FOCUS, COHERENCE, AND RIGOR

A group of educators at Rutgers University led by Dr. Roberta Shorr (Young, 2009) conducted a seven-year study on problem solving with minority and low-income students in low performing schools. They found that giving conceptually challenging problems to students caused frustration, but at the same time students were engaged and motivated. Working through the frustration in an emotionally safe environment, students gained "satisfaction, pride, and a willingness to work harder next time." This work also resulted in higher standardized test scores (e.g., average scores for Newark fourth graders rose from 45 to 79 percent.

At the Center for Mathematics and Teaching, we know that all students have potential to achieve in mathematics, we believe that the development of mathematics should reflect the connectedness of Big Ideas into a coherent whole, and we aim to make mathematics inviting and inclusive to more students. One of the ways we put these beliefs and intentions into action is by attending to focus, coherence, and rigor in program development.

FOCUS

According to Achieve the Core (2014), "Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness."

Achieve the Core organizes mathematics clusters into three categories: major clusters, supporting clusters, and additional clusters. This table identifies the clusters for 6th grade and their alignment to *MathLinks* lessons.

FOCUS IN *MATHLINKS* BASED ON MAJOR, MINOR, AND SUPPORTING CLUSTERS

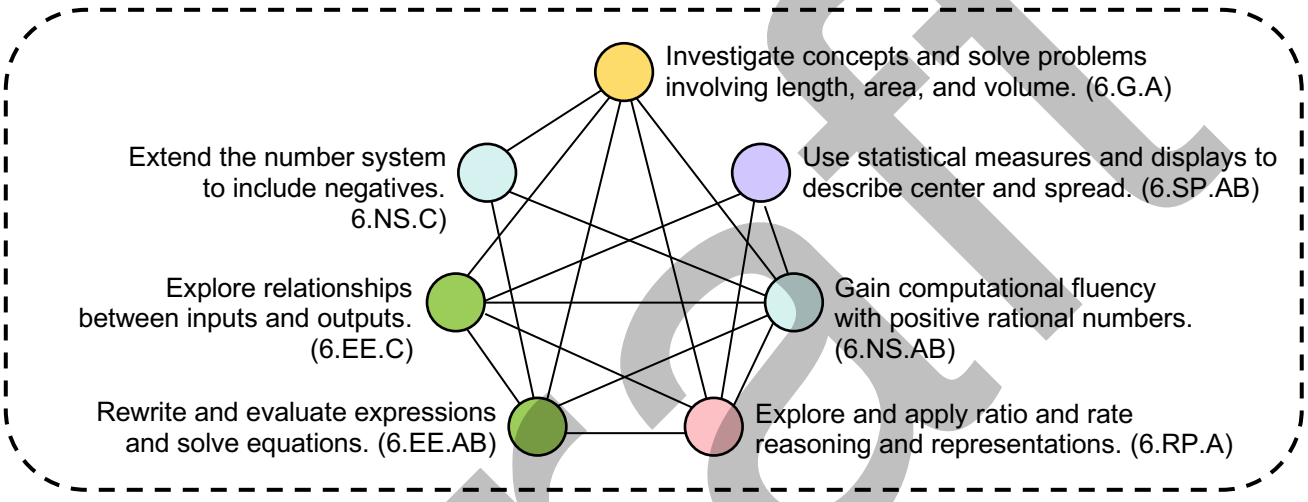
CLUSTER	Standards	Lesson Focus	Additional Lessons, Spiral Review Packets
Major Clusters			
• 6.RP.A Understand ratio concepts and use ratio reasoning to solve problems.	6.RP.1, 2, 3abcd	3.1, 3.2, 3.3, 3.4, 5.1, 5.2, 5.3	4.2, 4.3, 7.2, 7.3, 9.3 Spiral Rev 4,5,6,7,8,9,10
• 6.NS.A Apply and extend previous understandings of multiplication and division to divide fractions by fractions	6.NS.1	4.3, 4.4	Spiral Rev 5, 7, 9, 10
• 6.NS.C Apply and extend previous understandings of numbers to the system of rational numbers	6.NS.5, 6abc, 7abcd, 8	10.1, 10.2, 10.3, 10.4	<end of course, no spiral review>
• 6.EE.A Apply and extend previous understandings of arithmetic to algebraic operations.	6.EE.1, 2abc, 3, 4	6.1, 6.2, 6.3	7.1 7.2, 7.3, 9.1, 9.3 Spiral Rev 7,8, 9, 10
• 6.EE.B Reason about and solve one-variable equations and inequalities	6.EE.5, 6, 7, 8	6.2, 6.3, 8.1, 8.2, 8.3	7.1, 7.2, 7.3, 9.1, 9.3, 10.1, 10.2 Spiral Rev 9, 10
• 6.EE.C Represent and analyze quantitative relationships between dependent and independent variables	6.EE.9	7.1, 7.2, 7.3	8.3, 9.1, 9.3
Supporting Clusters			
• 6.G.A Solve real-world and mathematical problems involving area, surface area, and volume	6.G.1, 2, 3, 4	9.1, 9.2, 9.3, 10.4	<end of course, no spiral review>
Additional Clusters			
• 6.NS.B Compute fluently with multidigit numbers and find common factors and multiples	6.NS.2, 3, 4	2.1, 2.2, 2.3 4.1, 4.2, 6.1	Spiral Rev (all)
• 6.SP.A Develop understanding of statistical variability	6.SP.1, 2, 3	1.1, 1.2, 1.3	5.3
• 6.SP.B Summarize and describe distributions	6.SP.4, 5abcd	1.1, 1.2, 1.3	5.3, 7.2 Spiral Rev 9

