PACKET 7
STUDENT PACKET

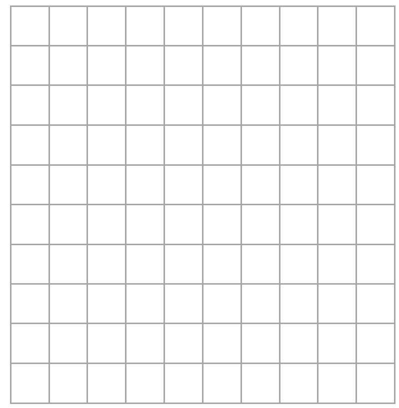
MathLinks GRADE 6

INPUTS AND OUTPUTS

Parent (or Guardian) signature ___________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. See **Student Resources** for mathematical vocabulary.

coordinate plane / coordinate axes / origin	
	
dependent variable	independent variable
input-output rule	unit rate/unit price

PICK A DOT

Follow your teacher's directions for (1) – (4).

(1)
(2)
(3)
(4)

5. Record the meaning of coordinate plane in **My Word Bank**.

VISUAL PATTERNS

We will review graphing ordered pairs. We will use words, tables of numbers, graphs, and algebraic input-output rules (equations) to describe visual patterns. We will identify what is typically the difference between independent variables and dependent variables.

[6RP3a, 6EE2ab, 6EE6, 6EE9; SMP1, 2, 3, 6, 7, 8]

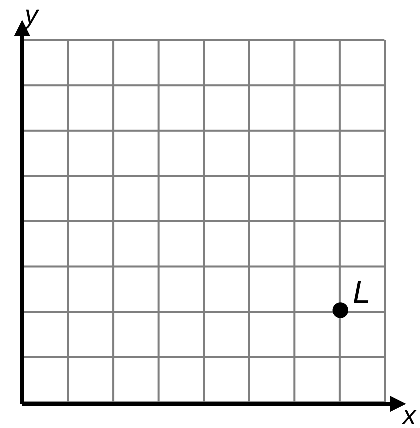
GETTING STARTED

Each small square on the grid represents 1 square unit.

1. Graph and label the following ordered pairs. As an example, the ordered pair (7, 2) is graphed and labeled point *L*.

   A (0, 0) *B* (4, 4) *C* (1, 5) *D* (5, 1)

   E (3, 0) *F* (0, 3) *G* (8, 6) *H* (6, 8)



2. How can you remember that we count the *x*-coordinate in the direction along the *x*-axis first and the *y*-coordinate in the direction along the *y*-axis second when graphing ordered pairs?

Use the word list below to fill in the blanks. Some words are used more than once. Use the coordinate plane below for reference or notes.

coordinate plane	horizontal	ordered pairs	origin	vertical

3. A __________ __________ is a plane with a horizontal axis and a vertical axis meeting at the point (0, 0), called the __________.

4. The __________ axis is typically referred to as the *x*-axis.

5. The __________ axis is typically referred to as the *y*-axis.

6. Points in the coordinate plane are named by pairs of numbers called __________ __________. They are written in the form (*x*, *y*).

7. From the origin to the point located at (3, 5), move 3 units in the __________ direction and 5 units in the __________ direction.

WHAT COMES NEXT?

Follow your teacher's directions.

(1)

Mateo

(2) Picture

(3) Table

1	
2	
3	
4	
5	

(4) Graph

Anita

(2) Picture

(3) Table

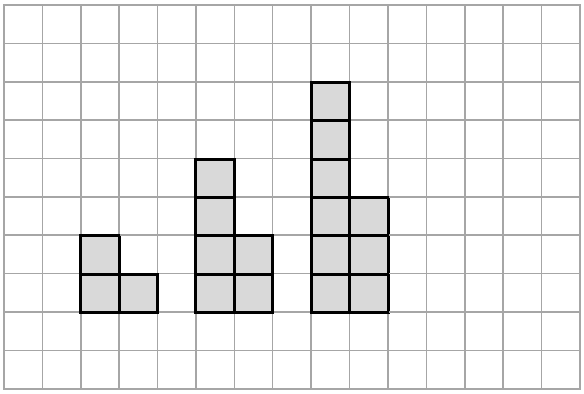
1	
2	
3	
4	
5	

(4) Graph

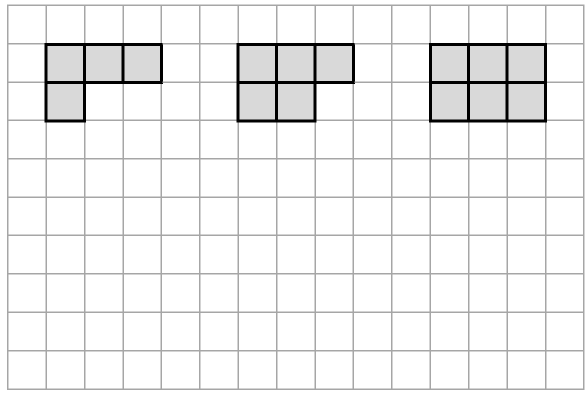
PRACTICE 1

1. Build steps 1 – 4 for tile patterns A and B if needed. Draw step 4 for each pattern. Complete the tables and draw the graphs with titles and labels.

Tile Pattern A

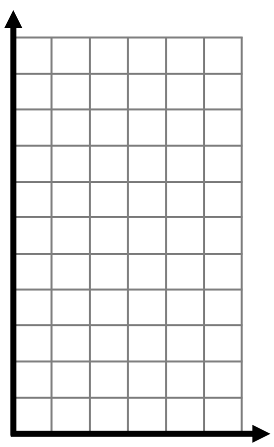


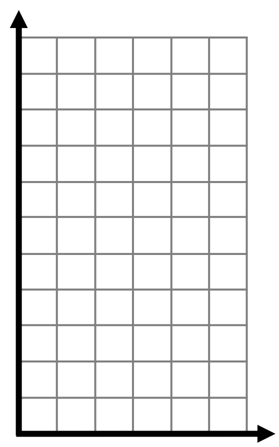
Tile Pattern B



1	
2	
3	
4	
5	

1	
2	
3	
4	
5	





2. Why is it appropriate to leave the points unconnected?

INPUT-OUTPUT RULES

Follow your teacher's instructions for (1) – (2).

(1)	(2)

3. Record the meaning of <u>input-output rule</u> in **My Word Bank**.

4. Write an input-output rule for Pattern A on **Practice 1** in words and symbols.

5. Write an input output rule for Pattern B on **Practice 1** in words and symbols.

> The input values (x) are sometimes referred to as the "independent variable," and output values (y) are sometimes referred to as the "dependent variable."

Fill in the missing numbers and blanks based on the suggested numerical patterns. In the tables below, the x-value is considered the input value and the y-value is the output value.

