PACKET 7
TEACHER EDITION

MathLinks GRADE 6

INPUTS AND OUTPUTS

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, ~~tape diagrams~~, double number line diagrams, or equations:
a.	Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b.	Solve unit rate problems including those involving unit pricing and constant speed.
c.	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
6.EE.A	**Apply and extend previous understandings of arithmetic to algebraic expressions.**
6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers:
a.	Write expressions that record operations with numbers and with letters standing for numbers.
b.	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems.
6.EE.B	**Reason about and solve one-variable equations and inequalities.**
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.C	**Represent and analyze quantitative relationships between dependent and independent variables.**
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*
6.SP.A	**Develop understanding of statistical variability.**
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

PACKET PLANNING

Starred (*) resources can be accessed on the Teacher Portal.

Packet Pacing* Up to 14 class periods	7.0 Opening Problem: Pick a Dot (< 1 period) 7.1 Visual Patterns (3 periods) 7.2 Comparing Prices (2-3 periods) 7.3 Rate Applications (2-3 periods) Review (3 periods) Assessment (1 period)
Packet Resources* Up to 3 class periods	• Extra Problems • Essential Skills • Math Talks • Nonroutine Problems • Tasks • Projects (in some packets) • Technology Activities
Assessment Options*	• On the Teacher Portal ✓ Packet Quizzes ✓ Comprehensive Tests ✓ Tasks ✓ Projects ✓ Using the *MathLinks* Rubric (an Activity Routine) • In the Student Packet ✓ Monitor Your Progress ✓ Packet Reflection • In the Teacher Edition ✓ References to Journals ✓ Suggested problems for the *MathLinks* Rubric
Materials	• Square tiles [7.1] • Poster paper, butcher paper, or board space [Review] • Markers [Review] • General supplies (e.g., colored pencils, rulers, tape, scissors, graph paper, calculators)
Slide Decks*	S7.0 Pick a Dot S7.1a What Comes Next? S7.1b Input-Output Rules S7.2 The Keychain Fundraiser S7.3 Raising Money for Music
Reproducibles*	R7-1 Match and Compare Sort Cards
Prepare Ahead	• Organize the distribution of square tiles, cups, and counters (baggies are recommended). Use positive/negative counters as a substitute for tiles if they are not unavailable. • See Activity Routines in the Teacher Portal for instructions for Using the *MathLinks* Rubric, **Poster Problems**, **Match and Compare Sort**, and **Why Doesn't it Belong?** [7.2, Review]
Other Resources*	• Parent Letter (English and Spanish) • Videos

COMPONENTS FOR DIFFERENT USERS

These resources can be accessed on the Teacher Portal.

For all students	• Student Packet (copyright protected, for viewing and projecting) • Resource Guide (Complete) • Extra Problems • Math Talks • Nonroutine Problems • Tasks • Projects (in some packets) • Technology Activities • Carole's Puzzles and Games
For English learners	• Student Packet Text File for Translation
For struggling learners	• Essential Skills • Extra Problems • Skill Boosters (Options: Decimal and Percent Concepts)
For advanced learners	• Student Packet (speed up instruction when possible) • Nonroutine Problems
For teachers	• Teacher Edition (this document) • Resource Guide (Complete) • Program Information • Activity Routines (Explanations and Introductory Examples)
For substitutes	• Previous Student Packets (unfinished work) • Practice 1 – 9 (may be completed independently any time after instruction of those topics in the packet) • Spiral Review • Vocabulary Review • Extra Problems • Carole's Puzzles and Games
For parents	• Resource Guide (Complete) • Parent Letter (English and Spanish)

MATH BACKGROUND

Independent and Dependent Variables

Independent variables are under our control, in the sense that we may specify their values. Once the values of the independent variables have been specified, the values of the dependent variables are completely determined. We have no control over them.

When two variables are in a proportional relationship, the values of either variable completely determine the values of the other. Either variable could be regarded as the independent variable, and the remaining variable would then be regarded as the dependent variable. Which of the two variables is the independent variable, and which is the dependent variable, depends on the context of the problem.

Consider this situation: Suppose text messages cost \$0.05 each.

Let n = the number of text messages sent.
Let C = the cost of a text message bill (in dollars).

- If we know the number of text messages sent, then we can determine the cost of the text message bill as $C = 0.05n$. In this case, n is the natural independent variable and C is the dependent variable because the total cost depends on the number of text messages sent.
- If we are operating within a budget and we have a limit C on what we can spend for text messages, then we can determine the number of text messages we can send as $n = \frac{C}{0.05}$. In this case, C is the natural independent variable and n is the dependent variable because the number of text messages depends on the total cost.

TEACHING TIPS

Applying Standards for Mathematical Practice (SMP)		
Here is an abbreviated version of the SMPs and some ways they are applied in this packet.		
SMP1	**Make sense of problems and persevere in solving them.** • Understand a problem and look for entry points. • Consider simpler or analogous problems. • Monitor progress and alter solution course as needed. • Make connections between multiple representations. • Check answers with a different method.	[All Lessons] Multiple representations are used extensively throughout this packet to help students make connections between numbers in tables, graphs, double number lines, equations, and the contexts from which these are all derived.
SMP2	**Reason abstractly and quantitatively.** • Use numbers and quantities flexibly in computations. • Attend to the meaning of quantities. • Decontextualize a problem using symbols, manipulate them, and then interpret based on the context.	[All Lessons] Students decontextualize situations and manipulate algebraic representations for the purpose of problem solving.
SMP3	**Construct viable arguments and critique the reasoning of others.** • Use assumptions, definitions, established results, examples, and counter examples to analyze an argument and discuss its merits or flaws. • Make and test conjectures based on evidence. • Analyze situations by breaking them into cases. • Understand and analyze the approaches of others.	[7.1] Students analyze diagrams made by two students. [Review] For **Match and Compare Sort** and **Why Doesn't it Belong?**, students must pay close attention to other's explanations or descriptions, offer critiques or advice, and revise work.
SMP4	**Model with mathematics.** • Attach meaningful mathematics to everyday problems and questions of interest. • Make reasonable assumptions and approximations to simplify a situation. • Identify quantities, use mathematical tools (such as multiple representations, formulas, equations) to analyze relationships. • Interpret results and draw conclusions in the context of the situation.	[7.2] Students make a recommendation to a fundraising committee based upon pricing a survey. [7.3] In **A Committee Decision**, students use unit prices, data, and statistics to make a fundraising recommendation.

Applying Standards for Mathematical Practice (SMP) Continued		
SMP5	**Use appropriate tools strategically.** • Select and use tools strategically (and flexibly) to visualize, explore, and compare information. • Use technological tools and resources to solve problems and deepen understanding.	[7.2] In **A Committee Decision**, students may choose to use a calculator for computations to focus more on the modeling/decision-making aspect of the problem.
SMP6	**Attend to precision.** • Calculate accurately and efficiently. • Explain thinking using mathematical vocabulary. • Use symbols appropriately • Specify units of measure.	[7.0] **Pick a Dot** requires students to use precise, descriptive mathematical language. This activity demonstrates the value of mathematical notation and symbols. [7.1, 7.2, 7.3] Key mathematical vocabulary is underlined throughout the packet, precise definitions are provided, and students are expected to make sense of, and use, mathematical language. [7.3] The **Raising Money for Music** series of problems require attention to what is being asked to produce accurate calculations. [Review] **Match and Compare Sort** requires close attention to the meaning of new academic vocabulary.
SMP7	**Look for and make use of structure.** • Recognize the structure of a symbolic representation and generalize it. • See complicated objects as composed of chunks of simpler objects.	[7.1, 7.2] Templates provided for **Visual Patterns and Comparing Prices** lessons help students identify structure within representations and generalize processes.
SMP8	**Look for and make use of repeated reasoning.** • Identify repeated calculations and patterns. • Generalize procedures based on repeated patterns or calculations. • Find shortcuts based on repeated patterns or calculations.	[All Lessons] Creating an input-output rule from given information is a shortcut for finding information quickly. For example, in the **Poster Problems**, students create input-output rules and then use them to find the number of tiles in step 100, or the step number when there are 120 tiles.

Strategies to Support Diverse Populations

Classrooms typically include students with different learning styles and needs. Here are some specific ways that *MathLinks* supports special populations. Strategies essential to the academic success of English learners are noted with a star (*). See General Program Information for more details.

	General Examples	***MathLinks* Examples**
Know your Learner	✓ Understand student attributes that support or interfere with learning ✓ Determine preferred learning and interaction styles ✓ Assess student knowledge of prerequisite mathematics content ✓ Check for understanding continuously ✓ Provide differentiation opportunities for intervention or enrichment to reach more learners ✓ Encourage students to write about their attitudes and feelings towards math ✓ Use contexts that link to students' cultures*	Built into the *MathLinks* Design: SP: Getting Started, Spiral Review, Monitor Your Progress, Packet Reflection TE: References to Journals PR: Extra Problems, Essential Skills, Nonroutine Problems, Projects OR: Skill Boosters, Assessment Options [7.0] Use the **Pick a Dot** opening problem to assess student prior knowledge of coordinate graphing. [Review] Group students by language proficiency to increase interaction on activities that require cooperative learning.
Increase Academic Language through Mathematics	✓ Provide opportunities for students to read, write, or speak about their mathematical learning ✓ Explain the academic vocabulary needed to access mathematical ideas, providing both examples and non-examples ✓ Use strategically organized groups that attend to language needs* ✓ Use rich mathematical contexts and sophisticated language to help ELs progress in their linguistic development* ✓ Use cognates and root words (when appropriate) to link new math terms to students' background knowledge*	Built into the *MathLinks* Design: SP: Word Bank, Vocabulary Review, Student Resources TE: Grouping suggestions, References to Journals, Suggested problems for the *MathLinks* Rubric PR: Math Talks OR: Critique student work on Slide Decks [All] Ask students to share translations of mathematical terms and write them next to key vocabulary in **My Word Bank**. Point out that many mathematical terms have Latin roots. [7.0] Students with special needs may need a review of the definitions of horizontal and vertical. [Review] **Match and Compare Sort** requires students to engage in rich mathematical discussions with new terminology.

Components cited: Student Packet (SP), Teacher Edition (TE), Packet Resources (PR), Other Resource (OR)

<table>
<tr><th colspan="3">Strategies to Support Diverse Populations (Continued)</th></tr>
<tr><th></th><th>General Examples</th><th>MathLinks Examples</th></tr>
<tr><td>Increase Comprehensible Input</td><td>✓ Link concepts to past learning

✓ Make concepts meaningful through hands-on activities, visuals, demonstrations, and color-coding

✓ Use a think-aloud strategy to model appropriate thinking processes and academic language use

✓ Use graphic organizers to help students record information and data, see patterns, and generalize them

✓ Use multiple representations (pictures, numbers, symbols, words, contexts) of math ideas to create meaning and make connections

✓ Strategically sequence and scaffold to make mathematics accessible

✓ Simplify written instructions, rephrase explanations, and use verbal and visual clues*</td><td>Built into the MathLinks Design:
SP: Structured workspace
TE: Slide Deck Alternatives, Reproducibles, Materials
OR: Slide Decks

[All] Use an explicit think-aloud process to model appropriate language to make connections between numbers in tables, ordered pairs, graphs, or equations.

[7.1] Provide students with square tiles to build patterns.

[Review] Match and Compare Sort includes a Venn diagram to help students see similarities and differences between two related words. Check in frequently with English learners during this language-intense activity.</td></tr>
<tr><td>Promote Student Interaction</td><td>✓ Use flexible group configurations that support content objectives

✓ Use strategies and activities that promote teacher/student and student/student interactions (e.g., think-pair-share, Poster Problems)

✓ Encourage elaborate responses through questioning

✓ Allow processing time and appropriate wait time, recognizing the importance of the different requirements for speaking, reading, and writing in a new language*

✓ Allow alternative methods to express mathematical ideas (e.g., visuals, students' first language)*</td><td>Built into the MathLinks Design:
SP: Lesson and Review activities
TE: References for Journals, Suggested problems for the MathLinks Rubric
PR: Math Talks, various games and puzzles
OR: Slide Decks, Activity Routines

[All Lessons] Use think-pair-share strategies to encourage communication in a safe environment for all learners, especially those who are learning English as a second language.

[7.0] For Pick a Dot, allow students to be creative in terms of how they want to express themselves (e.g., hand gestures) if language is a barrier.

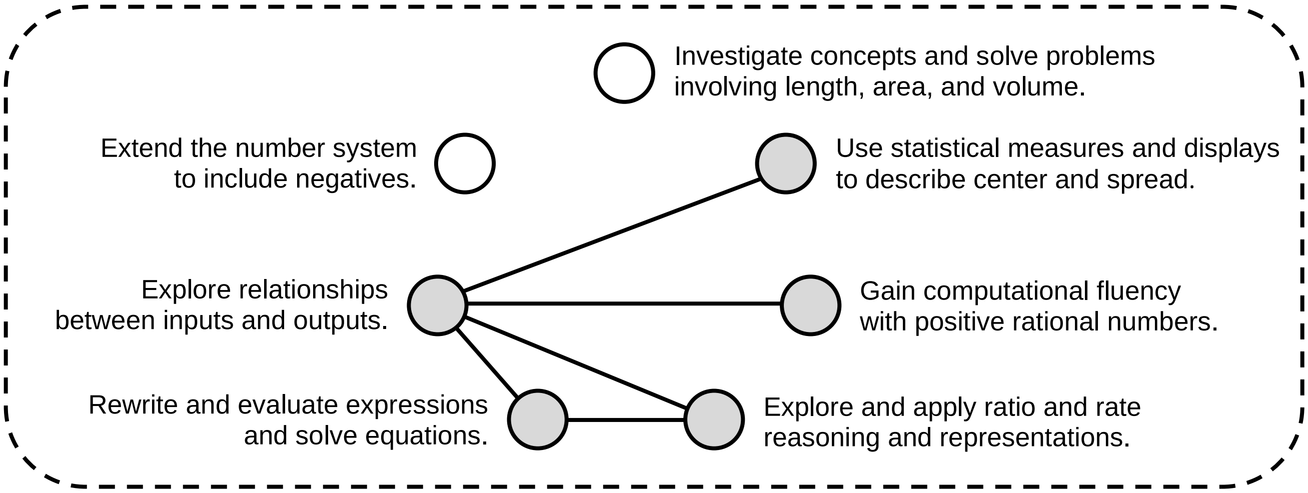
[Review] Match and Compare Sort are specifically designed to encourage student interaction and deemphasize competition. Organize students into pairs or groups that support language and content needs.</td></tr>
</table>

Components cited: Student Packet (SP), Teacher Edition (TE), Packet Resources (PR), Other Resources (OR)

Big Ideas and Connections

The Center for Mathematics and Teaching is dedicated to igniting and nurturing passion for mathematics in middle school students. We see the classroom as a place of joy and wonder, collaboration and purpose, perseverance and empowerment. We want all students to succeed in mathematics, as they explore its beauty in patterns, concepts, connections, and applications.

