

7-3 NONROUTINE PROBLEMS

RUNNING RATES

Lilia, Chris, Braxton, and Trinity run on the cross-country team.

1. Select all tables that represent a proportional relationship between the runners' times and distances ran.

a. Lilia

Time (min)	0	7.5	15	18.75
Distance (mi)	0	1	2	2.5

b. Chris

Time (min)	0	14.6	36.25	50
Distance (mi)	0	2	5	7

c. Braxton

Time (min)	0	3	12	19.5
Distance (mi)	0	$\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{4}$

d. Trinity

Time (min)	0	1.75	5.25	10.5
Distance (mi)	0	$\frac{1}{4}$	$\frac{3}{4}$	$1\frac{1}{2}$

2. Jason just joined the team and runs 3 miles in 18.45 minutes. Circle all statements below that that are proportional to the relationship between these two values.

- a. Jason runs half a mile in 3.075 minutes.
- b. Jason runs 4.5 miles in 19.95 minutes.
- c. Jason runs about 1.63 miles in 10 minutes. (if we accept slight rounding)
- d. Jason runs about 6 miles in 21.45 minutes.

3. How long would it take Jason to run a mile at his rate of 3 miles in 18.45 minutes?
How long is this in minutes and seconds? 6.15 min = 6 min 9 sec.

RUNNING RATES

Continued

4. Andrea runs a 3-mile race in 24.15 minutes. Assuming Andrea runs at a constant rate of speed, circle **all** the statements below that are true.

a. Andrea runs at a rate of 8.05 miles/minute.

b. Andrea runs at a rate of 8.05 minutes/mile.

c. The equation $y = 8.05x$ relates the time in minutes (x) and distance in miles (y) of Andrea's run.

d. The equation $y = 8.05x$ relates the distance in miles (x) and time in minutes (y) of Andrea's runs.

5. Caleb ran $\frac{5}{6}$ of a mile in 7.5 minutes.

a. If he continues at the same rate of speed, how long will it take Caleb to run a mile?
9 minutes

b. Draw a double number line or tape diagram to support your answer to part a.

Answers will vary. One possible answer:

$6(1.5) = 9 \text{ miles}$					
1.5	1.5	1.5	1.5	1.5	1.5
7.5 miles					

6. Braxton runs a 3-mile race and averages 7 min/mile. Drayton runs the same race runs each mile in exactly 7 minutes? Explain how are Braxton's and Drayton's rates the same, and potentially different too.


Answers may vary. One possible answer:

They both ran at the same average speed. However, we don't know that Braxton ran each mile at the exact same pace the way Drayton did.

PROTEIN DRINKS

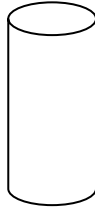
(Using the MathLinks Rubric) See Activity Routines in the Teacher Portal for directions.

Suppose you are shopping for your favorite protein drink. You find there are two different size drinks available.



Regular Size

\$4.80 for
12 ounces



Super Size

21 ounces
for \$6.30

- Which drink is the better buy? Justify your answer using a graph **and one more representation** (tables, equations, double number lines, etc.). Label and scale your graph appropriately.

Representations will vary. One possible method is to calculate the unit rate per ounce.

Regular size is \$0.40 per ounce.

Super Size is \$0.30 per ounce. The Super Size is the better buy based on price. However, someone may not want that much protein drink.

- Find the points (1, y) on the graphs. Explain what each of these coordinate pair means in the context of the problem.

<p>Regular: (1, <u>0.4</u>) <i>The Regular is 40 cents per ounce.</i></p>	<p>Super: (1, <u>0.3</u>) <i>The Super is 30 cents per ounce.</i></p>
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- After doing the math in this problem, is there anything else you would take into consideration in choosing one of the protein drink sizes? Explain.

Answers will vary. If 21 ounces of protein drink is too much, the Super would be a waste.

STRATEGIES FOR SIMPLIFYING COMPLEX FRACTIONS

A complex fraction is a fraction whose numerator or denominator is a fraction.

