantities in a proportional relationship.

Furthermore,

Let x = the number of gallons
Let y = the number of miles

The data fits the equation $y = 25x$ (an equation in the form $y = kx$), which is an equation that represents a proportional relationship.

Finally, if the points for (gallons, miles) are graphed, they will fall on a line through the origin (0,0).



Multiple Representations and Proportional Relationships

Suppose 4 balloons cost $6.00 and each balloon is the same price. Here are some strategies for representing this proportional relationship.

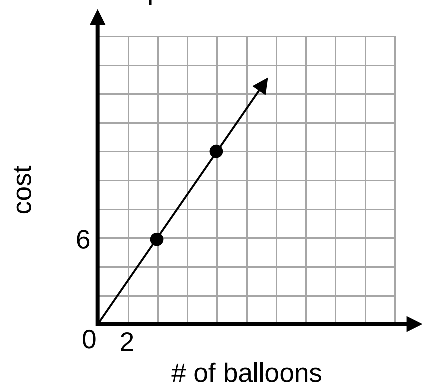
Strategy 1: Tables

Create a table to calculate unit rates. If the unit rates are the same, the variables are in a proportional relationship.

Number of Balloons	Cost	Unit Price
4	$6.00	$1.50
2	$3.00	$1.50
1	$1.50	$1.50
8	$12.00	$1.50

Strategy 2: Graphs

A straight line through the origin indicates quantities in a proportional relationship.



Strategy 3: Equations

An equation of the form $y = kx$ indicates quantities in a proportional relationship. In this case,

y = cost in dollars
x = number of balloons
k = cost per balloon (unit price)

To determine the unit price, create a ratio whose value is: $\frac{6 \text{ dollars}}{4 \text{ balloons}} = 1.50 \frac{\text{dollars}}{\text{balloons}}$

Therefore, k = $1.50 per balloon, and $y = 1.50x$.

This equation expresses the output as a constant multiple of the input, showing that the relationship is proportional.

Sense-Making Strategies to Solve Proportional Reasoning Problems

How much will 5 pencils cost if 8 pencils cost $4.40?

Strategy 1: Use a "halving" strategy

If 8 pencils cost $4.40, then
4 pencils cost $2.20,
2 pencils cost $1.10, and
1 pencil costs $0.55.

Therefore, 5 pencils cost

$0.55 + $2.20 = $2.75.

Strategy 2: Find unit prices

First, find the cost of one pencil.

$$\frac{\$4.40}{8} = \$0.55$$

Then, multiply by 5 to find the cost of 5 pencils,

($0.55)(5) = $2.75.

Sammie can crawl 12 feet in 3 seconds. At this rate, how far can she crawl in $1\frac{1}{2}$ minutes?

Strategy 1: Make a table

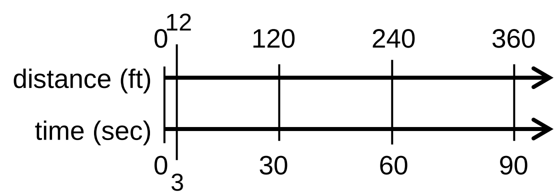
Distance	Time
12 ft	3 seconds
4 ft	1 second
240 ft	60 sec = 1 min
120 ft	30 sec = $\frac{1}{2}$ min
360 ft	90 sec = $1\frac{1}{2}$ min

Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

Strategy 2: Make a Double Number Line

12 feet in 3 seconds is equivalent to
120 feet in 30 seconds

$1\frac{1}{2}$ minutes = 90 seconds.

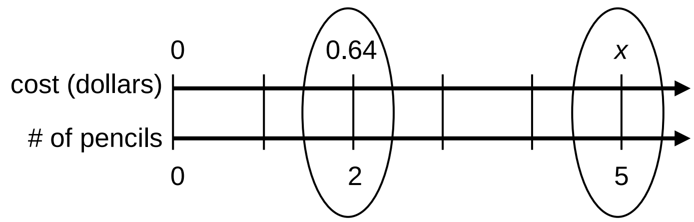


Sammie can crawl 360 feet in $1\frac{1}{2}$ minutes.