6.

x	1	2	3	4		6
y	5	6	7		9	

a. Rate of change: for every increase of x by 1, y increases by ___.

b. Input-output rule (words): add ___ to an x-value (independent variable) to get its corresponding y-value (dependent variable).

c. Input-output rule (equation): $y = x +$ ___

7.

x	1	2	3	4		6
y	4	8	12	16	20	

a. Rate of change: for every increase of x by 1, y increases by ___.

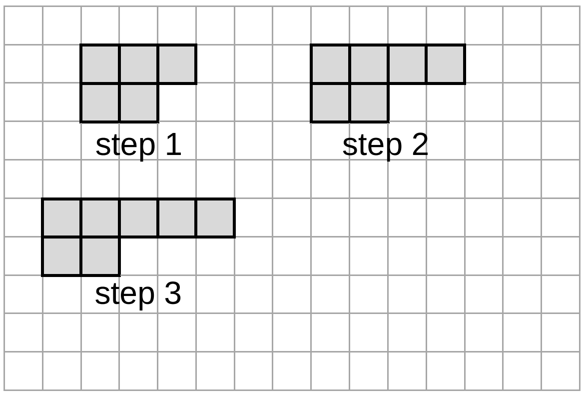
b. Input-output rule (words): Multiply an x-value (independent variable) by ___ to get its corresponding y-value (dependent variable).

c. Input-output rule (equation): $y =$ ___ $\bullet$ x

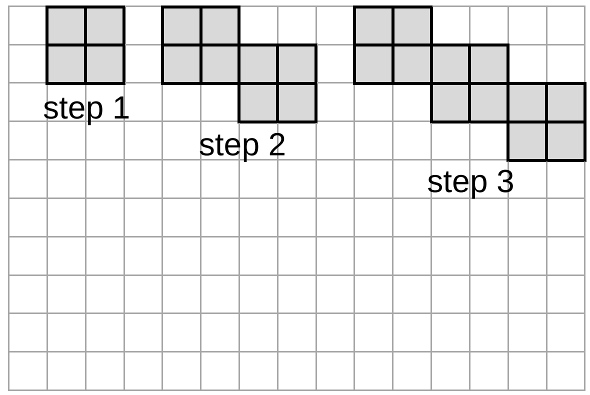
PRACTICE 2

1. Build steps 1–4 for tile patterns C and D if needed. Draw step 4 for each. Complete the tables and make graphs with titles and labels. Write an input-output rule (equation).

Tile Pattern C

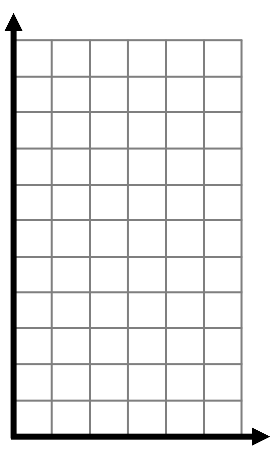


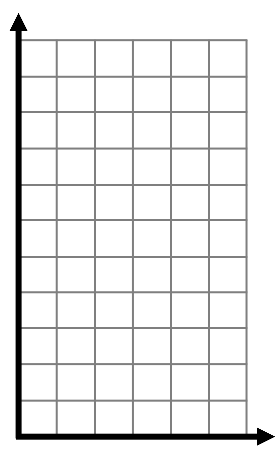
Tile Pattern D



1	
2	
3	
4	
5	

1	
2	
3	
4	
5	





Rule for C: __________

Rule for D: __________

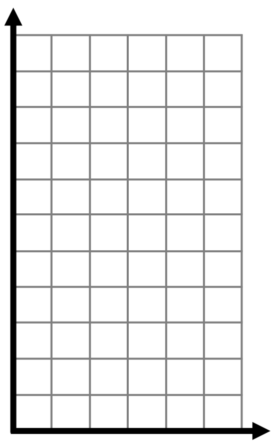
2. True or false: for these patterns, typically the step number is the independent variable.

PRACTICE 3

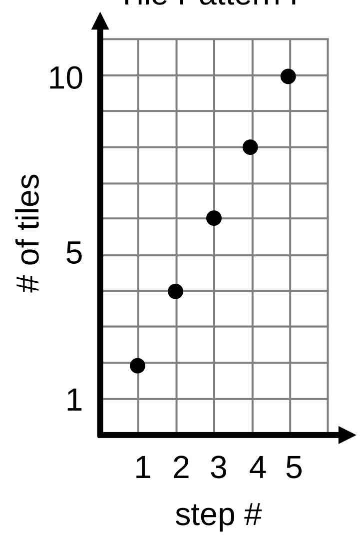
1. Pattern E is described with a table. Pattern F is described with a graph. Complete the other representations in any order.

Tile Pattern E	
step # (x)	# of tiles (y)
1	3
2	4
3	5
4	6
5	7

1	
2	
3	
4	
5	



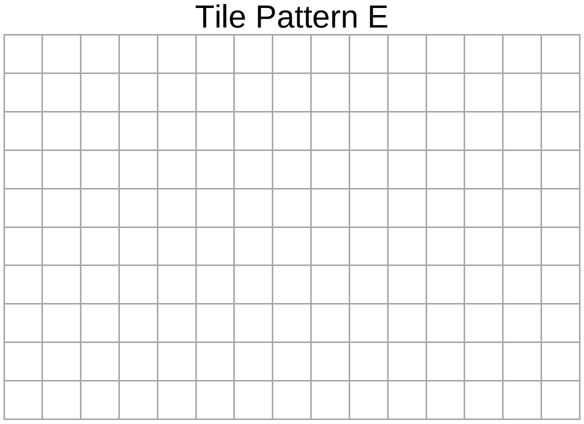
Tile Pattern F



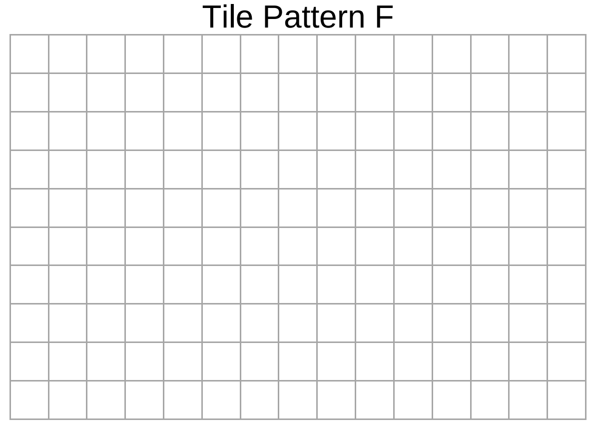
Rule for C: ___________

Rule for D: ___________

Tile Pattern E



Tile Pattern F



Write the increase in number of tiles for each step for each pattern. E: ___ F: ___

2. True or false: For these patterns, typically the step number is the dependent variable.

PRACTICE 4: EXTEND YOUR THINKING

1. Fill in the chart based upon the work you did for tile patterns A – F.
   - Column I: Copy each rule (make sure you have the correct rules before proceeding).
   - Columns II-IV: Find the numbers of square tiles for the given step numbers.
   - Column V: Find each step number when the number of square tiles is 60.