1. Here are two mathematical strategies for simplifying complex fractions. Fill in the missing parts of the fractions.

<p style="text-align: center;">Strategy 1:</p> <p>Write the complex fraction as a division problem.</p> $\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \div \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{1}{2} \cdot \frac{4}{3} = \frac{4}{6}$ <p style="text-align: center;"> step 1 step 2 step 3 </p>	<p style="text-align: center;">Strategy 2:</p> <p>Multiply by a form of the “big one” to create a denominator equal to one.</p> $\frac{\frac{1}{2}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}} = \frac{\frac{1}{2} \cdot \frac{4}{3}}{\frac{3}{4} \cdot \frac{4}{3}} = \frac{\frac{4}{6}}{1} = \frac{4}{6}$ <p style="text-align: center;"> step 1 step 2 step 3 step 4 </p>
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Circle all “step numbers” that apply. Note that a procedure may occur in one strategy or both.

	Strategy 1 Step #	Strategy 2 Step #
2. The complex fraction is rewritten using the “÷” symbol for division.	1 2 3	1 2 3 4
3. A form of the “big one” (multiplicative identity) is used to create a denominator equal to one.	1 2 3	1 2 3 4
4. The “multiply by the reciprocal” method for fraction division is applied.	1 2 3	1 2 3 4
5. The “multiply across” method for fraction multiplication is applied.	1 2 3	1 2 3 4
6. Describe where it is clear that Strategy 1 and Strategy 2 will give the same result.	1 2 3	1 2 3 4

STRATEGIES FOR SIMPLIFYING COMPLEX FRACTIONS

Continued

Simplify each fraction by rewriting it as a division problem (strategy 1).

<p>7. $\frac{\frac{3}{5}}{\frac{1}{5}}$</p> $\frac{3}{5} \div \frac{1}{5} = \frac{3}{5} \cdot \frac{5}{1} = 3$	<p>8. $\frac{\frac{2}{3}}{\frac{5}{6}}$</p> $\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \cdot \frac{6}{5} = \frac{4}{5}$
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Simplify each fraction by using a form of the “big one” to create a denominator equal to one (strategy 2).

<p>9. $\frac{\frac{1}{4}}{\frac{5}{8}}$</p> $\frac{\frac{1}{4} \cdot \frac{8}{5}}{\frac{5}{8} \cdot \frac{8}{5}} = \frac{\frac{8}{20}}{1} = \frac{2}{5}$	<p>10. $\frac{\frac{5}{9}}{\frac{3}{6}}$</p> $\frac{\frac{5}{9} \cdot \frac{6}{3}}{\frac{3}{6} \cdot \frac{6}{3}} = \frac{\frac{30}{27}}{1} = \frac{10}{9} \text{ or } 1\frac{1}{9}$
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5. Explain which strategy you prefer. *Answers will vary.*
6. Blakely said, “I think I know a shortcut for simplifying complex fractions,” and then she drew the picture below. Is her work correct? Explain what she did.

$$\frac{\frac{1}{2}}{\frac{3}{4}} \Rightarrow \frac{4}{6} = \frac{2}{3} \quad \text{Yes, because she multiplied the first fraction by the reciprocal of the second.}$$

Simplify each complex fraction. Use a strategy of your choice.

<p>11. $\frac{\frac{5}{3}}{\frac{1}{9}}$</p> <p style="text-align: center; color: red; font-weight: bold;">15</p>	<p>12. $\frac{\frac{2}{15}}{\frac{2}{5}}$</p> <p style="text-align: center; color: red; font-weight: bold;">$\frac{1}{3}$</p>
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MIXED PROBLEMS

1. A newspaper recently published a story that was titled: "Should People with Bigger Feet Pay More for Shoes?" *Answers will vary. One possible answer is below for each.*
 - a. Based on what you've learned so far about ratios and proportional relationships, what evidence or arguments might this article be trying to make about why people with bigger feet should pay more for shoes?
It takes more material to make bigger shoes, so it should cost more.
 - b. What might be a counterargument for why people with bigger feet should NOT pay more for shoes?
It would be difficult to price every size differently. The company could just find the average price to make all of the shoes (no matter the size) and use that for pricing. That way, all of the shoes could cost the same amount. There are many more factors that go into pricing shoes than just cost of materials (to name a few: rent for building space, cost of wages for workers, insurance, marketing, and sales costs).