MathLinks: Grade 6 is organized around seven big ideas. This graphic provides a snapshot of the ideas in Packet 7 and their connections to each other.



These ideas build on past work and prepare students for the future. Some of these include:

Prior Work	What's Ahead
• Generate and analyze patterns and relationships (Grades 4, 5) • Graph points in the coordinate plane to solve real-world and mathematical problems (Grade 5) • Understand ratio concepts and use ratio reasoning to solve problems (Grade 6) • Extend understanding of arithmetic to algebraic expressions (Grade 6) • Represent and interpret data (Grades 4, 5) • Summarize and describe distributions (Grade 6) • Fluently perform operations with positive numbers (Grades 4, 5, 6)	• Analyze proportional relationships and use them to solve problems (Grade 7) • Use properties of operations to generate equivalent expressions (Grade 7) • Solve problems involving numerical expressions and equations (Grade 7) • Define, evaluate, and compare functions (Grade 8) • Use functions to model relationships between quantities (Grade 8) • Continue to explore the world of functions (HS)

Students May Wonder…

- ***Why is it important to identify the independent and dependent variables?*** For most problems at the 6th grade level it really doesn't matter. In fact, for any "invertible function", either variable could be the independent variable. But as students learn about different functions, the input (independent variable) becomes more natural, and general properties of function may only apply if the independent variable is defined properly. For example, quite often, we graph situations over time. In these cases, time is the independent variable.

The Algebra Progression in *MathLinks*: Grade 6

The Expressions and Equations standards are the focus of four packets in *MathLinks*: Grade 6.

- In Packet 6, **Expressions**, students worked with both numerical and algebraic expressions. They learned to manipulate, simplify, and evaluate expressions using the distributive property and the conventions for order of operations. They also translated between words, numbers, and symbols.

- In Packet 7 (this packet), **Inputs and Outputs**, students use visuals and contexts to analyze and solve problems with multiple representations. Concepts related to proportional reasoning are reviewed and emphasized. Students' knowledge of expressions enables them to generate equations for relationships relating two variables, called "input-output rules."

  Without explicitly defining "function" (this is done in grade 8), students begin to develop flexibility when working with variables, expressions, and equations. The problems introduced set the stage for solving a linear equation in one variable since these equations are of the form $x + p = q$ and $px = q$ (i.e., "one-step" equations") for cases in which p and q, and are nonnegative rational numbers.

- In Packet 8, **Solving Equations**, students will learn mental math and substitution strategies and formalize the equation-solving processes, using balance and tape diagrams to work toward traditional equation-solving procedures, as the need grows.

- In Packet 10, **The Number Line and the Coordinate Plane**, students will graph solutions to simple inequalities.

The Proportional Reasoning Progression in *MathLinks*: Grade 6

The Ratio and Proportional Relationships standards are the focus of four packets in *MathLinks*: Grade 6.

- In Packet 3, **Ratio Representations**, students began the transition from additive to multiplicative thinking by learning to create and interpret ratios, tape diagrams, tables, equivalent ratios, and double number lines in a variety of contexts.

- In Packet 4, **Division**, students explored the structure of rate problems and solved rate problems as they gained fluency with division.

- In Packet 5, **Percent**, students used sense-making strategies, computational procedures, and double number lines to make sense of percent and percent applications.

- In Packet 7 (this packet), **Inputs and Outputs**, students informally expand their notion of proportional relationships, specifically the use of unit rates, into the world of input-output rules (functions).

REPRODUCIBLES

R7-1 MATCH AND COMPARE SORT CARDS

I △ UNIT RATE	I ○ UNIT PRICE
II △ INPUT-OUTPUT RULE	II ○ EQUATION
III △ INDEPENDENT VARIABLE	III ○ DEPENDENT VARIABLE
IV △ DOUBLE NUMBER LINE	IV ○ GRAPH IN COORDINATE PLANE
A △ ✓ a variable whose value may be specified ✓ typically, the input	A ○ ✓ on a horizontal and a vertical number line ✓ ordered pairs in the form (x, y) may be graphed
B ✓ a diagram made of two parallel number lines ✓ two quantities can be compared (like a ratio)	B ○ ✓ a variable whose value is determined by the values of the independent variable ✓ typically, the output
C △ ✓ the value of a ratio ✓ example: 45 miles per hour	C ○ ✓ a statement that asserts that two expressions are equal ✓ example: 20 = 15 + 5
D △ ✓ an equation that establishes a specific output value for each input value ✓ example: $y = 2.5x$	D ○ ✓ the price for one unit of measure ✓ example: $1.10 per orange

PACKET 7
ANSWER KEY

MathLinks GRADE 6

INPUTS AND OUTPUTS

Materials | Grouping | Reproducibles | Slide Deck | Journal Idea

Parent (or Guardian) signature ___________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

coordinate plane / coordinate axes / origin	
dependent variable	independent variable
input-output rule	unit rate/unit price

PICK A DOT

[SMP6]

Follow your teacher's directions for (1) – (4). Descriptions will vary. Below is an example for one point.

(1)	My dot is right in the middle of the slide.
(2)	Go a little to the right of the dot on the x-axis, then pick the second dot going up vertically.
(3)	From where the darker lines (rays) intersect (the origin), count 8 units to the right and then 7 units up.
(4)	My dot is point O.

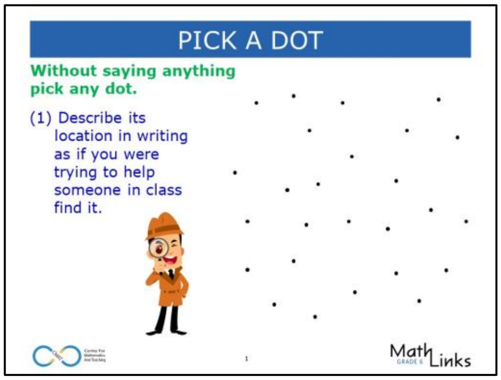
5. Record the meaning of coordinate plane in **My Word Bank**.

LESSON NOTES S7.0: PICK A DOT

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

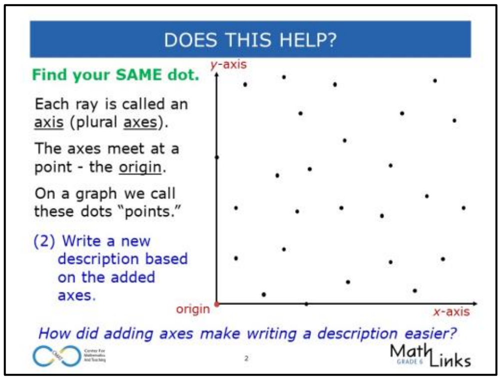
The activity is intended to create the intellectual "need to know" for the coordinate plane. Graphing in one quadrant was introduced in a grade 5 (Common Core Standard 5.G.1). Graphing in four quadrants will be introduced in Packet 10.

- Slide 1: For (1), students pick a dot and keep their choice to themselves while writing the location description. Ask a few students to read their descriptions aloud to the class, while other selected classmates try to guess the point's location. Or use this as a pair activity where students read each other's descriptions and make guesses.



- Slide 2: This slide reviews some important vocabulary. For (2), students write their new descriptions. Make sure students know that they must describe the SAME point as before.

  Why might these coordinate axes make describing the location of a point easier? Encourage the use of academic language so that students refer to specific axes or the origin as reference points.



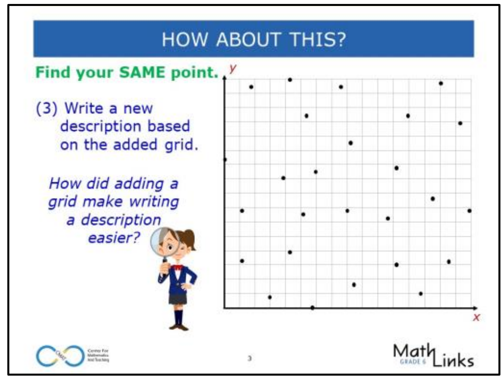
LESSON NOTES S7.0: PICK A DOT

Continued

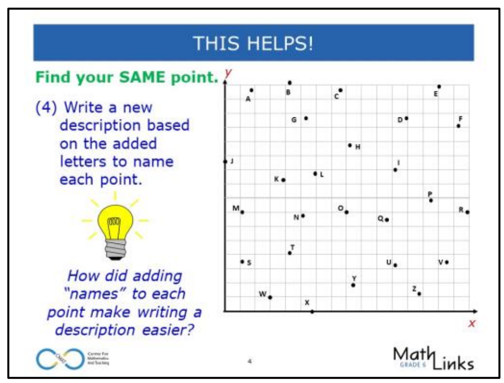
- Slide 3: For (3), students write a new description for the location of the SAME point.

  How does the grid help to make your description even easier? Regardless of prior experiences with the coordinate plane, now students can count specific numbers of units from reference points like the origin or the axes. Use this as a formative assessment to see if students remember that graphing in the first quadrant involves starting at the origin and counting to the right and then up. Have students share as they did for the previous slides.

  Informally discuss additional vocabulary associated with the coordinate plane as desired (e.g., ordered pairs, x-axis, y-axis, x-coordinate, y-coordinate)



- Slide 4: For (4), students write their final description, which can be as simple as stating the letter for the point. Collectively, these slides should highlight the utility of the coordinate graphing system.

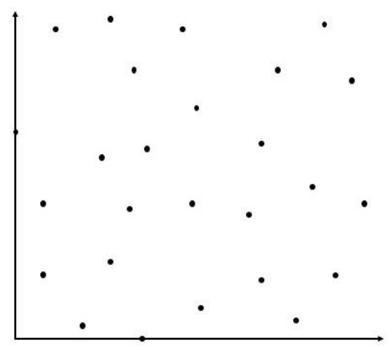


SLIDE DECK ALTERNATIVE S7.0: PICK A DOT

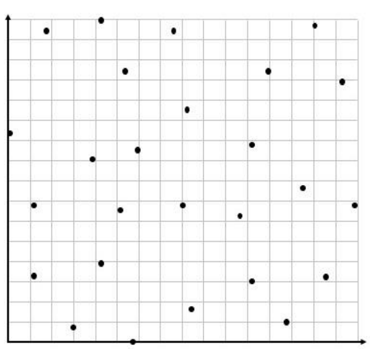
Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1-4

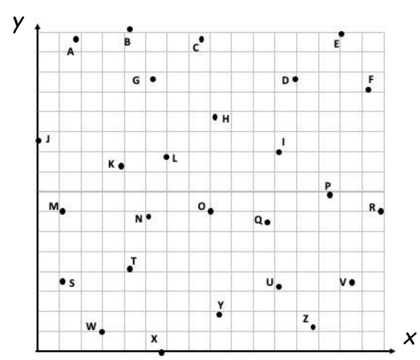
(1) Pick a dot. Describe its location in writing as if you were trying to help someone in the class find it.



(2) Write a new description based on the added axes.



(3) Write a new description based on the added grid.



(4) Write a new description based on the added letters to name each point.

VISUAL PATTERNS

We will review graphing ordered pairs. We will use words, tables of numbers, graphs, and algebraic input-output rules (equations) to describe visual patterns. We will identify what is typically the difference between independent variables and dependent variables.

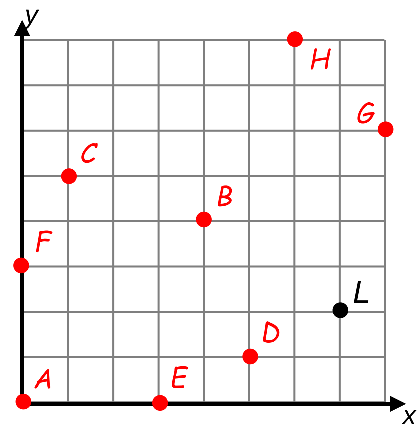
[6RP3a, 6EE2ab, 6EE6, 6EE9; SMP1, 2, 3, 6, 7, 8]

GETTING STARTED

Each small square on the grid represents 1 square unit.

1. Graph and label the following ordered pairs. As an example, the ordered pair (7, 2) is graphed and labeled point *L*.

A (0, 0)	*B* (4, 4)	*C* (1, 5)	*D* (5, 1)
E (3, 0)	*F* (0, 3)	*G* (8, 6)	*H* (6, 8)



2. How can you remember that we count the *x*-coordinate in the direction along the *x*-axis first and the *y*-coordinate in the direction along the *y*-axis second when graphing ordered pairs?
Answers will vary. One way to remember is that since *x* comes before *y* in the alphabet, we count in the *x* direction first.

Use the word list below to fill in the blanks. Some words are used more than once. Use the coordinate plane below for reference or notes.

coordinate plane	horizontal	ordered pairs	origin	vertical

3. A __coordinate__ __plane__ is a plane with a horizontal axis and a vertical axis meeting at the point (0, 0), called the __origin__.

4. The __horizontal__ axis is typically referred to as the *x*-axis.

5. The __vertical__ axis is typically referred to as the *y*-axis.

6. Points in the coordinate plane are named by pairs of numbers called __ordered__ __pairs__. They are written in the form (*x*, *y*).

7. From the origin to the point located at (3, 5), move 3 units in the __horizontal__ direction and 5 units in the __vertical__ direction.

WHAT COMES NEXT?

[SMP1, 3]

Follow your teacher's directions.

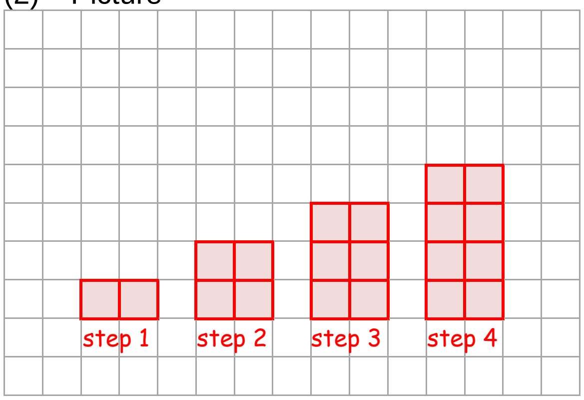
(1) ***Describe the way your pattern is growing.*** Answers may vary. Two possible answers:

- Start with 2 tiles and add 2 more each time. (Mateo's illustration below)
- Start with 2 tiles and double the amount each time. (Anita's illustration below)

(2) – (4) For both patterns: Draw steps 1-4 (build if desired); make tables; make graphs.