I	II	III	IV	V
Pattern	**Step 10**	**Step 100**	**Step 1,000**	**Step Number for 60 tiles**
A →				
B →				
C →				
D →				
E →				
F →				

2. Complete the table and fill in the blanks.

   a.

x	1	2	3	4	5	6	8	11	13
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$				$6\frac{1}{2}$

   b. Rate of change: for every increase of x by 1, y increases by _____.

   c. Input-output rule (words): Multiply an x-value by _____ to get its corresponding y-value;

      OR divide an x-value by _____ to get its corresponding y-value.

   d. Input-output rule (equation): $y =$ _____ $\bullet$ x; OR $y = \frac{x}{\square}$

   e. If $x = 100$, then $y =$ _____.

   f. If $y = 100$, then $x =$ _____.

COMPARING PRICES

We will use tables, double number lines, graphs, unit prices, equations, and words to compare prices.

[6RP3abc, 6NS3, 6EE2a, 6EE6, 6EE9, 6SP1, 6SP3; SMP1, 2, 4, 5, 7]

GETTING STARTED

Suppose you were shopping for groceries at Barter Jack's and have to make some choices. Explain which choices you'd make and why.

Choice 1: energy bars	**Choice 2: fruit**
Healthy Crunch → 2 bars for $2.50 OR Super Bar → 3 bars for $2.50	bananas → 2 pounds for $4.10 OR apples → 2 pounds for $4.95

THE KEYCHAIN FUNDRAISER

Follow your teacher's directions for (1) and (2).

(1a)

HI-TOPS

(1b) Table

2	
4	
6	
1	
3	

(1c) Rule: ___________

(2a)

DONUTS

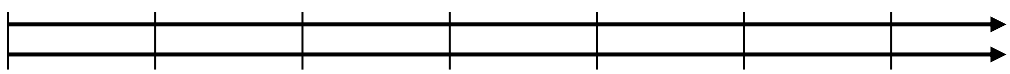
(2b) Table

3	
6	
9	
1	
2	

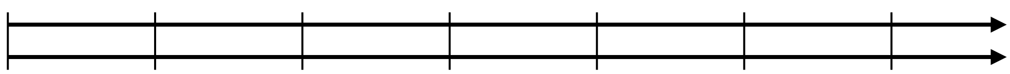
(2c) Rule: ___________

Double Number Lines

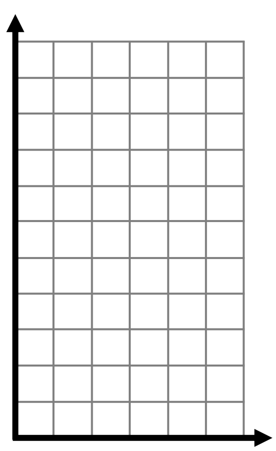
(1d)



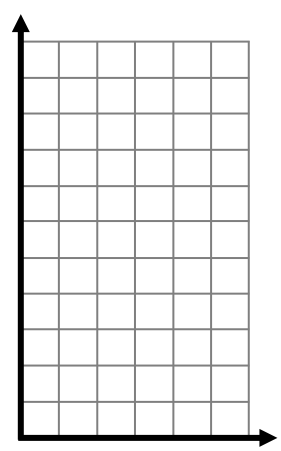
(2d)



(1e) Graph



(2e) Graph



3. Record the meanings of unit rate and unit price in **My Word Bank**.

PRACTICE 5

Here are two more keychain packages. Complete the representations for both.

GOOGLIES
5 for $6

EMOJIS
6 for $5

1a. Table

quantity (x)	Cost in $ (y)
5	
10	
1	
2	
3	

2a. Table

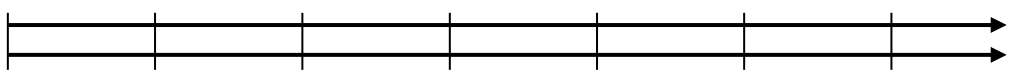
quantity (x)	Cost in $ (y)
6	
12	
3	
1	
2	

1b. Rule: __________

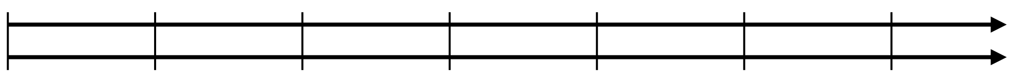
2b. Rule: __________

Double Number Lines

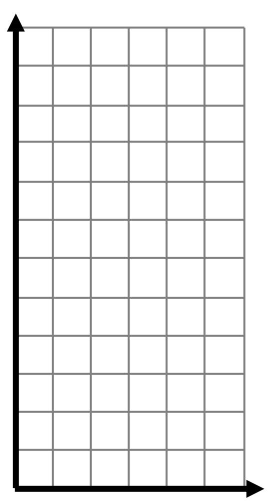
1c.



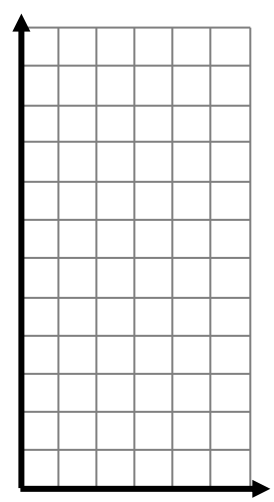
2c.



1d. Graph



2d. Graph



3. _________ have the lower unit price because…

PRACTICE 6

Here are two more keychain packages.

LOCKS
3 for $6

CUBES
2 for $5

1. Complete each table below.

LOCKS	
quantity (x)	Cost in $ (y)
3	
6	
1	
2	
5	

CUBES	
quantity (x)	Cost in $ (y)
2	
4	
6	
1	
3	

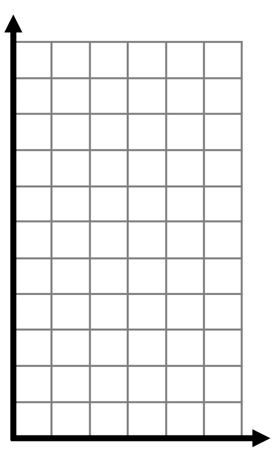
2. Explain how you know which is the cheaper purchase based on unit price.

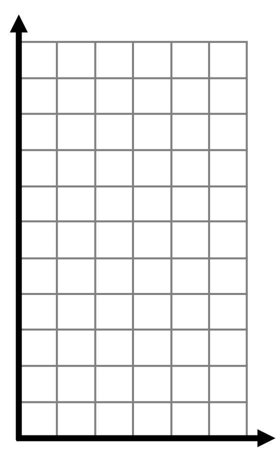
3. Explain how you know which is cheaper based on the entries with $x = 3$.