Writing Equations Based on Rates

Here are some ways to set up an equation to solve a rate problem. An equation in the form $\frac{a}{b} = \frac{c}{d}$ is commonly referred to as a “proportion.” Double number lines help make sense of this process. (See boxes on the next page for equation solving strategies.)

If 2 pencils cost $0.64, how much will 5 pencils cost?



Strategy 1: Compare rates (“between” two different units)

Create two rates from ratios that compare dollars to pencils. Equate expressions and solve for x.

$$\frac{x}{5} = \frac{0.64}{2}$$

x = 1.60 dollars for 5 pencils.

Note: The equation $\frac{5}{x} = \frac{2}{0.64}$ is another valid “between” equation for this problem.

Strategy 2: Compare like units (‘within” the same units)

Create one rate based on corresponding cost ratios and another rate based on the corresponding numbers of pencils ratios. Then, equate expressions and solve for x.

$$\frac{\text{cost}_{\text{case 1}}}{\text{cost}_{\text{case 2}}} = \frac{0.64}{x} \qquad \frac{\text{pencils}_{\text{case 1}}}{\text{pencils}_{\text{case 2}}} = \frac{2}{5}$$

$$\frac{0.64}{x} = \frac{2}{5}$$

x = 1.60 dollars for 5 pencils.

Note: The equation $\frac{x}{0.64} = \frac{5}{2}$ is another valid “within” equation for this problem.

Some Properties Relevant to Solving Equations

Here are some important properties of arithmetic and equality related to solving equations.

- The multiplication property of equality states that equals multiplied by equals are equal. Thus, if $a = b$ and $c = d$, then $ac = bd$.

  Example: If $1 + 2 = 3$ and $5 = 9 - 4$, then $(1 + 2)(5) = 3(9 - 4)$.

- The cross-multiplication property for equations states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ ($b \neq 0$, $d \neq 0$).

  This can be remembered with the diagram: $\frac{a}{b} \times \frac{c}{d}$.

  Example: If $\frac{5}{7} = \frac{12}{x}$, then $5 \bullet x = 7 \bullet 12$.
  To see that this property is reasonable, try simple numbers:
  If $\frac{3}{4} = \frac{6}{8}$, then $3 \bullet 8 = 4 \bullet 6$.

Applying Properties to Solve Proportion Equations

Strategy 1: Multiplication Property of Equality	**Strategy 2: Cross-Multiplication Property**
Solve for x: $\frac{x}{12} = \frac{3}{8}$ (Multiplication Property of Equality) $(8 \bullet 12) \bullet \frac{x}{12} = \frac{3}{8} \bullet (8 \bullet 12)$ $8x = 36$ $x = \frac{36}{8}$ $x = 4\frac{1}{2}$	Solve for x: $\frac{x}{12} = \frac{3}{8}$ (Cross-multiplication property) $8 \bullet x = (3 \bullet 12)$ $8x = 36$ $x = \frac{36}{8}$ $x = 4\frac{1}{2}$

Simplifying Complex Fractions

Strategy 1: A complex fraction can be written as a division problem.

Example: $$\frac{\frac{1}{4}}{\frac{3}{8}} = \frac{1}{4} \div \frac{3}{8} = \frac{1}{4} \bullet \frac{8}{3} = \frac{8}{12} = \frac{2}{3}$$

Strategy 2: A complex fraction can be multiplied by a form of the “big one” to create a denominator equal to one. Multiply the numerator and denominator each by the reciprocal of the denominator (in this case since the reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$). This process leaves a multiplication problem to compute.

Example: $$\frac{\frac{1}{4}}{\frac{3}{8}} \bullet \frac{\frac{8}{3}}{\frac{8}{3}} = \frac{\frac{1 \bullet 8}{4 \bullet 3}}{\frac{3 \bullet 8}{8 \bullet 3}} = \frac{\frac{8}{12}}{1} = \frac{8}{12} = \frac{2}{3}$$

While Strategy 2 seems to require more steps, this strategy makes more transparent the properties involved in writing the complex fraction in a more usable form.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
7.RP.A	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*
7.RP.2	Recognize and represent proportional relationships between quantities:
a	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c	Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*
d	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.
7.NS.3	Solve real-world and mathematical problems involving the four operations with rational numbers.
7.EE.B	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
7.G.A	**Draw, construct, and describe geometrical figures and describe the relationships between them.**
7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.