COHERENCE

According to a draft of the CA Math Framework (2022), "instructional materials should primarily involve tasks that invite students to make sense of big ideas, elicit wondering in authentic contexts, and necessitate mathematical investigation. Big ideas in mathematics are central to the learning of mathematics, link numerous mathematical understandings into a coherent whole, and provide focal points for students' investigations."

Inspired by the work of Jo Boaler (Boaler, Munson, Williams, What is Mathematical Beauty? Teaching through Big Ideas and Connections, youcubed.org, 2017) and based on the Content Domains, the *MathLinks* team synthesized the work of 6th grade into seven big ideas. These ideas and their connections within the entire program are shown here.

GRADE 6: BIG IDEAS AND CONNECTIONS



The work in *MathLinks* is organized into ten packets. An expanded dive into the specific big ideas and connections are included in the front section of each Teacher Edition packet. Towards the end of each Student Packet, students identify the big ideas within the packet and reflect upon how they are connected to each other.

RIGOR

According to the Common Core State Standards Initiative (2022), "a rigorous math program will pursue with equal intensity conceptual understanding, procedural skill and fluency, and applications." We first examine the MathLinks: Grade 6 program through this lens.

Conceptual Understanding in *MathLinks*

Conceptual understanding, the bedrock of a *MathLinks* course, frequently drives the other two components of rigor. It's a *MathLinks* philosophy to make sure all students have the opportunity to make meaning for every concept presented, and we aim to focus on the conceptual development of big ideas in depth, and make them plausible through investigations, activities, and practice. This is commonly done throughout lessons in Student Packets, oftentimes with the help of teacher Lesson Notes and Slide Decks (or Slide Deck Alternatives).

This table identifies some examples of concept development in *MathLinks:* Grade 6 Student Packets (lesson location in parentheses). Typically, these lessons include guided instruction with a slide deck, lesson notes, class discussions, or group work. Opportunities for independent work within a Student Packet appear as "Practice" pages within lessons, in the "Review" section as activities, and as spiral review in subsequent packets. Packet Resources on the Teacher Portal also contain problems, tasks, and projects to support conceptual development.

EXAMPLES OF CONCEPT DEVELOPMENT LESSONS IN *MATHLINKS*: GRADE 6	
CLUSTER DESCRIPTION	**CONCEPT DEVELOPMENT WITH TEACHER OR GROUP**
Major Clusters	
• 6.RP.A (RP 1, 2, 3) Understand ratio concepts and use ratio reasoning to solve problems.	(3.1) Paint Mixtures (5.3) Percent and Double Number Lines
• 6.NS.A (NS 1) Apply and extend previous understandings of multiplication and division to divide fractions by fractions	(4.3) Exploring Divide Across (4.4) Exploring Multiply by the Reciprocal
• 6.NS.C (NS 5, 6, 7, 8) Apply and extend previous understandings of numbers to the system of rational numbers	(10.1) Opposites (10.1) Distance and Absolute Value
• 6.EE.A (EE 1, 2, 3, 4) Apply and extend previous understandings of arithmetic to algebraic operations.	(6.1) GCF and the Distributive Property (6.3) Perimeter of a Rectangle
• 6.EE.B (EE 5, 6, 7, 8) Reason about and solve one-variable equations and inequalities	(8.2) Solving Equations with Mental Math and Substitution (8.2) Solving Equations with Tapes and Balance
• 6.EE.C (EE 9) Represent and analyze quantitative relationships between dependent and independent variables	(7.1) What Comes Next (7.1) Input-Output Rules
Supporting and Additional Clusters	
• 6.G.A (G 1, 2, 3, 4) Solve real-world and mathematical problems involving area, surface area, and volume	(9.1) Area of a Parallelogram
• 6.NS.B (NS 2, 3, 4) Compute fluently with multidigit numbers and find common factors and multiples	(2.1) Finding the Greatest Common Factor
• 6.SP.A (SP 1, 2, 3) Develop understanding of statistical variability	(1.2) Name Scores Revisited
• 6.SP.B (SP 4, 5) Summarize and describe distributions	(1.3) Interpreting Box Plots