4. Explain how you know which is cheaper based on the entries with $y = 10$.

5. Write a rule for each. LOCKS: ____________ CUBES: ____________

6. Complete a graph for each.





7. Which graph illustrates a greater cost increase per each additional keychain? How can you see this when comparing the graphs?

A COMMITTEE DECISION

Help the Lincoln Middle School fundraising committee decide which keychains to sell for the fundraiser. The six different keychains analyzed on the previous pages are listed below. In addition, a small survey was taken, the results of which are in the table below.

1. Complete the table.

Keychain	**Price**	**Unit price** (price per keychain)	**Students polled who preferred this keychain:**	
			Number	**Percent**
Hi-Tops	2 for $3		18	
Donuts	3 for $4		10	
Googlies	5 for $6		20	
Emojis	6 for $5		6	
Locks	3 for $6		1	
Cubes	2 for $5		5	
			Total:	Total:

Consider unit prices from the table.

2. What is the range of prices?	3. What is the median price?	4. What is the mean price?

Write one statistical question based on each.

5. Unit prices	6. Students polled

7. Recommend one or more keychains to the committee based on data from the table.

RATE APPLICATIONS

We will deepen our understanding of independent and dependent variables. We will use rates in money contexts and in time-distance contexts. We will identify unit rate in tables, graphs, and equations.

[6RP3abc, 6NS3, 6EE2ab, 6EE6, 6EE9; SMP1, 2, 4, 5, 6, 7, 8]

GETTING STARTED

Julie is saving $10 each week from her allowance.

1. Fill in the table for the total amount saved at each week through week 6.

Week # (W)	1		3		5	6			
Total $ (*T*)		20		40					

2. Rate of change: for every increase of one week, the total increases by _______

3. Complete an input-output rule that relates week number to total dollars saved: *T*= _______

4. For the last three entries in the table let *W* = 10, 25, and 37. Use your rule to find the corresponding values for *T*.

5. For problem 2 above, typically, which quantity is the independent variable?

6. After how many weeks of allowance will Julie have saved each amount?

a. $400	b. $500
c. $450	d. $425

RAISING MONEY FOR MUSIC

Follow your teacher's directions for (1) and (2).

The Springfield Education Foundation is trying to raise $100,000 for its music programs. They have fundraisers throughout the year.

(1) The SEF sells holiday trees for its December fundraiser. Each tree sells for _______.

Table I	
# of trees sold (*t*)	**Money earned in $ (*m*)**
1	
2	
5	
10	
50	
100	

Table II	
Money earned in $ (*m*)	**# of trees sold (*t*)**
50	
100	
200	
600	
1,000	
4,000	

(2)

Table I: The amount of money earned depends on ________________________________.

Table II: The number of trees sold depends on ________________________________ .

3. Write two different equations that relate t and m.

$m =$ _____ $t =$ _____

Use the equations from question 3 to complete problems 4 – 5.

4. If they sell 1,000 trees, how much will they earn?

What percent of the way would they be to their goal?

5. How many trees sold raises $60,000?

If they did this, how many more trees would they have to sell to reach their goal?

PRACTICE 7

1. The Springfield Education Foundation hosts a pancake breakfast for its May fundraiser. They sell each breakfast for $5.50. Complete Tables III and IV.

Table III

# of breakfasts sold (*b*)	Money earned in $ (*m*)
1	
2	
4	
12	
40	
100	

Table IV

Money earned in $ (*m*)	# of breakfasts sold (*b*)
5.50	
11	
55	
110	
440	
660	

2. Write two different equations that relate *b* and *m*.

   a. $m =$ ___________ b. $b =$ ___________

3. In Table III, money earned in dollars depends on…

4. If they sell 1,000 breakfasts, how much will they earn?

5. The Springfield Education Foundation sponsors a walk-a-thon for its August fundraiser. Each entry fee is $25. Complete Tables V and VI.

Table V

# of entries sold (*w*)	Money earned in $ (*m*)
1	
2	
3	
5	
10	
100	

Table VI

Money earned in $ (*m*)	# of entries sold (*w*)
25	
50	
125	
500	
1,000	
2,000	

6. Write two different equations that relate *m* and *w*.

   a. $m =$ ___________ b. $w =$ ___________

7. In Table VI, number of entries sold depends on…

8. How many entries sold accomplishes a $6,500 goal?

RUNNING

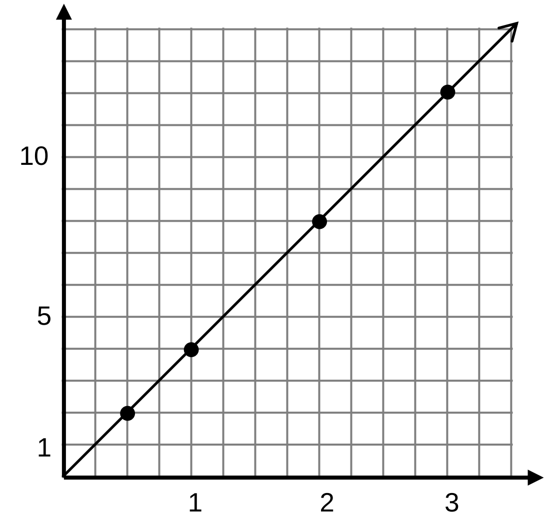
Martino is training for a marathon. Today he ran for 3 hours and 15 minutes at a constant rate of speed. A graph of his run is shown.

1. Write 3 hours and 15 minutes as a decimal.

2. Write a title and axis labels. The input values are time (in hours). The outputs values are distance (in miles).

3. Why is it appropriate to connect the points on the graph?

4. Explain what the point (0,0) on the graph means in the context of this problem.



5. Complete the table below.

Time in hours (*t*)	1	2	3	0.5	0.25	1.5	3.25
Distance in miles (*d*)							

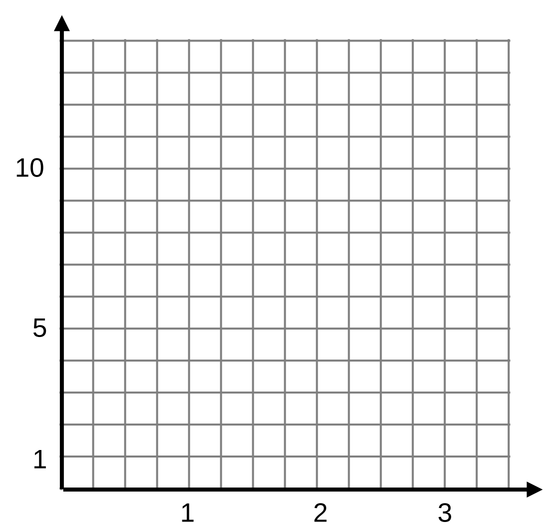
6. Write an equation for distance in terms of time.