Mateo

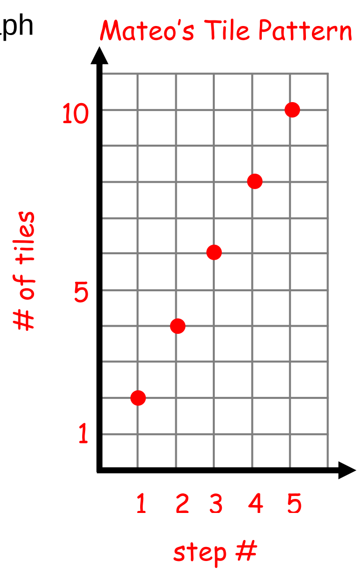
(2) Picture



(3) Table

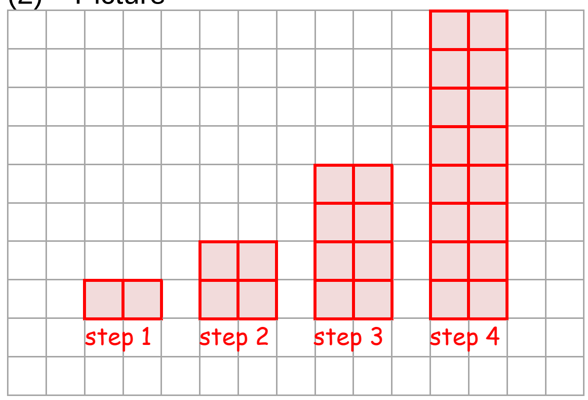
Mateo's Tile Pattern	
step # (x)	# of tiles (y)
1	2
2	4
3	6
4	8
5	10

(4) Graph



Anita

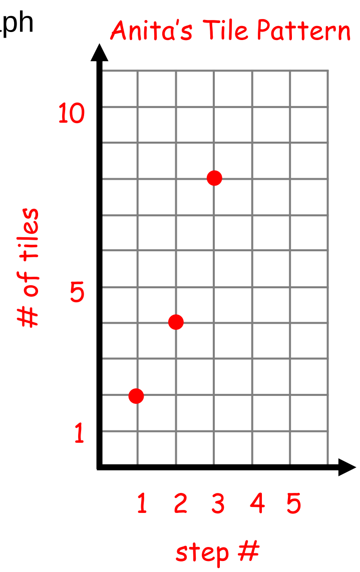
(2) Picture



(3) Table

Anita's Tile Pattern	
step # (x)	# of tiles (y)
1	2
2	4
3	8
4	16
5	32

(4) Graph



LESSON NOTES S7.1a: WHAT COMES NEXT?

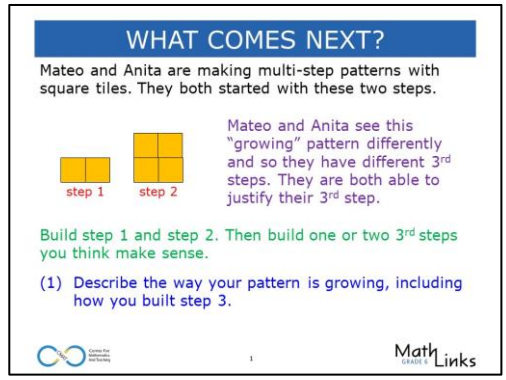
On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

We focus on growing geometric patterns for a review of multiple representations (i.e., recording numerical data in tables and graphs), a context that shows a relation between two variables, and a connection to rates. This work will continue in grade 7, and becomes formalized in grade 8 with a study of linear functions. Many students benefit from building each pattern with square tiles prior to drawing it.

- Slide 1: Show the growing pattern. Ask students to build the first two steps, and then a third step that makes sense to them.

  What is changing from step 1 to step 2? Two tiles are added, the number of tiles is doubled.

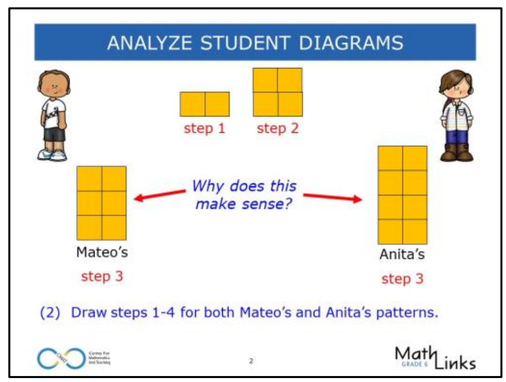
  For (1), students write descriptions of the growing pattern. Ask students to describe their 3rd steps (one or more). Share as desired.



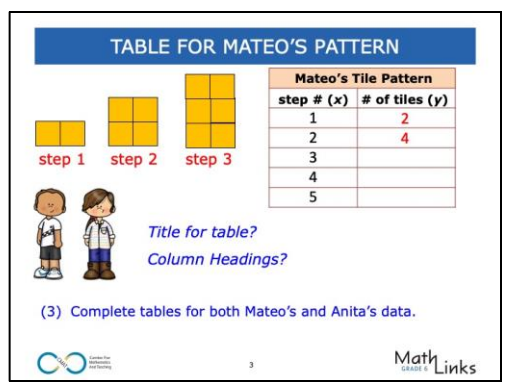
- Slide 2: Reveal Mateo's and Anita's 3rd steps, even if students have already come up with these options.

  Why can both of these be correct? Mateo sees the pattern as starting with 2 tiles and adding 2 more at each step. Anita sees it as starting with 2 tiles and doubling at each step.

  For (2), ask students to draw steps 1 – 4 for both patterns.



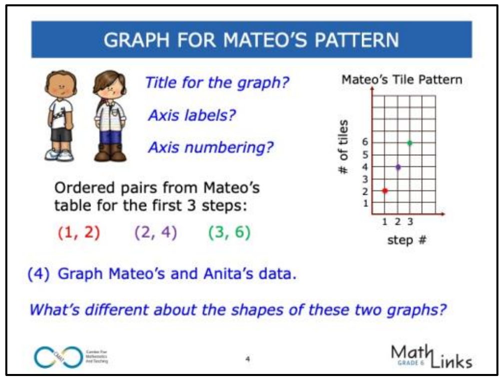
- Slide 3: For (3), ask students to complete tables for both. Discuss appropriate titles and column headings as needed.



LESSON NOTES S7.1a: WHAT COMES NEXT?

Continued

- Slide 4: For (4), ask students to complete graphs for both. If needed, discuss good title, axis labels, and axis numbering, and remind students that entry pairs from the table act as ordered pairs.



  Do we need to plot all data points in the table? No. Some may not fit. We only need enough points to determine the shape of the graph.

  Should we "connect the dots" for these graphs? This pattern context only makes sense for whole numbers of steps and tiles. It is permissible to draw a "trend line/curve" to show an overall shape.

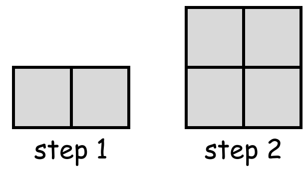
  What's different about the shapes of these graphs?
  - Mateo: points appear on a straight line; data points increase at a constant rate.
  - Anita: points appear on a curve; data points increase at increasing rate.

  How is the change from one step to the next shown on the different representations for Mateo? On the graph and in the table, as each step number increases by 1, the number of tiles increases by 2. For each increase by 2 tiles upward, the step number increases by 1 to the right.

SLIDE DECK ALTERNATIVE 7.1a: WHAT COMES NEXT?

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Mateo and Anita are making multi-step patterns with square tiles. They both started with the two steps. They see the patterns growing differently so they have different 3rd steps.



Build steps 1 and 2. Then build one or two 3rd steps you think make sense.

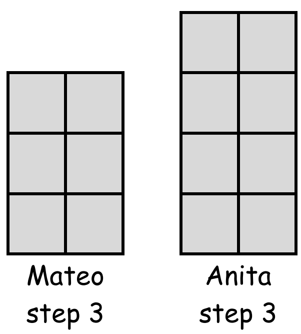
Slide 1

(1) Describe the way your pattern is growing.

Use your description to explain how you built step 3.

Slide 2

(2) Here is step 3 for Mateo's pattern and Anita's pattern. Draw steps 1-4 for both patterns.



Slide 3

(3) Complete tables for both Mateo's and Anita's data.

Slide 4

(4) Graph Mateo's and Anita's data.

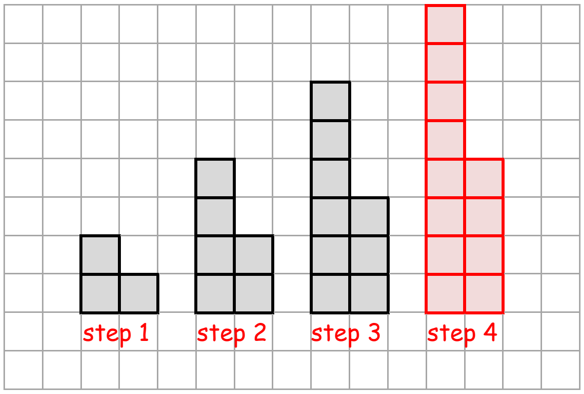
What is different about the shapes of these two graphs?

PRACTICE 1

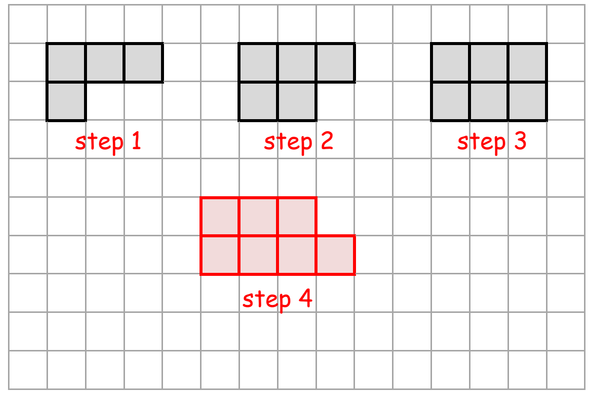
[SMP1]

1. Build steps 1 – 4 for tile patterns A and B if needed. Draw step 4 for each pattern. Complete the tables and draw the graphs with titles and labels.

Tile Pattern A



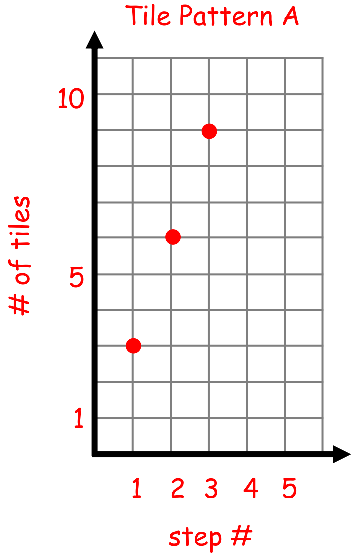
Tile Pattern B



Tile Pattern A	
step # (x)	# of tiles (y)
1	3
2	6
3	9
4	12
5	15

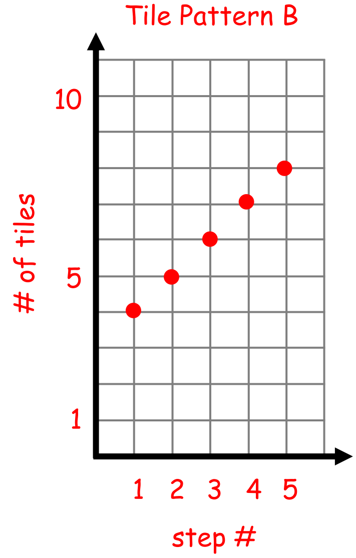
Tile Pattern B	
step # (x)	# of tiles (y)
1	4
2	5
3	6
4	7
5	8

Tile Pattern A



trend lines may be drawn to stress the linear relationships

Tile Pattern B



2. Why is it appropriate to leave the points unconnected? Step numbers and numbers of tiles must be whole number amounts, so we do not want to include points that represent fractional step numbers or numbers of tiles.

Square tiles

1+

INPUT-OUTPUT RULES

[SMP1, 2, 6]

Follow your teacher's instructions for (1) – (2).

(1) ***Input-output rule for Mateo in words.*** To get the number of tiles, multiply the step number by 2.	(2) ***Input-output rule for Mateo in symbols.*** $y = 2x$

3. Record the meaning of input-output rule in **My Word Bank**.

4. Write an input-output rule for Pattern A on **Practice 1** in words and symbols.
   To get the number of tiles, multiply the step number by 3.
   $y = 3x$

5. Write an input output rule for Pattern B on **Practice 1** in words and symbols.
   To get the number of tiles, add 3 to the step number.
   $y = x + 3$

The input values (x) are sometimes referred to as the "independent variable," and output values (y) are sometimes referred to as the "dependent variable."

Fill in the missing numbers and blanks based on the suggested numerical patterns. In the tables below, the x-value is considered the input value and the y-value is the output value.

6.

x	1	2	3	4	5	6
y	5	6	7	8	9	10

a. Rate of change: for every increase of x by 1, y increases by 1.

b. Input-output rule (words): add 4 to an x-value (independent variable) to get its corresponding y-value (dependent variable).

c. Input-output rule (equation): $y = x + 4$

7.

x	1	2	3	4	5	6
y	4	8	12	16	20	24

a. Rate of change: for every increase of x by 1, y increases by 4.

b. Input-output rule (words): Multiply an x-value (independent variable) by 4 to get its corresponding y-value (dependent variable).

c. Input-output rule (equation): $y = 4 \bullet x$

LESSON NOTES S7.1b: INPUT-OUTPUT RULES

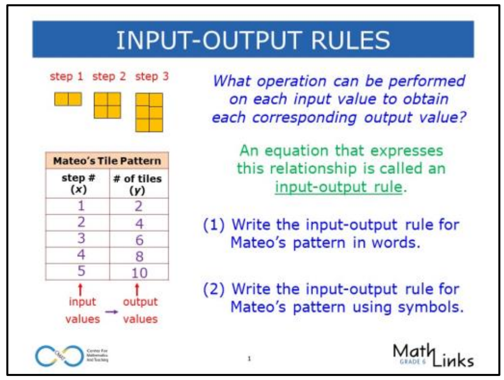
On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

We revisit three geometric patterns and their corresponding tables of values to focus on the relationship between the "independent variables" and "dependent variables." Students also begin to write these relationships in words and with symbols (equations) as input-output rules.

- Slide 1: Revisit Mateo's pattern. Focus on the relationship between the input values and the output values, and then discuss <u>input-output rules</u>.

