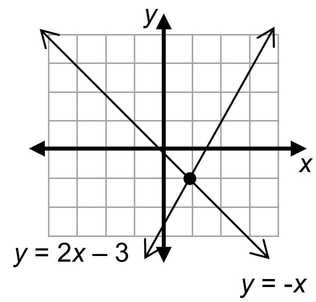
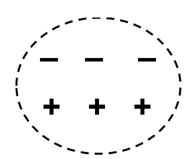
STUDENT RESOURCES

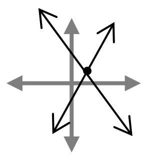
Word or Phrase	Definition
point of intersection	A <u>point of intersection</u> of two lines is a point where the lines meet. The two straight lines in the plane with equations $y = -x$ and $y = 2x - 3$ have point of intersection (1, -1). 
slope-intercept form	The <u>slope-intercept form</u> of the equation of a line is the equation $y = mx + b$, where m is the slope of the line, and b is the y-intercept of the line. The equation $y = 2x + 3$ determines a line with slope 2 and y-intercept 3.
solution to an equation	A <u>solution to an equation</u> involving variables consists of values for the variables which, when substituted, make the equation true. The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.
solve an equation	To <u>solve an equation</u> refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. To solve the equation $2x = 6$, one might think "two times what number is equal to 6?" Since 2(3) = 6, the only value for x that satisfies this condition is 3. Therefore 3 is the solution.
substitution	<u>Substitution</u> refers to replacing a value or quantity with an equivalent value or quantity. If $y = x + 5$, and we know that $x = 3$, then we may use substitution to rewrite the first equation to get $y = 3 + 5$. If $y = x + 10$, and we know also that $y = 2x + 4$, then we may use substitution to write one equation in x to get $x + 10 = 2x + 4$.
system of linear equations	A <u>system of linear equations</u> is a set of two or more linear equations in the same variables. An example of a system of linear equations in x and y: $\begin{cases} x + y = 1 \\ x + 2y = 4 \end{cases}$
zero pair	In the signed counters model, a positive and a negative counter together form a <u>zero pair</u>. Let "+" represent a positive counter, and let "−" represent a negative counter. Then the following is an example of a collection of (three) zero pairs. 

Systems of Linear Equations

A system of equations is a set of two or more equations in the same variables. A solution to the system of equations consists of values for the variables which, when substituted, make all equations simultaneously true.

A system of linear equations has exactly one solution, infinitely many solutions, or no solution.

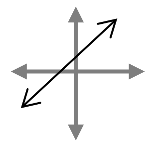
$$\begin{cases} y = -x + 3 \\ y = 2x - 3 \end{cases}$$



One solution at (2, 1)

These two lines intersect in aqexactly one point.This is the only pair of *x*- and *y*-values that statisfies these two equations simultaneously.

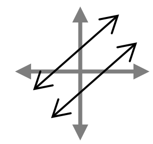
$$\begin{cases} y = x + 1 \\ 2y = 2x + 2 \end{cases}$$



Infinitely many solutions

Since the equations are equivalent, the two lines coincide. Every point on the line represents a solution.

$$\begin{cases} y = x - 2 \\ y = x + 2 \end{cases}$$



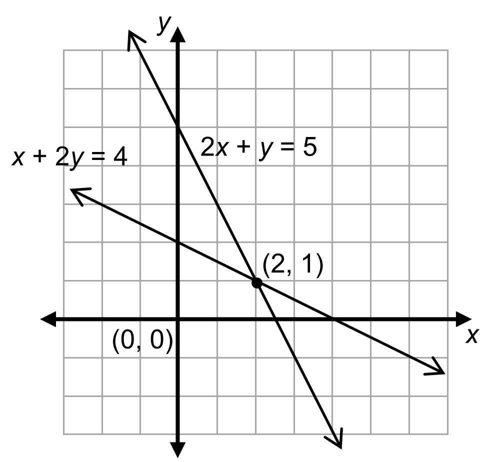
No solution

Since these two lines are parallel, they do not intersect. Thus these two equations have no solution in common.