2. A plumber is hired to install a bathroom. The plumber claims that the relationship between the number of hours worked and the total work fee is proportional. The fee for 6 hours of work is \$174. Select **all** combinations of values for the plumber's work hours and total work fee that support the claim that the relationship between the two values is proportional.
 - a. \$30 per hour
 - b. 8.75 hours and \$250
 - c. \$11 and 4 hours
 - d. 10.5 hours and \$304.50

3. Select all tables that represent a proportional relationship between x and y .

a.

x	0	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$
y	0	$\frac{1}{3}$	$\frac{2}{3}$	1

b.

x	0	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
y	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

c.

x	0	1	2	3
y	0	1	4	9

d.

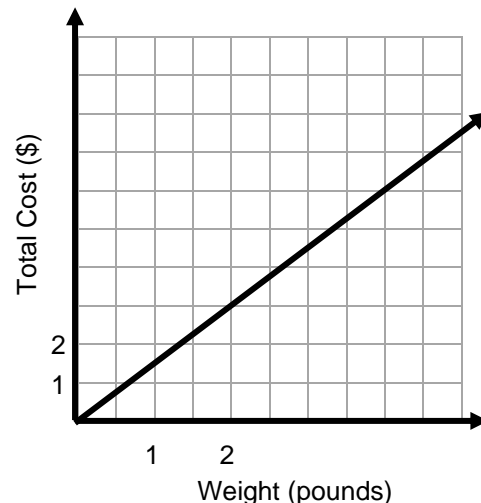
x	0	1	6	9
y	0	2.5	15	22.5

MIXED PROBLEMS
Continued

4. Lee wants to buy pies for her party for about 30 to 40 guests. One pie will serve 4 to 5 people. Each pie costs \$8.75.
- How many pies should Lee buy so that all guests gets pie? Justify your answer.
Answers will vary. One possible answer: The maximum number of people is 40 guests. Assuming one pie serves 4 people, Lee will need to buy 10 pies.
 - What will the total cost be?
If Lee buys 10 pies, the total cost is \$87.50.
 - If Lee has a budget of \$100 for the party, how much is left over, if any?
If Lee buys 10 pies, \$12.50
5. When playing a video game, Mason wins 6 out of every 8 games she plays. Select all of the statements that describe Mason’s situation.
- The ratio of Mason’s wins to losses is 3:4.
 - The ratio of Mason’s wins to games played is 3:4.
 - The equation $8x = 6y$ shows the relationship between the number of games Mason wins (x) and the number of games she loses (y).
 - The equation $8x = 6z$ shows the relationship between the number of games Mason wins (x) and the number of games she plays (y).
6. This graph shows the relationship between the pounds of apples bought at the market and the total cost, in dollars, for the apples.

Select each statement about the graph that is true.
Select all that apply.

- The point (0,0) shows the cost of 0 apples is \$0.00.
- The point (1, 1.5) shows the cost is \$1.00 for 1.5 pounds of apples.
- The point (1, 1.5) shows the cost is \$1.50 for 1 pound of apples.
- The point (2,3) shows the cost of 2 apples is \$3.00.
- The point (3, 4.5) shows the cost of 4.5 pounds of apples is \$3.00.



FROM THE MATH OLYMPIAD

1. 4 chefs require 10 minutes to prepare 20 desserts. At this rate, how many chefs are needed to prepare 75 desserts in 15 minutes?

"Chef-minutes" are the total time required by all the chefs to do a job.

To prepare 20 desserts requires $(4)(10) = 40$ chef-minutes.

So each dessert requires $\frac{40}{20} = 2$ chef-minutes.

To prepare 75 desserts would then require $(75)(2) = 150$ chef-minutes.

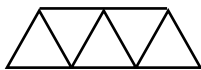
Therefore in 15 minutes, $\frac{150}{15} = 10$ chefs are required.

2. For 8 weeks of work, Melanie will receive \$600 and a new computer. After only 6 weeks of work, she would be entitled to a new computer but only an additional \$150. What is the value of the computer in dollars?

By leaving 2 weeks early, Melanie lost $\$600 - \$150 = \$450$. This is $\$225$ per week. At this rate in eight weeks, she would have earned a total of $\$1800$.

The value of the computer is $\$1800 - \$600 = \$1200$.

3. Avi used 11 toothpicks to form a row of five attached triangles, as shown. Suppose he continued this pattern, using 89 toothpicks in all. What is the total number of triangles formed?



The first three toothpicks form one triangle. After that every two additional toothpicks form another triangle. After the first three toothpicks are used, 86 are left. These 43 pairs of toothpicks form 43 more triangles. Including the first triangle, 44 triangles are formed.

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