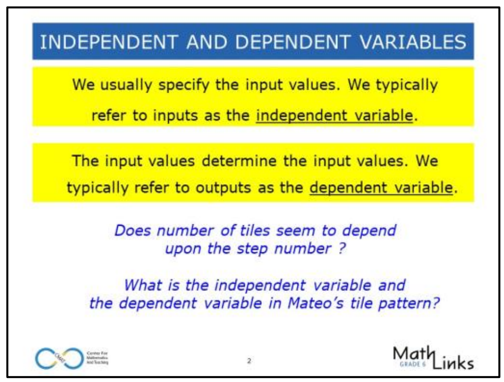
  What can we do to each x-value to obtain the corresponding y-value? Multiply the x-value by 2 to get the y-value. If students reply to add 2 each time, they are recognizing a recursive rule that describes successive output values, which is valid, but not the focus here.

  For (1) and (2), students write an input-output rule for Mateo's pattern in words and using symbols.



- Slide 2: Discuss terms that are associated with the input values (independent variable) and output values (dependent variable) as desired.

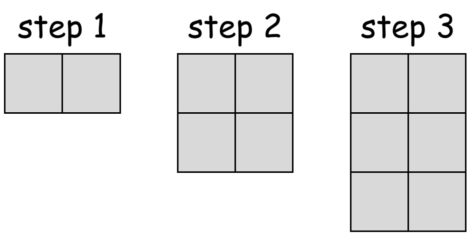
  Why does the number of tiles depend on the step number? If we know the step number, we can multiply it by 2 to determine the number of tiles. That is why step number is the independent variable and the number of tiles is the dependent variable.



SLIDE DECK ALTERNATIVE S7.1b: INPUT-OUTPUT RULES

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1



Mateo's Tile Pattern	
step # **(x)**	**# of tiles** **(y)**
1	2
2	4
3	6
4	8
5	10

↑ ↑

input values → output values

(1) Write the input-output rule for Mateo's pattern in words.

(2) Write the input-output rule for Mateo's pattern using symbols.

Slide 2

We usually specify the input values. We typically refer to the inputs as the <u>independent variable</u>. The output values are determined by the input values. We typically refer to the outputs as the <u>dependent variable</u>.

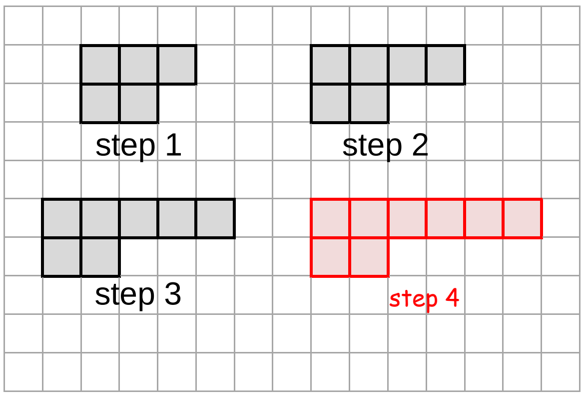
(3) For Mateo's tile pattern, specify the independent variable and the dependent variable.

PRACTICE 2

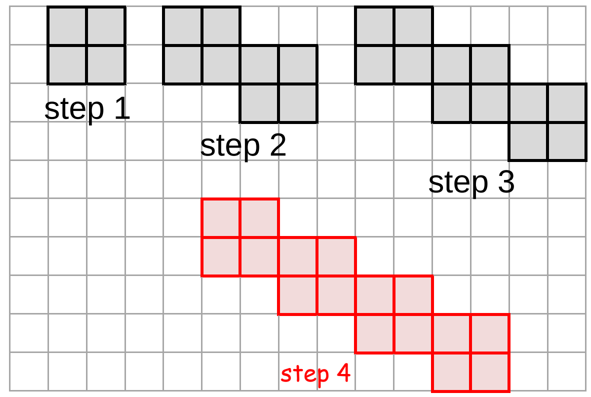
[SMP1, 2, 7]

1. Build steps 1–4 for tile patterns C and D if needed. Draw step 4 for each. Complete the tables and make graphs with titles and labels. Write an input-output rule (equation).

Tile Pattern C



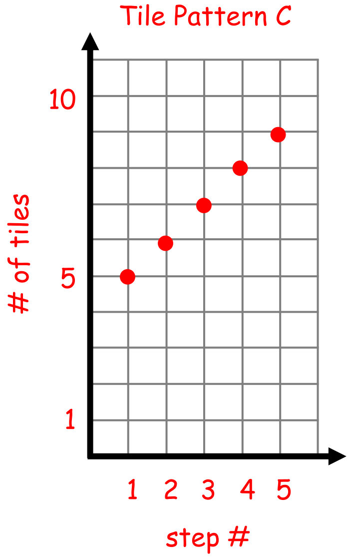
Tile Pattern D



Tile Pattern C	
step # (x)	# of tiles (y)
1	5
2	6
3	7
4	8
5	9

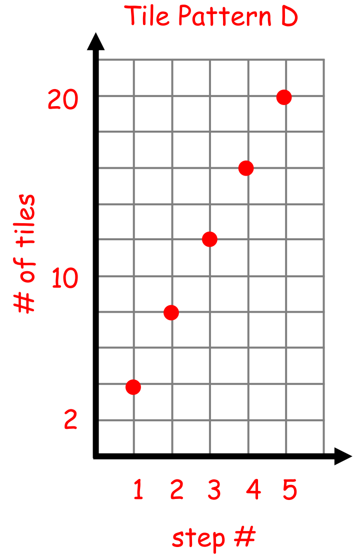
Tile Pattern D	
step # (x)	# of tiles (y)
1	4
2	8
3	12
4	16
5	20

Tile Pattern C



trend lines may be drawn to stress the linear relationships

Tile Pattern D



Rule for C: $y = x + 4$

Rule for D: $y = 4x$

2. True or false: for these patterns, typically the step number is the independent variable.
   true

Square tiles (optional)

1+

PRACTICE 3

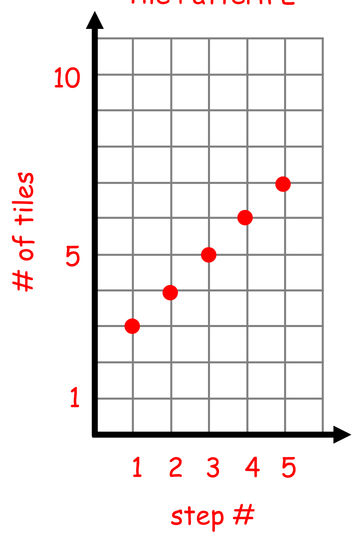
[SMP1, 2, 7]

1. Pattern E is described with a table. Pattern F is described with a graph. Complete the other representations in any order.

Tile Pattern E	
step # (x)	# of tiles (y)
1	3
2	4
3	5
4	6
5	7

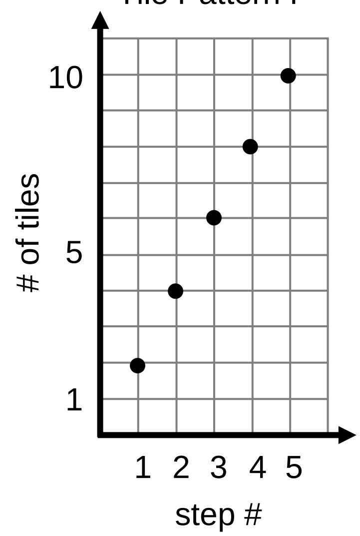
Tile Pattern F	
step # (x)	# of tiles (y)
1	2
2	4
3	6
4	8
5	10

Tile Pattern E



trend lines may be drawn to stress the linear relationships

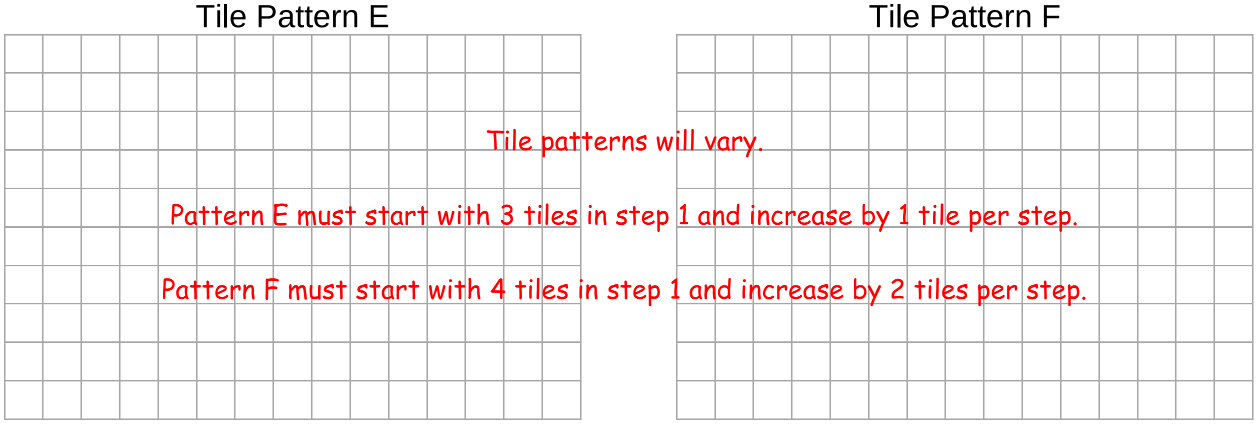
Tile Pattern F



Rule for C: $y = x + 2$

Rule for D: $y = 2x$

Tile Pattern E Tile Pattern F



Tile patterns will vary.

Pattern E must start with 3 tiles in step 1 and increase by 1 tile per step.

Pattern F must start with 4 tiles in step 1 and increase by 2 tiles per step.

Write the increase in number of tiles for each step for each pattern. E: 1 F: 2

2. True or false: For these patterns, typically the step number is the dependent variable. false

PRACTICE 4: EXTEND YOUR THINKING

[SMP1, 2, 8]

1. Fill in the chart based upon the work you did for tile patterns A – F.
   - Column I: Copy each rule (make sure you have the correct rules before proceeding).
   - Columns II-IV: Find the numbers of square tiles for the given step numbers.
   - Column V: Find each step number when the number of square tiles is 60.

I	II	III	IV	V
Pattern	**Step 10**	**Step 100**	**Step 1,000**	**Step Number for 60 tiles***
A → $y = 3x$	30	300	3,000	20
B → $y = x + 3$	13	103	1,003	57
C → $y = x + 4$	14	104	1,004	56
D → $y = 4x$	40	400	4,000	15
E → $y = x + 2$	12	102	1,002	58
F → $y = 2x$	20	200	2,000	30

* Students are expected to use any number-sense strategy that makes sense to them. More formal equation-solving strategies are introduced in Packet 8.

2. Complete the table and fill in the blanks.

a.

x	1	2	3	4	5	6	8	11	13
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	4	$5\frac{1}{2}$	$6\frac{1}{2}$

b. Rate of change: for every increase of x by 1, y increases by $\frac{1}{2}$.

c. Input-output rule (words): Multiply an x-value by $\frac{1}{2}$ to get its corresponding y-value;

OR divide an x-value by 2 to get its corresponding y-value.

d. Input-output rule (equation): $y = \frac{1}{2} \bullet x$; OR $y = \frac{x}{\boxed{2}}$

e. If $x = 100$, then $y = 50$.

f. If $y = 100$, then $x = 200$.

COMPARING PRICES

We will use tables, double number lines, graphs, unit prices, equations, and words to compare prices.

[6RP3abc, 6NS3, 6EE2a, 6EE6, 6EE9, 6SP1, 6SP3; SMP1, 2, 4, 5, 7]

GETTING STARTED

Suppose you were shopping for groceries at Barter Jack's and have to make some choices. Explain which choices you'd make and why. Answers may vary.

Choice 1: energy bars	Choice 2: fruit
Healthy Crunch → 2 bars for \$2.50 OR Super Bar → 3 bars for \$2.50 Answers will vary. Based upon quantity and price, you pay the same for Healthy Crunch and Super Bar (\$2.50), but get more Super Bars at that price, making them the better buy. However, taste or health considerations may factor into the decision.	bananas → 2 pounds for \$4.10 OR apples → 2 pounds for \$4.95 Answers will vary. Based upon quantity and price, you get the same number of pounds of apples as bananas (2 lb), but the bananas are cheaper, making them the better buy. However, taste or health considerations may factor into the decision.

THE KEYCHAIN FUNDRAISER

[SMP1, 2, 7]

Follow your teacher's directions for (1) and (2).

(1a) – (1e) and (2a) – (2e) Copy pricing information, and complete the various representations.

(1a)

HI-TOPS
2 for $3

(2a)

DONUTS
3 for $4

(1b) Table

HI-TOPS	
# of keychains (x) (quantity)	Cost in $ (y)
2	3
4	6
6	9
1	1.50
3	4.50

(2b) Table

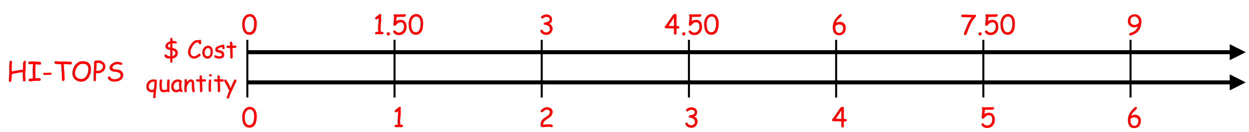
DONUTS	
# of keychains (x) (quantity)	Cost in $ (y)
3	4
6	8
9	12
1	1.33
2	2.66 or 2.67

(1c) Rule: $y = 1.50x$

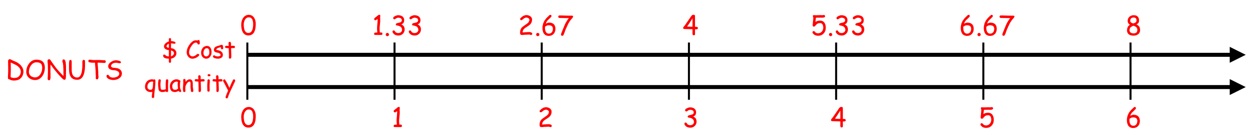
(2c) Rule: $y = 1.33x$

Double Number Lines Rounding may vary by one cent.