Applications in *MathLinks*

Problem solving is an important driver of instruction within *MathLinks* courses. In *MathLinks*, we aim to include engaging mathematical problems and applications with accessible entry points for all students, multiple approaches or solutions, and extensions to challenge and enrich. All Student Packets begin with an Opening Problem, which introduces a concept or establishes a "need to know." In many cases, students require more instruction throughout the packet before they are fully prepared to bring the problem to its conclusion. More "meaty" problems exist the packets as well.

This table identifies some examples of mathematical problems and applications in *MathLinks:* Grade 6 Student Packets (lesson location in parentheses). Some of these problems include guided instruction with a slide deck and lesson notes. Others are appropriate for independent work. Packet Resources on the Teacher Portal (e.g. Tasks, Projects) also contain mathematical problems and applications.

EXAMPLES OF PROBLEM SOLVING LESSONS IN *MATHLINKS*: GRADE 6	
CLUSTER DESCRIPTION	**MATHEMATICAL PROBLEMS AND APPLICATIONS**
Major Clusters	
• 6.RP.A (RP 1, 2, 3) Understand ratio concepts and use ratio reasoning to solve problems.	(3.0, 3.1) Nana's Chocolate Milk + Nana…Revisited (3.3) The Grain Grocer (5.0, 5.3) Growth Spurts, Growth Spurts Revisited
• 6.NS.A (NS 1) Apply and extend previous understandings of multiplication and division to divide fractions by fractions	(4.3) Practice 8 (Problems 1, 8) (4.4) Practice 11
• 6.NS.C (NS 5, 6, 7, 8) Apply and extend previous understandings of numbers to the system of rational numbers	(10.4) A Basketball Court, Practice 8
• 6.EE.A (EE 1, 2, 3, 4) Apply and extend previous understandings of arithmetic to algebraic operations.	(6.0, 6.1) The Problem of 4's + Problem of 4's Extended
• 6.EE.B (EE 5, 6, 7, 8) Reason about and solve one-variable equations and inequalities	(8.0) Lions and Tigers and Bears (8.1) Mobiles and Balance, Practice 1
• 6.EE.C (EE 9) Represent and analyze quantitative relationships between dependent and independent variables	(7.3) Raising Money for Music, Practice 7 (7.3) Practice 9
Supporting and Additional Clusters	
• 6.G.A (G 1, 2, 3, 4) Solve real-world and mathematical problems involving area, surface area, and volume	(9.0, 9.1) Which Rug is Bigger?, Practice 3 (Problem 5) (9.2) Who Needs More Paint? (9.3) The Food Drive
• 6.NS.B (NS 2, 3, 4) Compute fluently with multidigit numbers and find common factors and multiples	(2.1) The Factor Game
• 6.SP.A (SP 1, 2, 3) Develop understanding of statistical variability	(1.0, 1.1) Beach Cleanup, Practice 5 (Question 2)
• 6.SP.B (SP 4, 5) Summarize and describe distributions	(1.Review) Ages of Presidents (7.2) A Committee Decision

Procedural Skill and Fluency in *MathLinks*

Procedural skill and fluency are more than just computational speed and accuracy. In a 2014 position paper, NCTM described procedural fluency as "the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another.

In *MathLinks*, we aim to thoughtfully develop new procedural skills and provide opportunities for students to gain fluency throughout the year. Skills practice in Student Packets is located in:

- Practice pages – These pages support concept development.
- Review activities – These pages often include skills practice.
- Spiral Review – These pages review prior skills and concepts with distributed practice.
- Computational Fluency Challenge – This Activity Routine, which is in the Spiral Review section, aims to develop fluency with whole number and decimal operations, which are expectations for 6th grade.

Achieve the Core explicitly identifies two CCSS-M standards (6.NS.2 and 6.NS.3) where fluency is expected in 6th grade. The Ed Report Evidence Guide (2021) also includes 6.EE.1 as a fluency expectation. The table below shows some examples in 6th grade Student Packets where these procedural skills are developed and students have opportunities to gain fluency through independent practice.