7. What is the meaning of the coefficient of *t* in the equation? Circle it in the table and the graph.

8. At this rate, how far did Martino run in 2.5 hours?

9. At this rate, how many hours would it take Martino to run 17 miles?

10. Suppose Martino starts his run fast and then slows down. Make a properly labeled graph to illustrate this situation.



PRACTICE 8

Darryl is doing a 52-mile bicycle ride that takes him 4 hours. He rides at a constant rate.

1. Complete the table below.

Time in hours (t)	4	2	1	0.5	0.25	0.75	1.5	2.75	3.25
Distance in miles (d)									

2. Write an equation that relates distance and time.

3. Draw a graph below. Include at least four points from the table. Be sure to label it completely.

Linda is doing an 18-kilometer uphill hike that takes her 6 hours. She hikes at a constant rate.

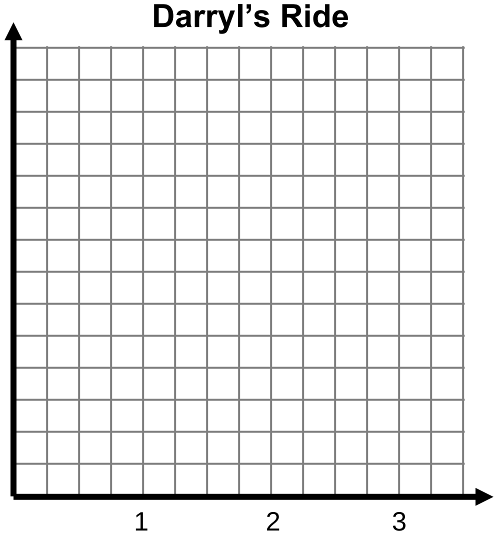
4. Complete the table below.

Time in hours (t)	6		2	0.5	0.25	0.75			
Distance in kilometers (d)		3					5.25	6.75	10.5

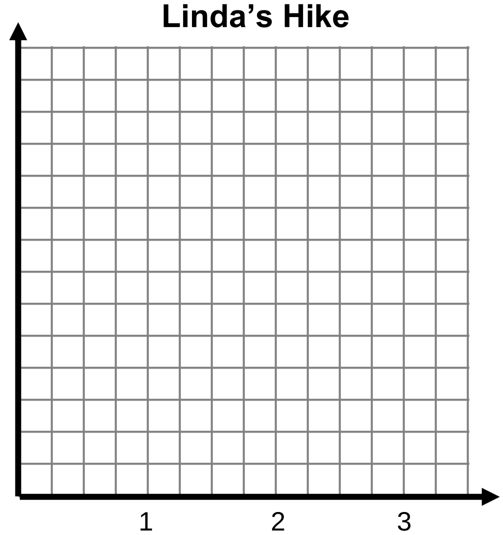
5. Write an equation that relates distance and time.

6. Draw a graph below for Linda's hike. Include at least four points from the table. Be sure to label it completely.

Darryl's Ride



Linda's Hike

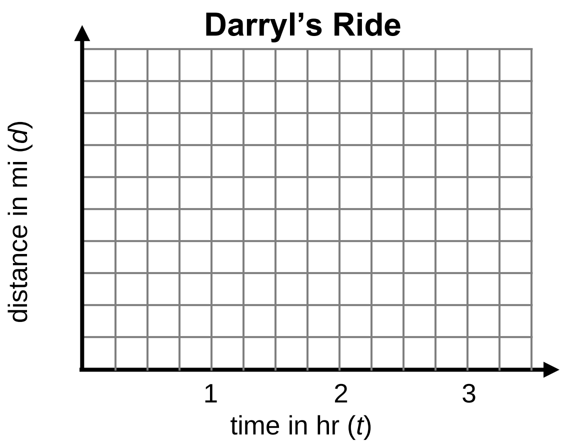


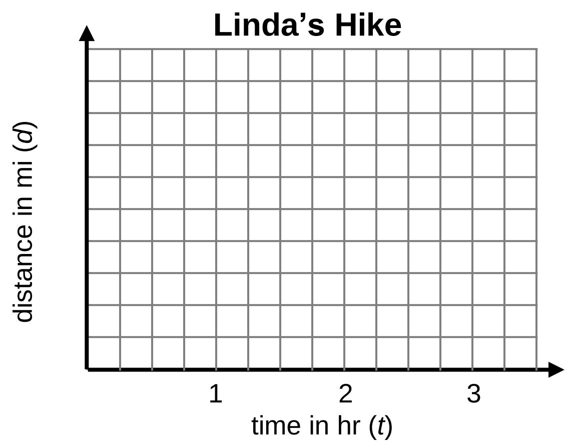
PRACTICE 9: EXTEND YOUR THINKING

Refer to Darryl's ride and Linda's hike from the previous page.

1. Sketch a graph for Darryl if, instead of riding at a constant rate the entire race, he started off slowly uphill, then fast downhill, then finished at medium speed on flat land.

2. Sketch a graph for Linda if she hikes at some constant rate the entire time, except for a 30-minute break somewhere in the middle of the hike.

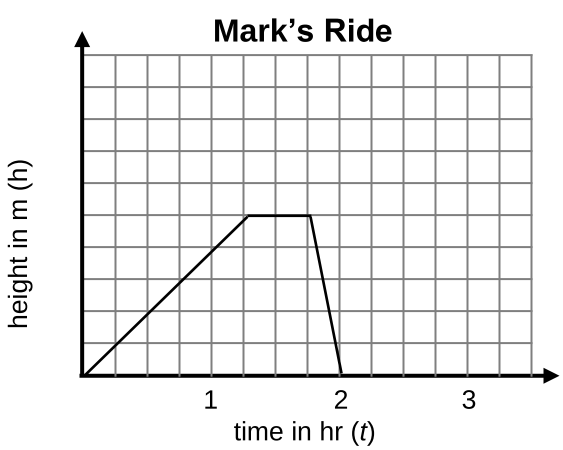


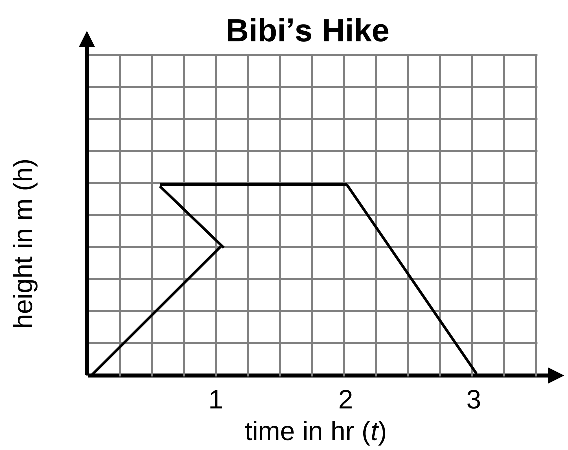


Notice that the graphs for Mark and Bibi below have height on the vertical axes.