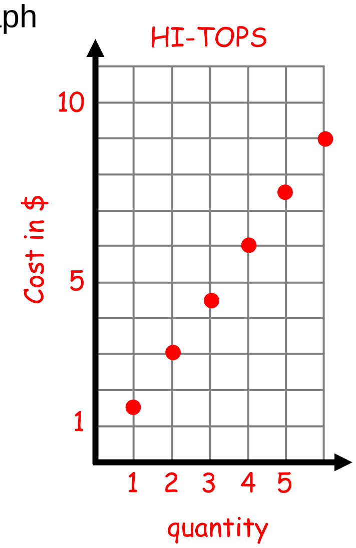
(1d)



(2d)

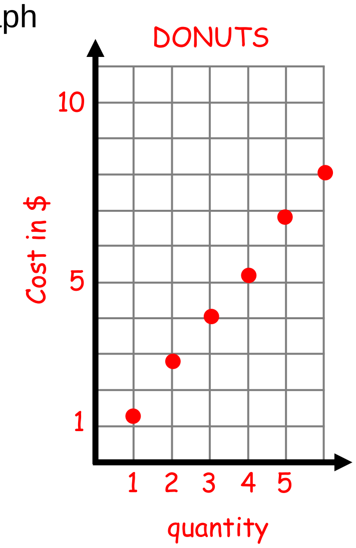


(1e) Graph



trend lines may be drawn to stress the linear relationships

(2e) Graph



3. Record the meanings of <u>unit rate</u> and <u>unit price</u> in **My Word Bank**.

LESSON NOTES S7.2: THE KEYCHAIN FUNDRAISER

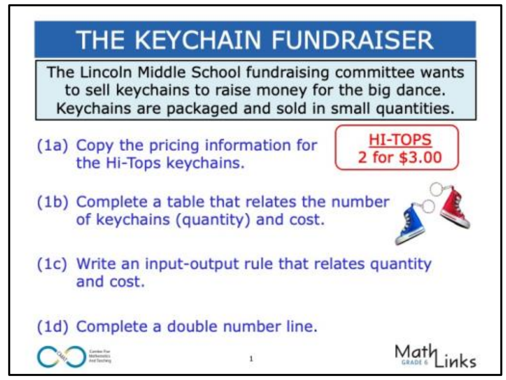
On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

In this lesson, we analyze prices using multiple representations including tables, equations, double number lines, graphs, and unit prices. Students will compare prices and other features to decide which keychains might be preferable for a fundraiser. They may wonder about various details. For example, must packages of keychains be sold intact or can they be split up? Is a unit price analysis reasonable? Is the quality the same? Are different options equally desirable? Entertain questions as they arise, and discuss assumptions to keep the analysis reasonable.

- Slide 1: For (1a) – (1d), ask students to copy the pricing information for Hi-Tops and create the three representations. This is review. Discuss as needed.

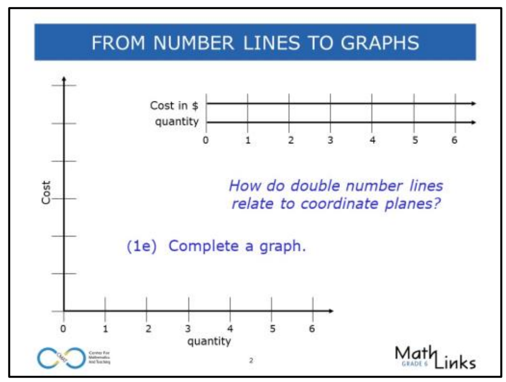
  Which appears to be the input value? Number of keychains (or quantity).

  Are any entries in the table helpful for creating an input-output rule (equation)? Students may realize that the price for 1 keychain represents the coefficient of x.



- Slide 2: Connect a double number line to a coordinate graph. For (1e), students complete a graph.

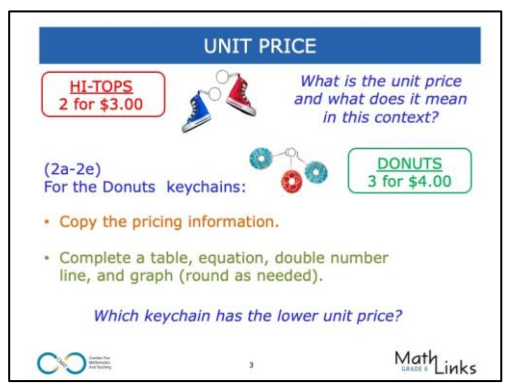
  How do double number lines relate to coordinate planes? One of the number lines becomes the x-axis and the other the y-axis. Pairs of entries can be graphed. A graph in a coordinate plane shows how one variable appears to be changing in relation to the other at a quick glance.



- Slide 3: ***What does the unit price mean in this context?*** It is the price for one Hi-Top keychain ($1.50). For this discussion it does not matter if they are sold separately.

  Reveal the pricing information for Donuts. For (2a) – (2e) ask students to complete the representations.

  Which keychain has a lower unit price? The DONUTS keychains are a better buy in terms of price per one keychain (<u>unit rate</u> or <u>unit price</u>), but this does not necessarily mean they are preferable for the fundraiser.



SLIDE DECK ALTERNATIVE S7.2: THE KEYCHAIN FUNDRAISER

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slides 1-2

The Lincoln Middle School fundraising committee wants to sell keychains to raise money for the big dance. Keychains are packaged and sold in small quantities.

Hi-Top Keychains are 2 for $3.00.

(1a) Copy the pricing information for the Hi-Top keychains.

(1b) Complete a table that relates the number of keychains (quantity) and cost.

(1c) Write an input-output rule that relates quantity and cost.

(1d) Complete a double number line.

(1e) Complete a graph.

Slide 3

Donut Keychains are 3 for $4.00

(2a-e) For the Donuts keychains:

- Copy the pricing information.
- Complete a table, equation, double number line, and graph (round as needed).

Which keychain has the lower unit price?

PRACTICE 5

[SMP1, 2, 7]

Here are two more keychain packages. Complete the representations for both.
Rounding may vary by about one cent on this page.

GOOGLIES
5 for $6

EMOJIS
6 for $5

1a. Table

GOOGLIES	
quantity (*x*)	Cost in $ (*y*)
5	6
10	12
1	1.20
2	2.40
3	3.60

2a. Table

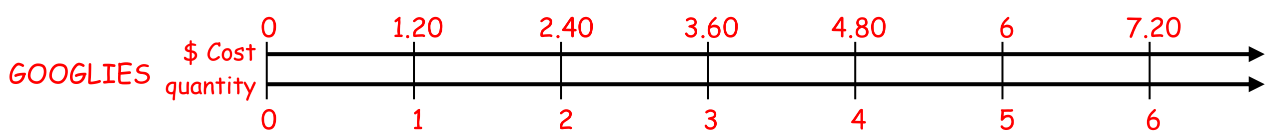
EMOJIS	
quantity (*x*)	Cost in $ (*y*)
6	5
12	10
3	2.50
1	0.83
2	1.66

1b. Rule: $y = 1.20x$

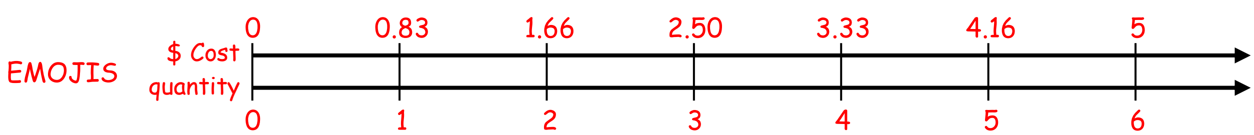
2b. Rule: $y = 0.83x$

Double Number Lines Rounding may vary by one cent.

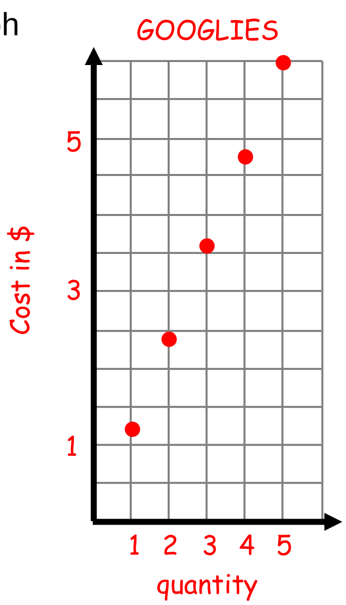
1c.



2c.

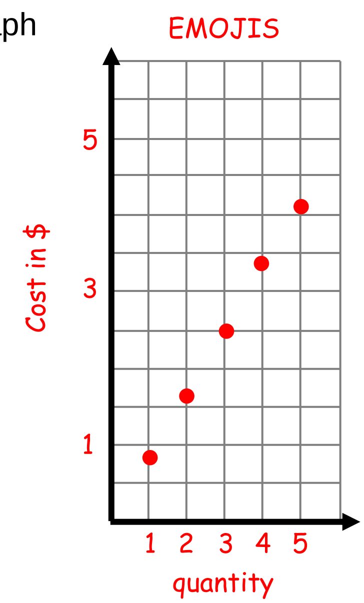


1d. Graph



trend lines may be drawn to stress the linear relationships

2d. Graph



3. EMOJIS have the lower unit price because… $0.83 per keychain < $1.20 per keychain.

PRACTICE 6

[SMP1, 2, 7]

Here are two more keychain packages.

LOCKS
3 for $6

CUBES
2 for $5

1. Complete each table below.

LOCKS	
quantity (x)	Cost in $ (y)
3	6
6	12
1	2
2	4
5	10

CUBES	
quantity (x)	Cost in $ (y)
2	5
4	10
6	15
1	2.50
3	7.50

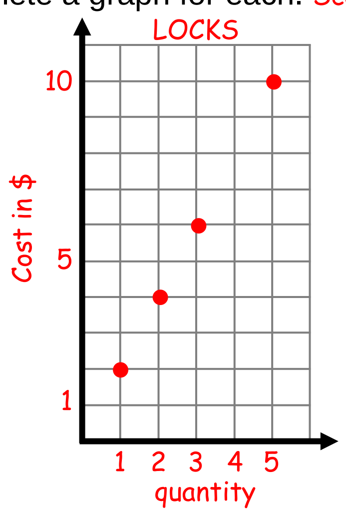
2. Explain how you know which is the cheaper purchase based on unit price.
   Locks are cheaper. For 1 keychain, $2 per keychain < $2.50 per keychain.

3. Explain how you know which is cheaper based on the entries with x = 3.
   Locks are cheaper. For 3 keychains, $6 < $7.50.

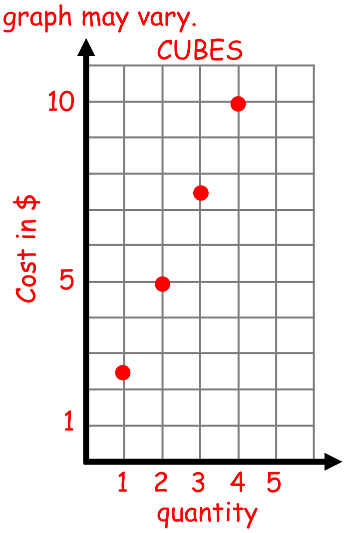
4. Explain how you know which is cheaper based on the entries with y = 10.
   Locks are cheaper. For $10, 5 keychains > 4 keychains.

5. Write a rule for each. LOCKS: $y = 2.00x$ CUBES: $y = 2.50x$

6. Complete a graph for each. Scaling and points chosen to graph may vary.



trend lines may
be drawn to
stress the linear
relationships



7. Which graph illustrates a greater cost increase per each additional keychain? CUBES
   How can you see this when comparing the graphs? The graph for CUBES is "steeper," (increase of $2.50 per additional keychain compared to an increase of $2.00 per additional keychain).

A COMMITTEE DECISION

(Using the *MathLinks* Rubric) See Activity Routines in the Teacher Portal for instructions. [SMP4, 5, 7]

Help the Lincoln Middle School fundraising committee decide which keychains to sell for the fundraiser. The six different keychains analyzed on the previous pages are listed below. In addition, a small survey was taken, the results of which are in the table below.

1. Complete the table. Percent rounded to the nearest whole percent.

Keychain	**Price**	**Unit price** (price per keychain)	**Students polled who preferred this keychain:**	
			Number	**Percent**
Hi-Tops	2 for $3	$1.50	18	30%
Donuts	3 for $4	$1.33	10	17%
Googlies	5 for $6	$1.20	20	33%
Emojis	6 for $5	$0.83	6	10%
Locks	3 for $6	$2.00	1	2%
Cubes	2 for $5	$2.50	5	8%
			Total: 60	Total: 100%

Consider unit prices from the table.

2. What is the range of prices?

   $2.50 - $0.83 = $1.67

3. What is the median price?

   $1.42

4. What is the mean price?

   $1.56

Write one statistical question based on each. Answers will vary. Examples:

5. Unit prices

   What is the typical unit price of a keychain?

6. Students polled

   What keychains do students like best?

7. Recommend one or more keychains to the committee based on data from the table.

   Answers will vary. Some possibilities:

   - Recommend emoji's because they are the cheapest
   - Recommend googlies because they are the most popular and 2nd cheapest

RATE APPLICATIONS

We will deepen our understanding of independent and dependent variables. We will use rates in money contexts and in time-distance contexts. We will identify unit rate in tables, graphs, and equations.

[6RP3abc, 6NS3, 6EE2ab, 6EE6, 6EE9; SMP1, 2, 4, 5, 6, 7, 8]

GETTING STARTED

[SMP8]

Julie is saving $10 each week from her allowance.

1. Fill in the table for the total amount saved at each week through week 6.

Week # (W)	1	2	3	4	5	6	10	25	37
Total $ (*T*)	10	20	30	40	50	60	100	250	370

2. Rate of change: for every increase of one week, the total increases by __$10__

3. Complete an input-output rule that relates week number to total dollars saved: $T =$ __$10W$__

4. For the last three entries in the table let W = 10, 25, and 37. Use your rule to find the corresponding values for T.

5. For problem 2 above, typically, which quantity is the independent variable? Week number

6. After how many weeks of allowance will Julie have saved each amount?

a. $400 40 weeks	b. $500 50 weeks
c. $450 45 weeks	d. $425 43 weeks 10(42.5) = 425, but allowances are not usually given for partial weeks

RAISING MONEY FOR MUSIC

[SMP1, 2, 6, 7, 8]

Follow your teacher's directions for (1) and (2).

The Springfield Education Foundation is trying to raise $100,000 for its music programs. They have fundraisers throughout the year.

(1) The SEF sells holiday trees for its December fundraiser. Each tree sells for $50.

Complete Tables I and II.

Table I

# of trees sold (t)	Money earned in $ (m)
1	50
2	100
5	250
10	500
50	2,500
100	5,000

Table II

Money earned in $ (m)	# of trees sold (t)
50	1
100	2
200	4
600	12
1,000	20
4,000	80

(2) *Write two different word statements that relate t and m.*

Table I: The amount of money earned depends on the number of trees sold.

Table II: The number of trees sold depends on the amount of money earned.

(or the money needed to earn for a particular fundraising goal).

3. Write two different equations that relate t and m.

$m = 50t$ $t = \frac{m}{50}$

Use the equations from question 3 to complete problems 4 – 5.