EXAMPLES OF COMPUTATIONAL FLUENCY WORK IN *MATHLINKS*: GRADE 6		
Standard / Description	**Development of Procedure**	**Opportunities to gain fluency**
6.NS.2 Fluently divide multidigit numbers using the standard algorithm	(4.1) Whole Number Division	(4.1) Practices 1- 4 Spiral Review (Packets 1, 2, 3, 5, 6, 7)
6.NS.3 Fluently perform operations with multidigit decimals using the standard algorithm.	(4.2) Decimal Division and Rate Problems (5.1) Percent as a Number	(4.2) Practices 5 – 7 (5.1) Practice 3 Spiral Review (Packets 1, 2, 3, 8, 9, 10)
6.EE.1 Write and evaluate numerical expressions involving whole number exponents.	(6.0) Problem of 4s (6.1) Numerical Expressions	(6.1) Practices 2 – 4 (6.1) Problem of 4s Extended Spiral Review (Packets 7, 8, 9, 10)

In addition to Student Packets, resources in the Teacher Portal offer opportunities to practice grade level skills and to fill in gaps where needed. Look in Packet Resources for:

- Essential Skills – this entire section reviews skills and concepts important for success in a given packet. Activity Routines such as Big Square Puzzles, Open Middle Problems, and Four-in-a-Row Games are also in these sections for some packets. They provide a practice alternative to "drill and kill."

- Extra Problems – Problems are available for every lesson.

- Nonroutine Problems – Some Big Square Puzzles, Open Middle Problems, and Four-in-a-Row Games are also located in this section for some packets.

Finally, for students who need to fill major gaps in skills, consider Skill Boosters, which are in the Teacher Portal. This Resource helps students catch up on below grade level work without losing extensive instructional time on grade level work. A Skill Booster routine is intended to take about 10 minutes per day.

Connecting the Three Aspects of Rigor

Interpretations of "rigor" are evolving. In 2022, the CA Framework defined rigor to mean that "conceptual understanding can be used to analyze a novel situation encountered in the world. Rigor means that students can flexibly apply methods to different situations, and connect mathematical ideas, approaches, and representations." A closer look at two major domains in 6th grade illustrates how this is done.

Ratio and Proportional Relationships

The Ratio and Proportional Relationships standards are the focus of four packets in *MathLinks*: Grade 6. In Packet 3, **Ratio Representations**, students begin the transition from additive to multiplicative thinking by learning to create and interpret ratios, tape diagrams, tables, equivalent ratios, and double number lines in a variety of contexts. In Packet 4, **Division**, students explore the structure of rate problems and solve rate problems as they gain fluency with division. In Packet 5, **Percent**, students use sense making strategies, computational procedures, and double number lines to make sense of percent and percent applications. In Packet 7, **Inputs and Outputs**, students informally expand their notion of proportional relationships, specifically the use of unit rates, into the world of input-output rules (functions). Problems such as Nana's Chocolate Milk and Growth Spurts provide contexts for extended explorations.

Expressions and Equations

The Expressions and Equations standards are the focus of four packets in MathLinks Grade 6. In Packet 6, **Expressions**, students work with both numerical and algebraic expressions. They learn to manipulate, simplify, and evaluate expressions using the distributive property and the conventions for order of operations. They also translate between words, numbers, and symbols. In Packet 7, **Inputs and Outputs**, students use visuals and contexts to analyze and solve problems with multiple representations. Concepts related to proportional reasoning are reviewed and emphasized. Students' knowledge of expressions enables them to generate equations for relationships relating two variables, called "input-output rules." Without explicitly defining "function" (this is done in grade 8), students begin to develop flexibility when working with variables, expressions, and equations. The problems introduced set the stage for solving a linear equation in one variable since these equations are of the form $x + p = q$ and $px = q$ (i.e., "one-step" equations") for cases in which p and q and are nonnegative rational numbers. In Packet 8, **Solving Equations**, students learn mental math and substitution strategies and formalize the equation-solving processes, using balance and tape diagrams to work toward traditional equation-solving procedures, as the need grows. In Packet 10, **Number Lines and the Coordinate Plane**, students graph solutions to simple inequalities. Problems such as The Problem of 4s and Raising Money for Music provide contexts for explorations and connections.

In Conclusion

The authors of MathLinks carefully researched and thoughtfully operationalized focus, coherence, and rigor as we designed the program. We aimed to integrate mathematical concepts, skills, and applications through contexts and problems in a natural way. The result is an efficient core program that contains just 10 Student Packets with 33 lessons that can be completed in about 100 days! This leaves ample time for review, intervention, enrichment, and choice for both teachers and students, using the resources available in Student Packets and on the Teacher Portal.