3. Explain Mark's graph in writing.

4. Explain the part of Bibi's hike that does not make sense.





REVIEW

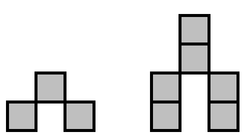
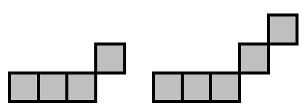
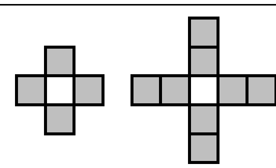
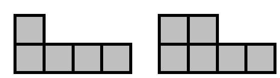
POSTER PROBLEMS: INPUTS AND OUTPUTS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Steps 1 and 2 of each pattern are given below.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
 Step 1 Step 2	 Step 1 Step 2	 Step 1 Step 2	 Step 1 Step 2

A. Copy steps 1 and 2 onto the poster and draw step 3. Explain your step 3 in words.

B. Make a table, label it appropriately, and record values for steps 1 through 5.

C. Make a graph and label it appropriately.

D. Write an input-output rule that relates the total number of tiles to the step number.

Part 3: Return to your seats. Work with your group, and show all work.

Use your "start problem."

1. Find the number of tiles in step 100.

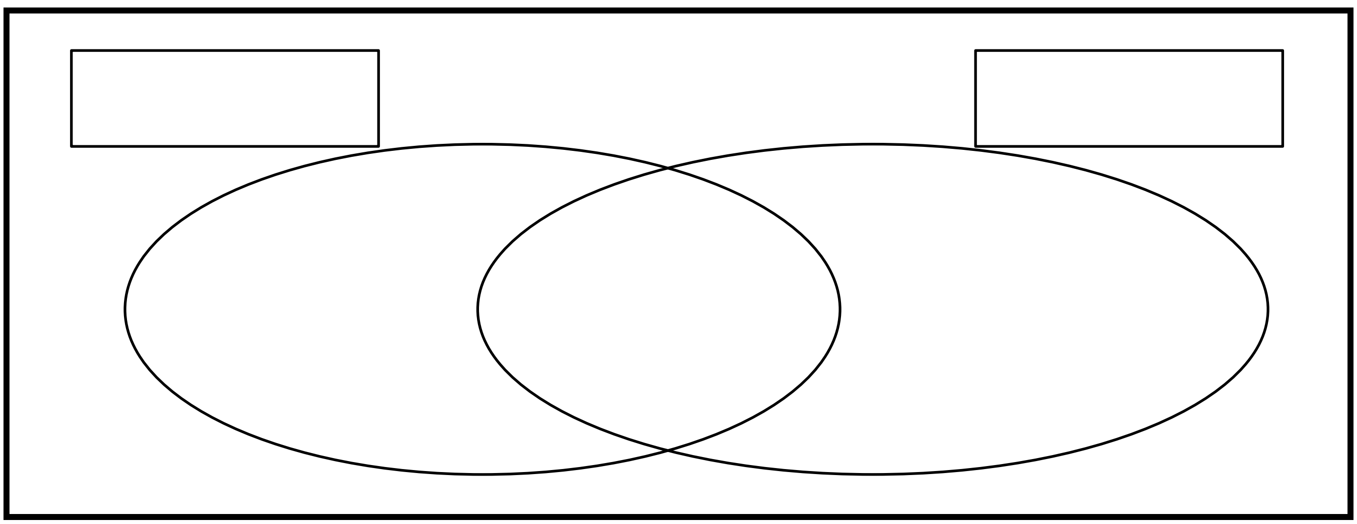
2. Find which step number has exactly 120 tiles.

MATCH AND COMPARE SORT: INPUTS AND OUTPUTS

1. Individually, using the match and compare sort cards, match words with descriptions. Record results.

Card set △			**Card set ◯**		
Card number	**word**	**Card letter**	**Card number**	**word**	**Card letter**
I			**I**		
II			**II**		
III			**III**		
IV			**IV**		

2. Partners, choose a pair of numbered matched cards and record the attributes that are the same and those that are different.



WHY DOESN'T IT BELONG?: INPUTS AND OUTPUTS

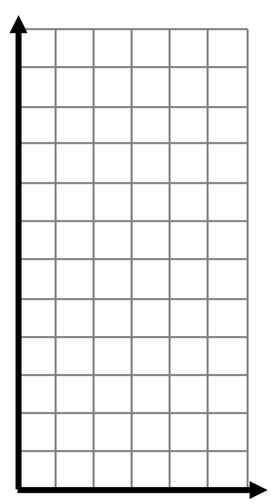
Four different stores near LaRonda's home sell yellowfin tuna by the pound. Though prices are listed differently, she can buy any number of pounds at each store at the given rate.

1. Choose one of these price rates and explain why it doesn't belong with the others. Then choose at least one more and explain why it doesn't belong.

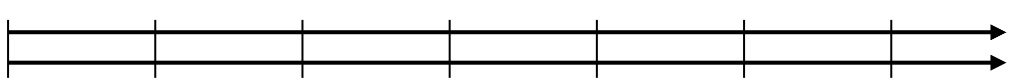
A \$3.50 for 1 pound	**B** \$24.00 for 6 pounds
C \$8.75 for 2.5 pounds	**D** 4 pounds for \$14.00

2. For the price that is common among three of the stores above, complete the following:
   - a table
   - a double number line
   - a graph
   - an input-output rule (equation)