4. If they sell 1,000 trees, how much will they earn?

$m = 50t = 50(1{,}000) = 50{,}000$ dollars

What percent of the way would they be to their goal? $\frac{50{,}000}{100{,}000} = 50\%$

5. How many trees sold raises $60,000?

$t = \frac{m}{50} = \frac{60{,}000}{50} = 1{,}200$ trees

If they did this, how many more trees would they have to sell to reach their goal?

This requires another $40,000 → $t = \frac{m}{50} = \frac{40{,}000}{50} = 800$ trees

LESSON NOTES S7.3: RAISING MONEY FOR MUSIC

On slides, blue italic text suggests discussion; blue numbered text suggests written responses.

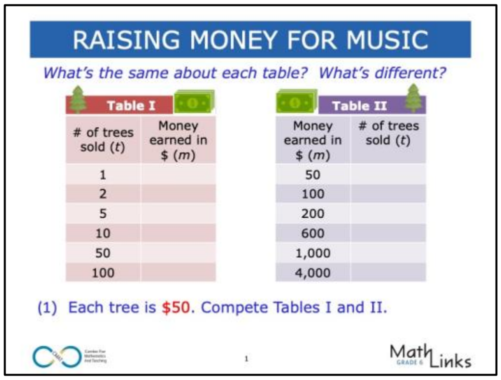
Here we analyze a fundraising effort using tables and equations. We also consider that the independent and dependent variables might change based upon our focus.

- Slide 1: Focus attention on the problem context and the tables started on this slide.

  What's the same about each table? They both keep track of the number of trees sold and money earned in dollars.

  What's different about each table? The variables are "switched." In Table I, number of trees sold is the input, and money earned in dollars is the output. Table II is the opposite.

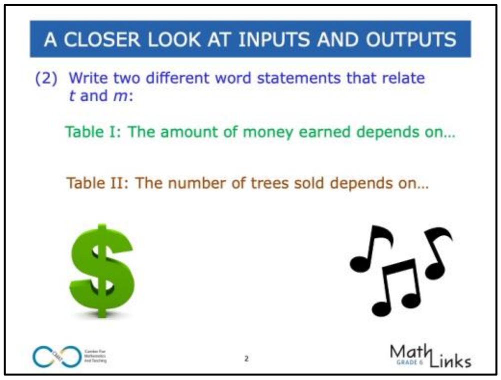
  For (1), students copy the cost of a tree and complete the tables.



- Slide 2: For (2), pose each sentence frame one at a time and discuss.

  When does it make sense that number of trees is the input and money earned in dollars is the output? The amount of money we earn will depend upon how many trees we sell. In this case, the trees are the independent variable and amount of money is the dependent variable.

  When does it make sense that money earned in dollars is the input and number of trees is the output? This may seem less obvious than the question above. We might think about a particular fundraising goal in such a way that the number of trees we want to sell depends upon the amount of money we want to earn. In this case, the amount of money is the independent variable and the number of trees is the dependent variable.



SLIDE DECK ALTERNATIVE 7.3: RAISING MONEY FOR MUSIC

Slide Decks and Lesson Notes are designed to provide teacher support for engaging guided instruction. The Slide Deck Alternative offers a modified option.

Slide 1

What is the same about each table? What is different?

(1) Each tree is $50. Complete Tables I and II.

Slide 2

(2) Complete two different word statements that relate *t* and *m*:

Table I: The amount of money earned depends on…

Table II: The number of trees sold depends on…

PRACTICE 7

[SMP1, 2, 7, 8]

1. The Springfield Education Foundation hosts a pancake breakfast for its May fundraiser. They sell each breakfast for $5.50. Complete Tables III and IV.

Table III

# of breakfasts sold (b)	Money earned in $ (m)
1	5.50
2	11
4	22
12	66
40	220
100	550

Table IV

Money earned in $ (m)	# of breakfasts sold (b)
5.50	1
11	2
55	10
110	20
440	80
660	120

2. Write two different equations that relate b and m.

   a. m = $5.50b$

   b. b = $\frac{m}{5.50}$

3. In Table III, money earned in dollars depends on… number of breakfasts sold.

4. If they sell 1,000 breakfasts, how much will they earn? $5,500

5. The Springfield Education Foundation sponsors a walk-a-thon for its August fundraiser. Each entry fee is $25. Complete Tables V and VI.

Table V

# of entries sold (w)	Money earned in $ (m)
1	25
2	50
3	75
5	125
10	250
100	2,500

Table VI

Money earned in $ (m)	# of entries sold (w)
25	1
50	2
125	5
500	20
1,000	40
2,000	80

6. Write two different equations that relate m and w.

   a. m = $25w$

   b. w = $\frac{m}{25}$

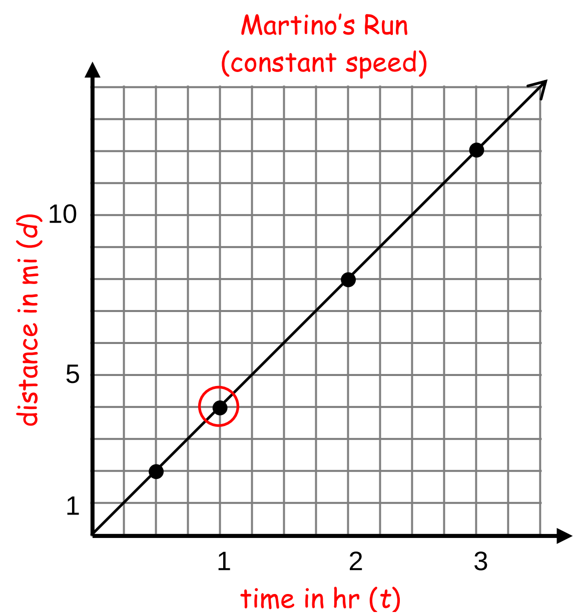
7. In Table VI, number of entries sold depends on… money earned in $ (their goal).

8. How many entries sold accomplishes a $6,500 goal? 260

RUNNING

[SMP1, 2, 7, 8]

Martino is training for a marathon. Today he ran for 3 hours and 15 minutes at a constant rate of speed. A graph of his run is shown.



1. Write 3 hours and 15 minutes as a decimal.
   3.25 hours

2. Write a title and axis labels. The input values are time (in hours). The outputs values are distance (in miles).

3. Why is it appropriate to connect the points on the graph?
   Time and distance are continuous measures. In other words, fractional portions make sense in the context.

4. Explain what the point (0,0) on the graph means in the context of this problem.
   Martino ran 0 miles in 0 hours.

5. Complete the table below.

Time in hours (*t*)	1	2	3	0.5	0.25	1.5	3.25
Distance in miles (*d*)	4	8	12	2	1	6	13

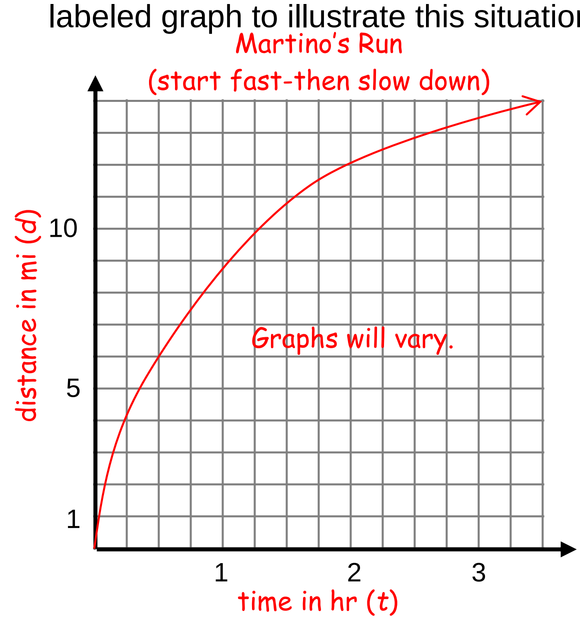
6. Write an equation for distance in terms of time.
   $d = 4t$

7. What is the meaning of the coefficient of *t* in the equation? Circle it in the table and the graph.
   The 4 represents the number of miles run per one hour, or the unit rate.

8. At this rate, how far did Martino run in 2.5 hours?
   10 miles

9. At this rate, how many hours would it take Martino to run 17 miles?
   4.25 hours

10. Suppose Martino starts his run fast and then slows down. Make a properly labeled graph to illustrate this situation.



PRACTICE 8

[SMP1, 2, 7, 8]

Darryl is doing a 52-mile bicycle ride that takes him 4 hours. He rides at a constant rate.

1. Complete the table below.

Time in hours (t)	4	2	1	0.5	0.25	0.75	1.5	2.75	3.25
Distance in miles (d)	52	26	13	6.5	3.25	9.75	19.5	35.75	42.25

2. Write an equation that relates distance and time. $d = 13t$

3. Draw a graph below. Include at least four points from the table. Be sure to label it completely. Axis scaling and graphed points may vary. Placement of points may be estimated.

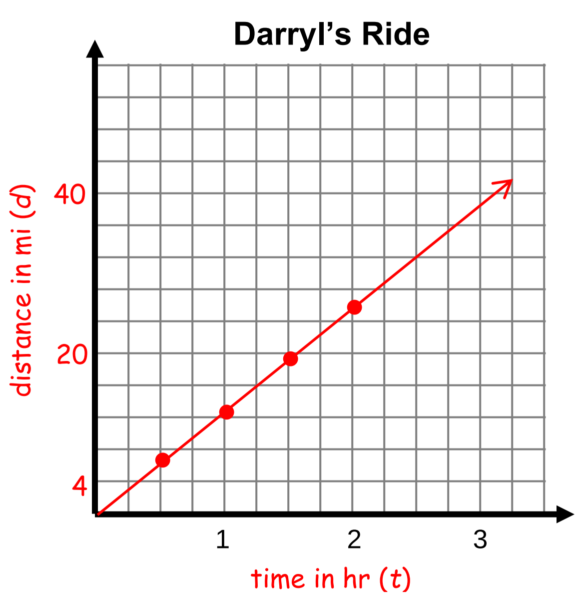
Linda is doing an 18-kilometer uphill hike that takes her 6 hours. She hikes at a constant rate.

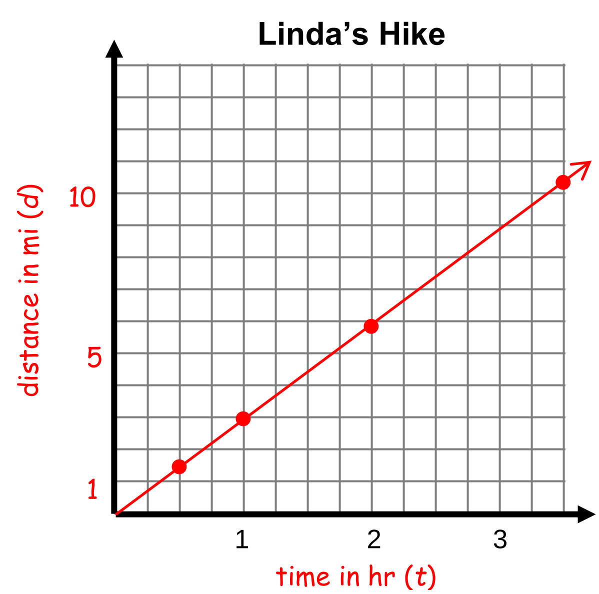
4. Complete the table below.

Time in hours (t)	6	1	2	0.5	0.25	0.75	1.75	2.25	3.5
Distance in kilometers (d)	18	3	6	1.5	0.75	2.25	5.25	6.75	10.5

5. Write an equation that relates distance and time. $d = 3t$

6. Draw a graph below for Linda's hike. Include at least four points from the table. Be sure to label it completely. Axis scaling and graphed points may vary. Placement of points may be estimated.





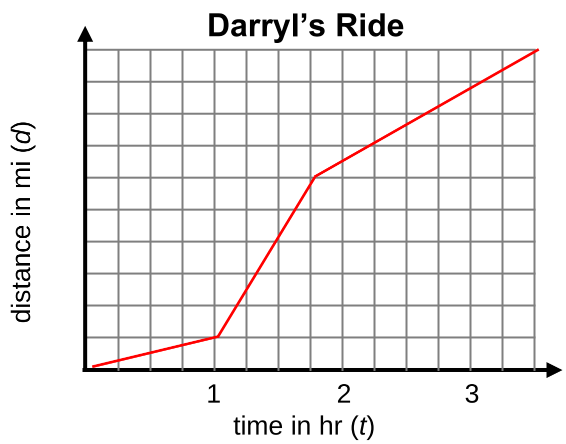
PRACTICE 9: EXTEND YOUR THINKING

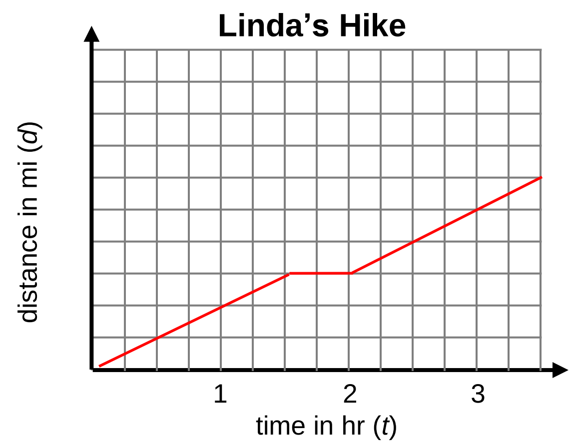
(Using the *MathLinks* Rubric) See Activity Routines in the Teacher Portal for instructions. [SMP1, 2, 4, 7]

Refer to Darryl's ride and Linda's hike from the previous page.

Graphs will vary. Numbers for distance are purposely omitted, but may be included.

1. Sketch a graph for Darryl if, instead of riding at a constant rate the entire race, he started off slowly uphill, then fast downhill, then finished at medium speed on flat land.
   One possible answer is shown below.
2. Sketch a graph for Linda if she hikes at some constant rate the entire time, except for a 30-minute break somewhere in the middle of the hike.
   One possible answer is shown below.