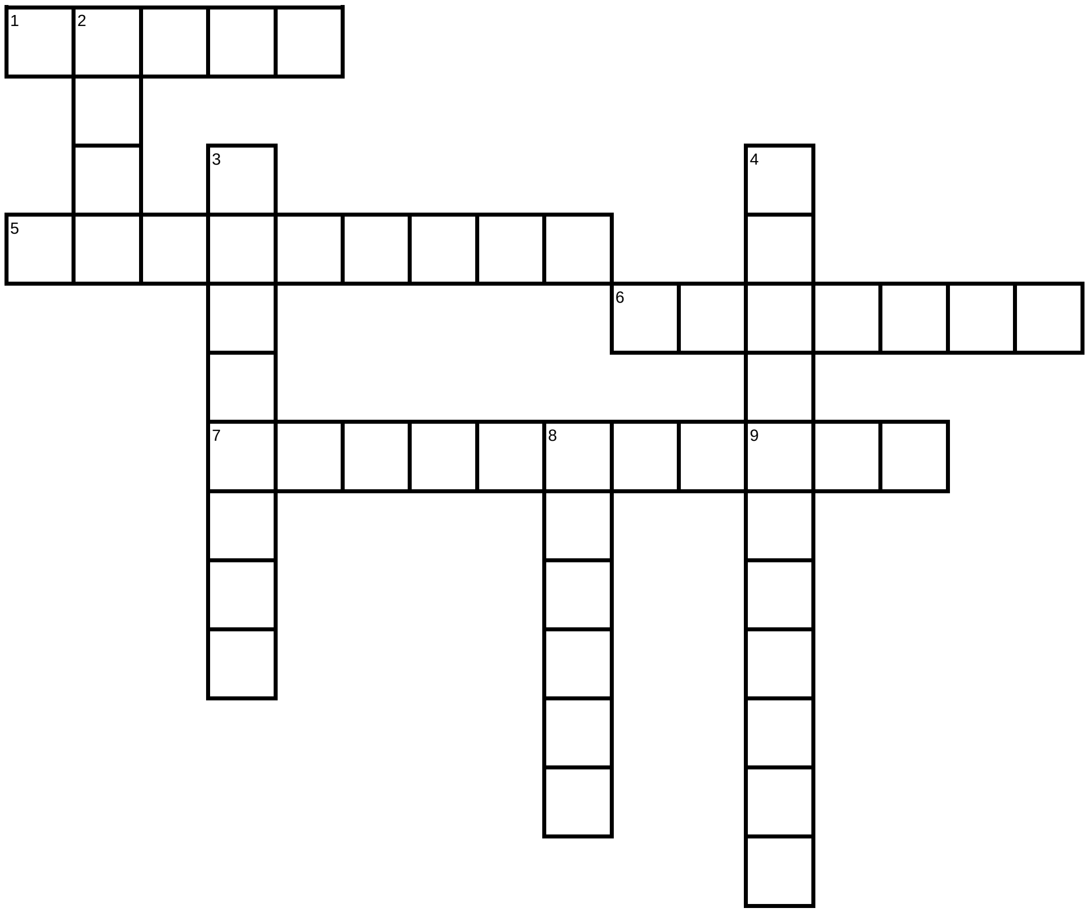
(*x*)	**(*y*)**
1	
2	
3	
4	
5	



Equation:



VOCABULARY REVIEW



Across

1 If 6 oranges cost $4.50, the unit ___ per orange is $0.75.

5 A variable that is typically thought of as the output

6 Pairs of numbers in the form (*x*, *y*) are called ___ pairs.

7 A rule where, given *x*, you can find *y* (2 words)

Down

2 If there are 12 pencils for a group of 4 students, the unit ___ is 3 pencils per each student.

3 Direction of the *y*-axis

4 A variable typically thought of as the input

6 Name of the point (0, 0)

SPIRAL REVIEW

1. **Computational Fluency Challenge.** This paper and pencil exercise will help you gain fluency with multiplication and division. Try to complete this challenge without any errors. No calculators!

   - Start with 2.5. Multiply by 4. Divide the result by 5. Multiply the result by 20. Multiply the result by 40. Now you have a "big number". My big number is _____.

   - Start with your big number. Divide it by 40. Divide the result by 10. Divide the result by 2. Multiply the result by 5. Divide the result by 4. What is the final result? _____

2. Evaluate each numerical expression below.

a. $26 - 4^2$	b. $2(6 + 4)$	c. $2(6 - 4)^2$
d. $2 + (6 + 4)^2$	e. $2(6^2) + 2(4)^2$	f. $6 + 4 \bullet 2$

3. Christopher needs to paint a rectangular picture that has an area of $12\frac{3}{8}$ square yards on a brick wall. The width of his painting is $2\frac{3}{4}$ yards. What does the height of the painting need to be?

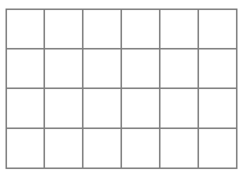
SPIRAL REVIEW
Continued

4. Ryann bought 3 drinks that cost $2 each and 3 hot dogs that cost $2.50 each at the snack bar. Circle the expressions that could be used to compute the total cost.

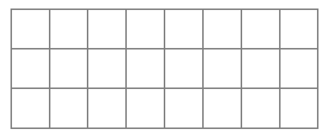
   2 + 2 + 2(2.50)   3(2) + 3(2.50)   (2 + 2.5)(3)   3 + 2 + 3 + 2.50

5. Denali and Jena created plans for a quilted blanket in sewing class, using patches that are 6 inches on each side. Patches are represented by the small squares below.

Denali's Plan



Jena's Plan



   a. Whose blanket covers a greater area? Explain.

   b. After they sew the patches together, they have to sew a ribbon around the edge of their blankets. Who will need to use a longer ribbon? Explain your answer in feet.

   c. Leo wants to make a square blanket using 16 patches. Write two equivalent expressions using multiplication and exponents to represent its area.

6. Some nutritionists recommend that teens drink 8 cups of water per day.

   a. Yesterday Helena drank 64 oz of water. Did Helena reach this goal? What percent of this goal did Helena drink?

   b. Yesterday Betina drank a quart of water. Did Betina reach this goal? What percent of this goal did Betina drink?

   c. Yesterday Caryn drank 6 cups of water. Did Caryn reach this goal? What percent of this goal did Caryn drink?

REFLECTION

1. **Big Ideas**. Shade all circles that describe big ideas in this packet. Draw lines to show connections that you noticed.

- Investigate concepts and solve problems involving length, area, and volume.
- Extend the number system to include negatives.
- Use statistical measures and displays to describe center and spread.
- Explore relationships between inputs and outputs.
- Gain computational fluency with positive rational numbers.
- Rewrite and evaluate expressions and solve equations.
- Explore and apply ratio and rate reasoning and representations.

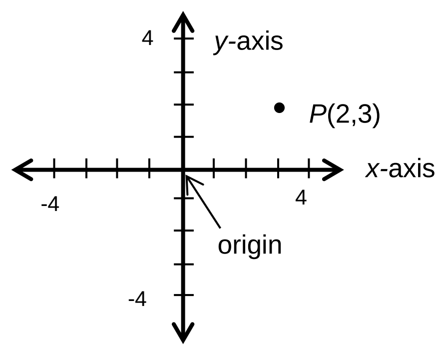
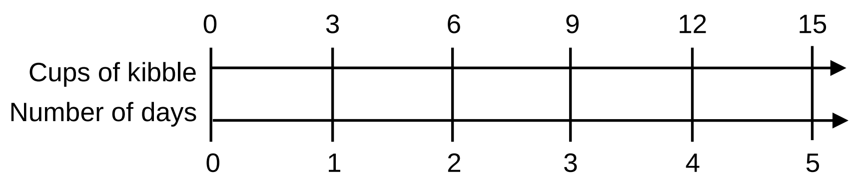
Give an example from this packet of one of the connections above.

2. **Packet Progress.** Go back to **Monitor Your Progress** on the cover and complete or update your responses. Explain something you understand better now than before or something you would still like to work on.

3. **Mathematical Practice.** Did you get stuck on any problems in this packet? What did you do to get unstuck (SMP1)?

4. **More Connections.** Give an example of how data along with your math skills helped you to make a decision or prediction.

STUDENT RESOURCES

Word or Phrase	Definition
coefficient	A <u>coefficient</u> is a number or constant factor in a term of an algebraic expression. In the expression $3x + 5$, 3 is the coefficient of the term $3x$, and 5 is the constant term.
coordinate plane	A <u>coordinate plane</u> is a plane with two perpendicular number lines (<u>coordinate axes</u>) meeting at a point (the <u>origin</u>). Each point *P* of the coordinate plane corresponds to an ordered pair (a, b) of numbers, called the <u>coordinates</u> of *P*. The point *P* may be denoted $P(a, b)$. The coordinate axes are often referred to as the *x*-axis and the *y*-axis respectively. The origin has coordinates (0,0). 
double number line	A <u>double number line</u> is a diagram made up of two parallel number lines that visually depict the relative sizes of two quantities. Double number lines are often used when the two quantities have different units, such as miles and hours. The proportional relationship “Wrigley eats 3 cups of kibble per day” can be represented in the following double number line diagram. 
dependent variable	A <u>dependent variable</u> is a variable whose value is determined by the values of the independent variables. See <u>independent variable</u>.
equation	An <u>equation</u> is a mathematical statement that asserts the equality of two expressions. $18 = 8 + 10$ is an equation that involves only numbers. This is a numerical equation. $18 = x + 10$ is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.
expression	A mathematical <u>expression</u> is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8 + x}{10}$, and $4v - w$.

<table>
<tr><th>Word or Phrase</th><th>Definition</th></tr>
<tr><td>independent variable</td><td>An independent variable is a variable whose value may be specified. Once specified, the values of the independent variables determine the values of the dependent variables.

For the equation $y = 3x$, y is the dependent variable and x is the independent variable. We may assign a value to x. The value assigned to x determines the value of y.</td></tr>
<tr><td>input-output rule</td><td>An input-output rule for a sequence of values is a rule that establishes explicitly an output value for each given input value.

<table><tr><td>input value (x)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>x</td></tr><tr><td>output value (y)</td><td>1.5</td><td>3</td><td>4.5</td><td>6</td><td>7.5</td><td>$1.5x$</td></tr></table>
In the table above, the input-output rule could be $y = 1.5x$. In other words, to get the output value, multiply the input value by 1.5. If $x = 100$, then $y = 1.5(100) = 150$.</td></tr>
<tr><td>rate</td><td>See unit rate.</td></tr>
<tr><td>unit price</td><td>A unit price is a price for one unit of measure.

If 4 apples cost $1.00, then the unit price is $\frac{\$1.00}{4} = \$0.25$ for one apple, or 0.25 dollars per apple or 25 cents per apple.</td></tr>
<tr><td>unit rate</td><td>The unit rate associated with a ratio $a : b$ of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. This is sometimes referred to as the value of the ratio.

The ratio of 40 miles for every 5 hours has a unit rate of $\frac{40}{5} = 8$ miles per hour.</td></tr>
<tr><td>variable</td><td>A variable is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to quantities that vary in a relationship (as in a formula or an input-output rule). They may refer to unknown quantities in expressions, equations, or inequalities. Finally, they may be used to generalize rules of arithmetic.

In the equation $d = rt$, the quantities d, r, and t are variables.
In the equation $2x = 10$, the variable x may be referred to as the unknown.
The equation $a + b = b + a$ generalizes the commutative property of addition for all numbers a and b.</td></tr>
</table>

The Coordinate Plane

A coordinate plane is determined by a horizontal number line (the *x*-axis) and a vertical number line (the *y*-axis) intersecting at the zero on each line. The point of intersection (0, 0) of the two lines is called the origin. Points are located using ordered pairs (*x*, *y*).

- The first number (*x*-coordinate) indicates how far the point is to the right of the *y*-axis.
- The second number (*y*-coordinate) indicates how far the point is above the *x*-axis.

Point, coordinates, and interpretation

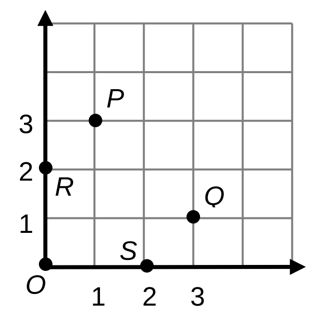
O (0, 0) → at the intersection of the axes.

P (1, 3) → start at the origin, move 1 unit right, then 3 units up

Q (3, 1) → start at the origin, move 3 units right, then 1 unit up

R (0, 2) → start at the origin, move 0 units right, then 2 units up

S (2, 0) → start at the origin, move 2 units right, then 0 units up.



Multiple Representations: Tables, Graphs, and Equations

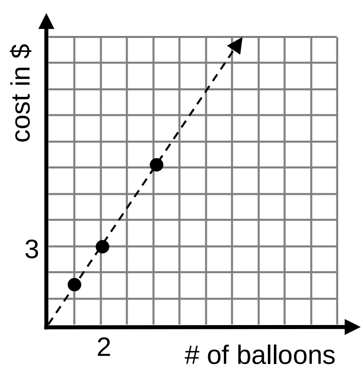
Suppose 4 balloons cost $6.00 and each balloon is the same price. Here are some representations for this relationship.

Table

Number of Balloons	Cost in $
4	6.00
2	3.00
1	1.50
8	12.00

Note that the unit price is $1.50 per balloon

Graph



Numbers of balloons must be discrete values (specifically, whole numbers), however a trend line may be drawn to show a growth pattern.

Equation (input-output rule)

Let *y* = cost in dollars and *x* = number of balloons.

We can see from the table that the unit price is 1.50 dollars per balloon.

It appears that multiplying any input value by 1.5 yields its corresponding output value.

Therefore, $y = 1.5x$.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, ~~tape diagrams~~, double number line diagrams, or equations:
a.	Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b.	Solve unit rate problems including those involving unit pricing and constant speed.
c.	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
6.EE.A	**Apply and extend previous understandings of arithmetic to algebraic expressions.**
6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers:
a.	Write expressions that record operations with numbers and with letters standing for numbers.
b.	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems.
6.EE.B	**Reason about and solve one-variable equations and inequalities.**
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.C	**Represent and analyze quantitative relationships between dependent and independent variables.**
6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*
6.SP.A	**Develop understanding of statistical variability.**
6.SP.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.