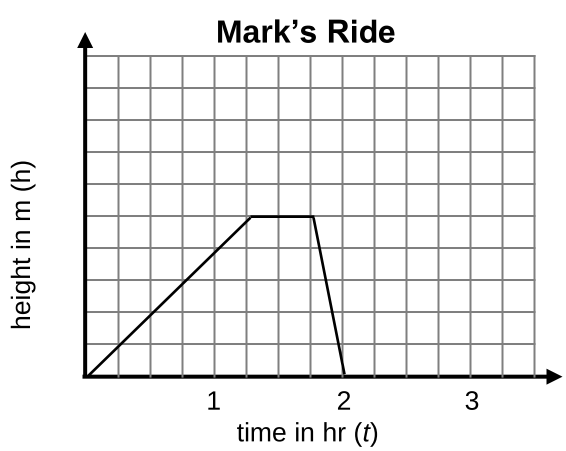


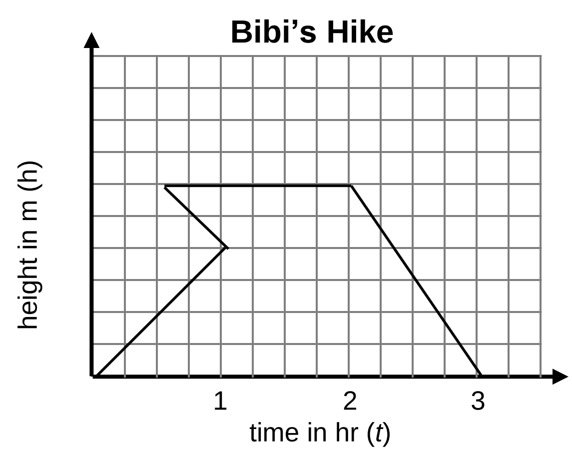


Notice that the graphs for Mark and Bibi below have height on the vertical axes.

3. Explain Mark's graph in writing.
   Mark starts at $h = 0$ m ("sea level"), travels uphill at a constant rate for 1 hour and 15 minutes, stays at that height for 30 minutes, and then travels back down to his start height in 15 minutes.

4. Explain the part of Bibi's hike that does not make sense.
   There is a line segment that shows time going "backwards." In other words, as the height increases, time cannot decrease.





REVIEW

POSTER PROBLEMS: INPUTS AND OUTPUTS

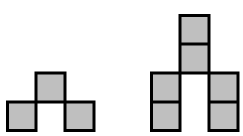
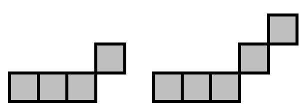
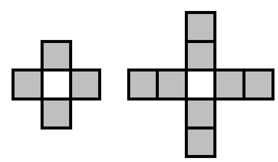
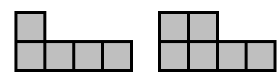
See Activity Routines in the Teacher Portal for directions. [SMP1, 2, 7, 8]

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Steps 1 and 2 of each pattern are given below.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
			
Step 1 Step 2	Step 1 Step 2	Step 1 Step 2	Step 1 Step 2

A. Copy steps 1 and 2 onto the poster and draw step 3. Explain your step 3 in words.

B. Make a table, label it appropriately, and record values for steps 1 through 5.

C. Make a graph and label it appropriately.

D. Write an input-output rule that relates the total number of tiles to the step number.

Part 3: Return to your seats. Work with your group, and show all work.

Use your "start problem."

1. Find the number of tiles in step 100.

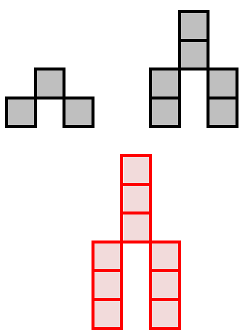
2. Find which step number has exactly 120 tiles.

POSTER PROBLEMS: INPUTS AND OUTPUTS

Answer Key

Likely patterns illustrated below, though they may vary if justified.

Poster 1 (or 5)



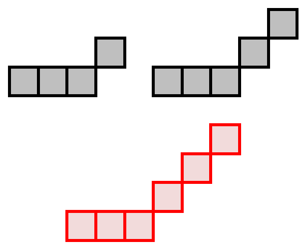
Step # (x)	# of tiles (y)
1	3
2	6
3	9
4	12
5	15

$y = 3x$

Step 100: 300 tiles

120 tiles: step 40

Poster 2 (or 6)



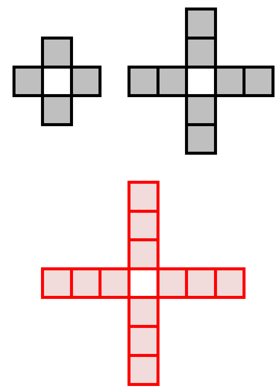
Step # (x)	# of tiles (y)
1	4
2	5
3	6
4	7
5	8

$y = x + 3$

Step 100: 103 tiles

120 tiles: step 117

Poster 3 (or 7)



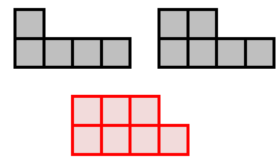
Step # (x)	# of tiles (y)
1	4
2	8
3	12
4	16
5	20

$y = 4x$

Step 100: 400 tiles

120 tiles: step 30

Poster 4 (or 8)

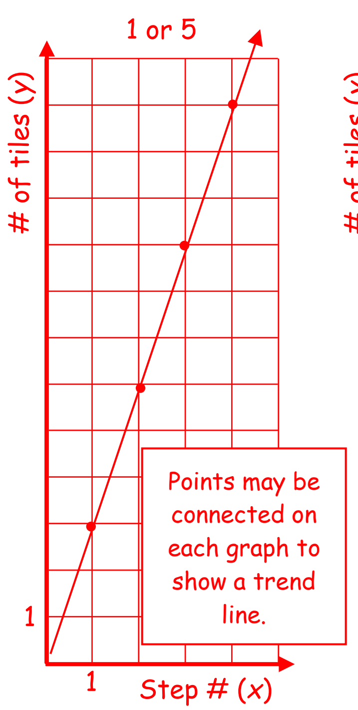


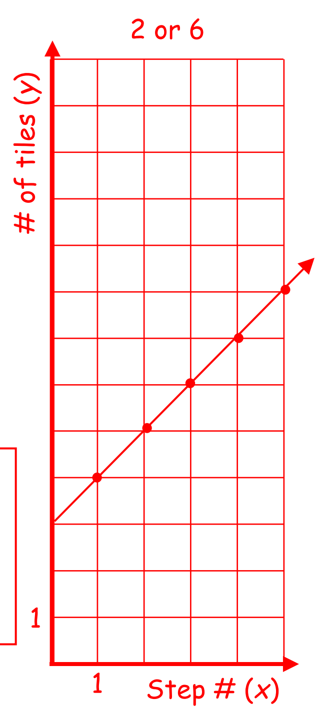
Step # (x)	# of tiles (y)
1	5
2	6
3	7
4	8
5	9

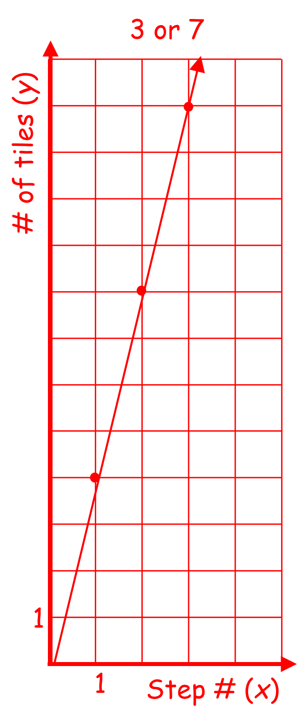
$y = x + 4$

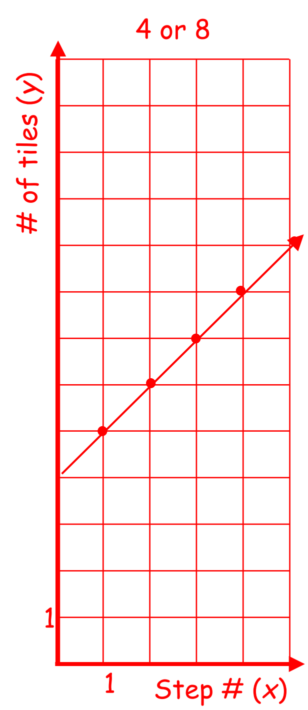
Step 100: 104 tiles

120 tiles: step 116









MATCH AND COMPARE SORT: INPUTS AND OUTPUTS

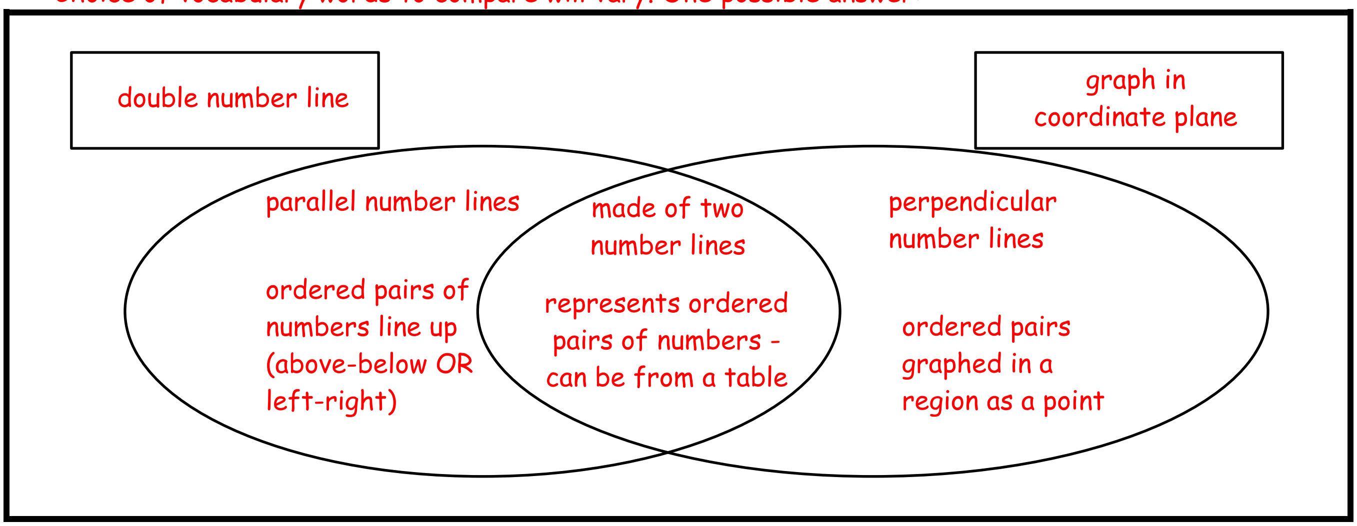
[SMP3, 6]

1. Individually, using the match and compare sort cards, match words with descriptions. Record results.

Card set △			Card set ◯		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I	unit rate	C	I	unit price	D
II	input-output rule	D	II	equation	C
III	independent variable	A	III	dependent variable	B
IV	double number line	B	IV	graph in coordinate plane	A

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.

Choice of vocabulary words to compare will vary. One possible answer:



WHY DOESN'T IT BELONG?: INPUTS AND OUTPUTS

[SMP1, 3]

Four different stores near LaRonda's home sell yellowfin tuna by the pound. Though prices are listed differently, she can buy any number of pounds at each store at the given rate.

1. Choose one of these price rates and explain why it doesn't belong with the others. Then choose at least one more and explain why it doesn't belong.

A $3.50 for 1 pound	**B** $24.00 for 6 pounds
C $8.75 for 2.5 pounds	**D** 4 pounds for $14.00

Possible responses:

A doesn't belong because it is the only one written as a unit price.

B doesn't belong because it is not $3.50 per pound like the others.

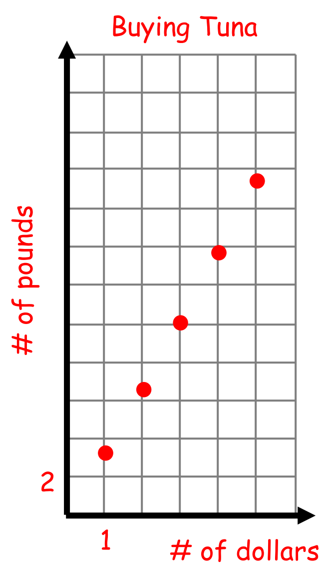
C doesn't belong because it is not a whole number of pounds.

D doesn't belong because the ratio is written in the opposite order (pounds per dollars).

2. For the price that is common among three of the stores above, complete the following:
   - a table
   - a double number line
   - a graph
   - an input-output rule (equation)

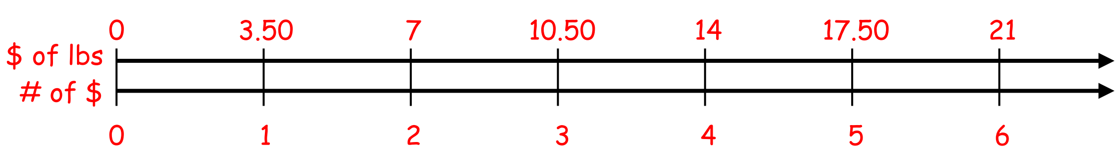
Labeling and scaling may vary for the visual representations. Points graphed are estimated locations. A trend line may be drawn.

(*x*) # of pounds	(*y*) # of dollars
1	3.50
2	7.00
3	10.50
4	14.00
5	17.50

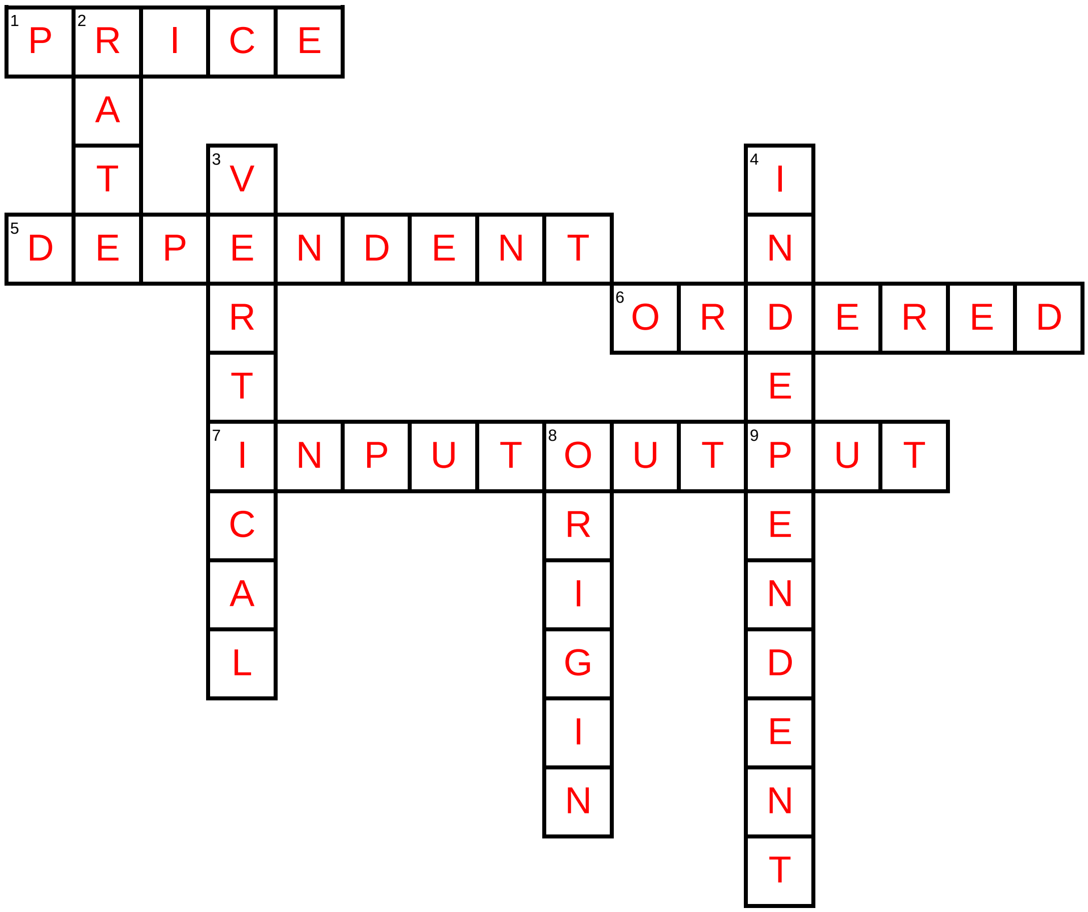


Equation:

$y = 3.5x$



VOCABULARY REVIEW



Across

1 If 6 oranges cost $4.50, the unit ___ per orange is $0.75.

5 A variable that is typically thought of as the output

6 Pairs of numbers in the form (*x*, *y*) are called ___ pairs.

7 A rule where, given *x*, you can find *y* (2 words)

Down

2 If there are 12 pencils for a group of 4 students, the unit ___ is 3 pencils per each student.

3 Direction of the *y*-axis

4 A variable typically thought of as the input

6 Name of the point (0, 0)

SPIRAL REVIEW

1. **Computational Fluency Challenge.** This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

   - Start with 2.5. Multiply by 4. Divide the result by 5. Multiply the result by 20. Multiply the result by 40. Now you have a "big number". My big number is _1600_.
   - Start with your big number. Divide it by 40. Divide the result by 10. Divide the result by 2. Multiply the result by 5. Divide the result by 4. What is the final result? _2.5_

2. Evaluate each numerical expression below.

a. $26 - 4^2$ 10	b. $2(6 + 4)$ 20	c. $2(6 - 4)^2$ 8
d. $2 + (6 + 4)^2$ 102	e. $2(6^2) + 2(4)^2$ 104	f. $6 + 4 \bullet 2$ 14

3. Christopher needs to paint a rectangular picture that has an area of $12\frac{3}{8}$ square yards on a brick wall. The width of his painting is $2\frac{3}{4}$ yards. What does the height of the painting need to be?

$12\frac{3}{8} \div 2\frac{3}{4} = 4\frac{1}{2}$ yd

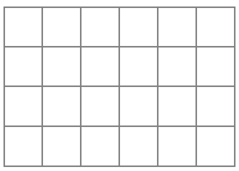
SPIRAL REVIEW
Continued

4. Ryann bought 3 drinks that cost \$2 each and 3 hot dogs that cost \$2.50 each at the snack bar. Circle the expressions that could be used to compute the total cost.

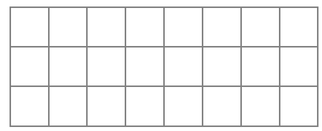
   $2 + 2 + 2(2.50)$ $\quad$ $3(2) + 3(2.50)$ $\quad$ $(2 + 2.5)(3)$ $\quad$ $3 + 2 + 3 + 2.50$

5. Denali and Jena created plans for a quilted blanket in sewing class, using patches that are 6 inches on each side. Patches are represented by the small squares below.

Denali's Plan



Jena's Plan



   a. Whose blanket covers a greater area? Explain.
   They both have the same area. $6 \times 4 = 8 \times 3 = 24$ square patches

   b. After they sew the patches together, they have to sew a ribbon around the edge of their blankets. Who will need to use a longer ribbon? Explain your answer in feet.
   Parker needs 10 feet of ribbon. Levi needs 11 feet of ribbon. Levi needs more.

   c. Leo wants to make a square blanket using 16 patches. Write two equivalent expressions using multiplication and exponents to represent its area.
   $4^2 = 4(4)$

6. Some nutritionists recommend that teens drink 8 cups of water per day.

   a. Yesterday Helena drank 64 oz of water. Did Helena reach this goal? What percent of this goal did Helena drink? Yes, 100%

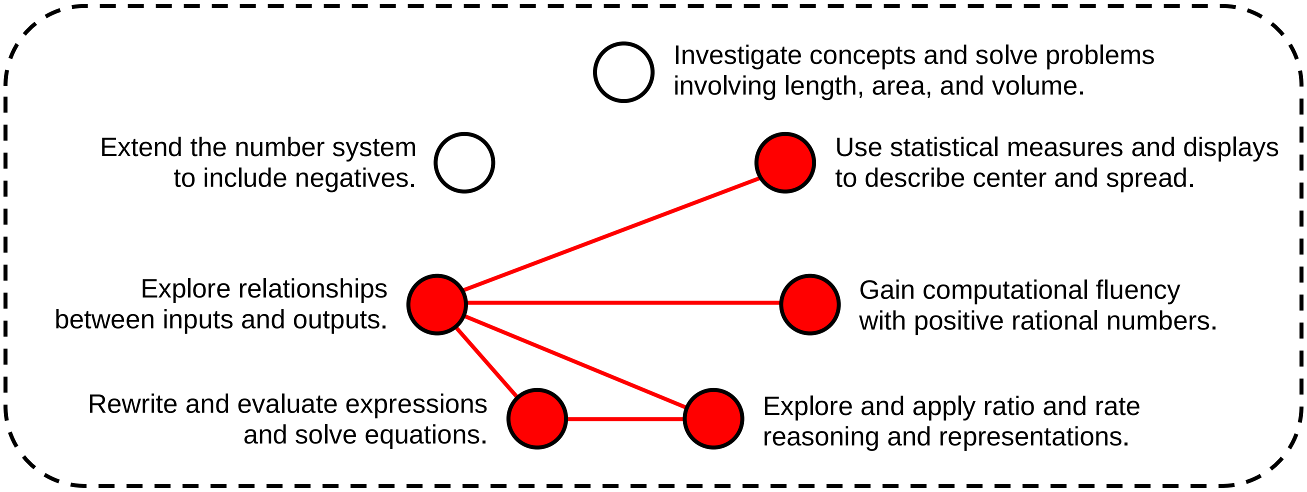
   b. Yesterday Betina drank a quart of water. Did Betina reach this goal? What percent of this goal did Betina drink? No, 50%

   c. Yesterday Caryn drank 6 cups of water. Did Caryn reach this goal? What percent of this goal did Caryn drink? No, 75%

REFLECTION

Answers will vary. Some possible answers:

1. **Big Ideas**. Shade all circles that describe big ideas in this packet. Draw lines to show connections that you noticed.



Give an example from this packet of one of the connections above.

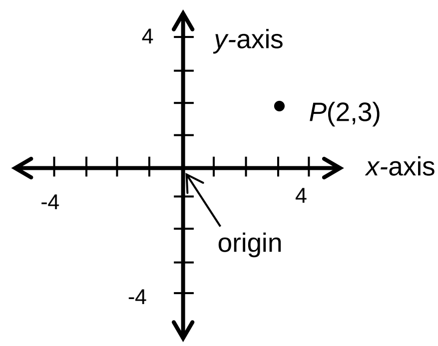
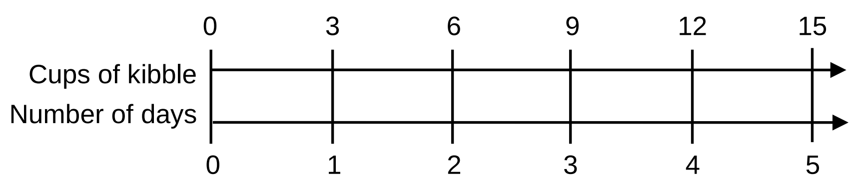
- Making a recommendation about the emoji purchase for the fundraiser might be done by using the given data and proportional reasoning.
- Input-output relationships are seen in equations and graphs.

2. **Packet Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Did you get stuck on any problems in this packet? What did you do to get unstuck (SMP1)?
Perseverance when solving problems is a growth mindset behavior. Some strategies might be to break apart problems into simpler problems, work on the parts understood first, try different representations, ask for help from friend or teacher.

4. **More Connections.** Give an example of how data along with your math skills helped you to make a decision or prediction.
   - For keychain fundraiser, polling data and costs are analyzed to make recommendation to the committee.
   - For music fundraiser, input-output data, tables, and equations are used to determine if a particular fundraiser would meet its fundraising goals.

STUDENT RESOURCES

Word or Phrase	Definition
coefficient	A coefficient is a number or constant factor in a term of an algebraic expression. In the expression $3x + 5$, 3 is the coefficient of the term $3x$, and 5 is the constant term.
coordinate plane	A coordinate plane is a plane with two perpendicular number lines (coordinate axes) meeting at a point (the origin). Each point P of the coordinate plane corresponds to an ordered pair (a, b) of numbers, called the coordinates of P. The point P may be denoted $P(a, b)$. The coordinate axes are often referred to as the x-axis and the y-axis respectively. The origin has coordinates $(0,0)$. 
double number line	A double number line is a diagram made up of two parallel number lines that visually depict the relative sizes of two quantities. Double number lines are often used when the two quantities have different units, such as miles and hours. The proportional relationship "Wrigley eats 3 cups of kibble per day" can be represented in the following double number line diagram. 
dependent variable	A dependent variable is a variable whose value is determined by the values of the independent variables. See independent variable.
equation	An equation is a mathematical statement that asserts the equality of two expressions. $18 = 8 + 10$ is an equation that involves only numbers. This is a numerical equation. $18 = x + 10$ is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.
expression	A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8 + x}{10}$, and $4v - w$.

<table>
<tr><th>Word or Phrase</th><th>Definition</th></tr>
<tr><td>independent variable</td><td>An independent variable is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables.

For the equation $y = 3x$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.</td></tr>
<tr><td>input-output rule</td><td>An input-output rule for a sequence of values is a rule that establishes explicitly an output value for each given input value.

<table>
<tr><td>input value (x)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>x</td></tr>
<tr><td>output value (y)</td><td>1.5</td><td>3</td><td>4.5</td><td>6</td><td>7.5</td><td>$1.5x$</td></tr>
</table>

In the table above, the input-output rule could be $y = 1.5x$. In other words, to get the output value, multiply the input value by 1.5. If $x = 100$, then
$y = 1.5(100) = 150$.</td></tr>
<tr><td>rate</td><td>See unit rate.</td></tr>
<tr><td>unit price</td><td>A unit price is a price for one unit of measure.

If 4 apples cost \$1.00, then the unit price is $\frac{\$1.00}{4} = \$0.25$ for one apple, or 0.25 dollars per apple or 25 cents per apple.</td></tr>
<tr><td>unit rate</td><td>The unit rate associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. This is sometimes referred to as the value of the ratio.

The ratio of 40 miles for every 5 hours has a unit rate of $\frac{40}{5} = 8$ miles per hour.</td></tr>
<tr><td>variable</td><td>A variable is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to quantities that vary in a relationship (as in a formula or an input-output rule). They may refer to unknown quantities in expressions, equations, or inequalities. Finally, they may be used to generalize rules of arithmetic.

In the equation $d = rt$, the quantities d, r, and t are variables.
In the equation $2x = 10$, the variable x may be referred to as the unknown.
The equation $a + b = b + a$ generalizes the commutative property of addition for all numbers a and b.</td></tr>
</table>

The Coordinate Plane

A coordinate plane is determined by a horizontal number line (the *x*-axis) and a vertical number line (the *y*-axis) intersecting at the zero on each line. The point of intersection (0, 0) of the two lines is called the origin. Points are located using ordered pairs (*x*, *y*).

- The first number (*x*-coordinate) indicates how far the point is to the right of the *y*-axis.
- The second number (*y*-coordinate) indicates how far the point is above the *x*-axis.

Point, coordinates, and interpretation

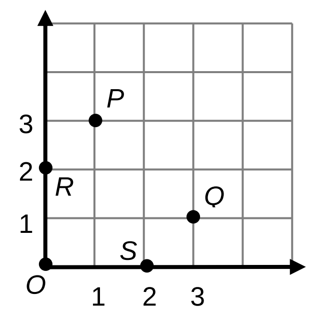
O (0, 0) → at the intersection of the axes.

P (1, 3) → start at the origin, move 1 unit right, then 3 units up

Q (3, 1) → start at the origin, move 3 units right, then 1 unit up

R (0, 2) → start at the origin, move 0 units right, then 2 units up

S (2, 0) → start at the origin, move 2 units right, then 0 units up.



Multiple Representations: Tables, Graphs, and Equations

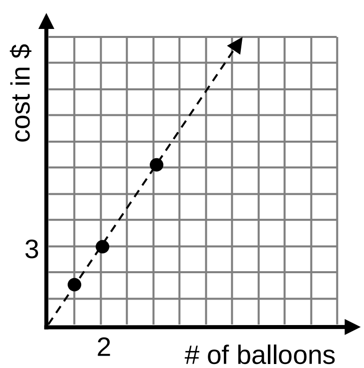
Suppose 4 balloons cost $6.00 and each balloon is the same price. Here are some representations for this relationship.

Table

Number of Balloons	Cost in $
4	6.00
2	3.00
1	1.50
8	12.00

Note that the unit price is $1.50 per balloon

Graph



Numbers of balloons must be discrete values (specifically, whole numbers), however a trend line may be drawn to show a growth pattern.

Equation (input-output rule)

Let *y* = cost in dollars and *x* = number of balloons.

We can see from the table that the unit price is 1.50 dollars per balloon.

It appears that multiplying any input value by 1.5 yields its corresponding output value.

Therefore, $y = 1.5x$.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, ~~tape diagrams~~, double number line diagrams, or equations:
a.	Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b.	Solve unit rate problems including those involving unit pricing and constant speed.
c.	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
6.EE.A	**Apply and extend previous understandings of arithmetic to algebraic expressions.**
6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers:
a.	Write expressions that record operations with numbers and with letters standing for numbers.
b.	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems.
6.EE.B	**Reason about and solve one-variable equations and inequalities.**
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.C	**Represent and analyze quantitative relationships between dependent and independent variables.**
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*
6.SP.A	**Develop understanding of statistical variability.**
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Do you know what mathematicians do after it snows? They make snow angles!

Draw a picture of your favorite animal.

Last page!