Solving a System of Linear Equations by Graphing

To solve a system of equations by graphing, graph both lines on the same set of axes and observe the point(s) of intersection, if any.

Solve by graphing: $\begin{cases} 2x + y = 5 \\ x + 2y = 4 \end{cases}$



1. Change each to slope-intercept form, $y = mx + b$.

   $2x + y = 5 \longrightarrow y = -2x + 5$

   $x + 2y = 4 \longrightarrow y = -\frac{1}{2}x + 2$

2. Graph each equation.

3. Observe the intersection of the lines, (2, 1). This represents the solution to the system. In other words, these are the x- and y- values that satisfy both equations. Remember that not every system of equations has exactly one solution.

4. Check by substituting solutions in the original equations to be sure they are correct.

   $2x + y = 5 \longrightarrow 2(2) + 1 = 5$ (true)

   $x + 2y = 4 \longrightarrow 2 + 2(1) = 4$ (true)

Properties of Arithmetic

Properties of arithmetic govern the manipulation of expressions (mathematical phrases).
For any three numbers *a*, *b*, and *c*:

- ✓ Associative property of addition
  $a + (b + c) = (a + b) + c$
- ✓ Commutative property of addition
  $a + b = b + a$
- ✓ Additive identity property
  (addition property of 0)
  $a + 0 = 0 + a = a$
- ✓ Additive inverse property
  $a + (-a) = -a + a = 0$
- ✓ Associative property of multiplication
  $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
- ✓ Commutative property of multiplication
  $a \bullet b = b \bullet a$
- ✓ Multiplicative identity property
  (multiplication property of 1)
  $a \bullet 1 = 1 \bullet a = a$
- ✓ Multiplicative inverse property
  $a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1 \quad (a \neq 0)$
- ✓ Distributive property relating addition and multiplication
  $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers *a*, *b*, and *c*.

Properties of Equality

Properties of equality govern the manipulation of equations (mathematical sentences).
For any three numbers *a*, *b*, and *c*:

- ✓ Addition property of equality
  (Subtraction property of equality)
  If $a = b$ and $c = d$, then $a + c = b + d$
- ✓ Multiplication property of equality
  (Division property of equality)
  If $a = b$ and $c = d$, then $ac = bd$
- ✓ Reflexive property of equality: $a = a$
- ✓ Symmetric property of equality: If $a = b$, then $b = a$
- ✓ Transitive property of equality: If $a = b$, and $b = c$, then $a = c$

Solving Equations Using a Model 1

Let **+** represent 1

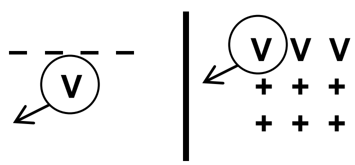
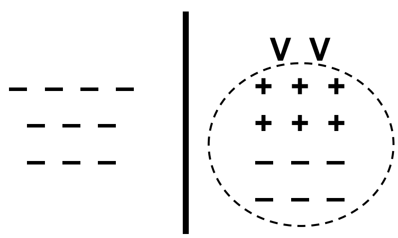
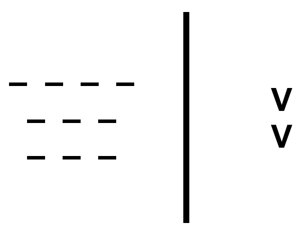
Let **–** represent -1

Let **V** represent the unknown (like x)

Let **Λ** represent the opposite of the unknown (like $-x$)

The following example illustrates one solution path. Other paths are possible to arrive at the same solution.

Solve: $-4 + x = 3(x + 2)$

Picture	Equation	What did you do?
	$-4 + x = 3x + 6$ $\underline{-x} \quad \underline{-x}$ $-4 = 2x + 6$	build the equation remove one x from each side
	$-4 = 2x + 6$ $\underline{+(-6)} \quad \underline{+(-6)}$ $-10 = 2x$	add -6 to each side remove zero pairs
	$\frac{-10}{2} = \frac{2}{2}x$ $-5 = x$	divide both sides by 2 put counters equally into cups (or do mentally) notice the use of the big 1

Check by substituting the solution into the original equation:

$-4 + x$	=	$3(x + 2)$	
$-4 + (-5)$	=	$3(-5 + 2)$	?
-9	=	$3(-3)$	?
-9	=	-9	true

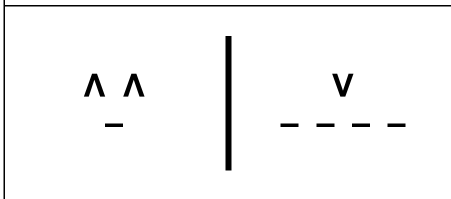
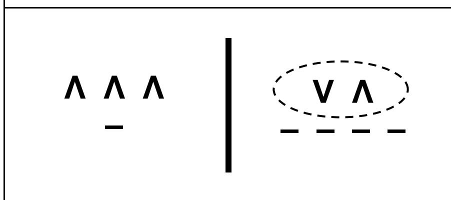
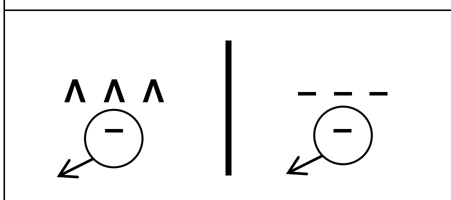
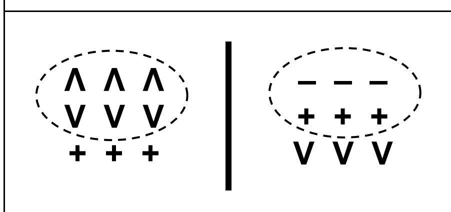
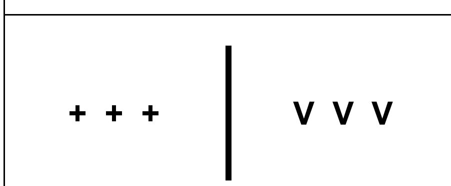
Solving Equations Using a Model 2

Let **+** represent 1 Let **V** represent the unknown (like x)

Let **–** represent -1 Let **Λ** represent the opposite of the unknown (like $-x$)

The following example illustrates one solution path. Other paths are possible to arrive at the same solution.

Solve: $-2x - 1 = x - 4$

Picture	Equation	What did you do?
	$-2x - 1 = x - 4$	build the equation
	$-2x - 1 = x - 4$ $\underline{+(-x)} = \underline{+(-x)}$ $-3x - 1 = -4$	add the opposite of x to both sides remove the zero pair
	$-3x - 1 = -4$ $\underline{-(-1)} = \underline{-(-1)}$ $-3x = -3$	remove -1 from both sides* *this gives the same result as adding 1 to each side
	$-3x = -3$ $\underline{(+3x) + 3} = \underline{+3 + (3x)}$ $3 = 3x$	add 3x to both sides AND add 3 positives to both sides remove zero pairs
	$\frac{3}{3} = \frac{3}{3}x$ $1 = x$	divide both sides by 3 put counters equally into cups (or do mentally) notice the use of the big 1

Check by substituting the solution into the original equation:

$$-2x - 1 = x - 4$$
$$-2(1) - 1 = 1 - 4 \quad ?$$
$$-2 - 1 = -3 \quad ?$$
$$-3 = -3 \quad \text{true}$$

Using Algebraic Techniques to Solve Equations

To solve equations using algebra:

- Use the properties of arithmetic to simplify each side of the equation (e.g., associative properties, commutative properties, inverse properties, distributive property).
- Use the properties of equality to isolate the variable (e.g., addition property of equality, multiplication property of equality).

Solve: $3 - x + 3 = 5x - 2x - 2$

Equation	What did you do?	Property
$3 - x + 3 = 5x - 2x - 2$ $6 - x = 3x - 2$	arithmetic collect like terms	distributive property $(5 - 2)x = 3x$
$6 - x = 3x - 2$ $+ 2 \quad\quad + 2$ $8 - x = 3x$	add 2 to both sides arithmetic	addition property of equality additive inverse/identity properties
$8 - x = 3x$ $+ x \quad + x$ $8 = 4x$	add x to both sides collect like terms	addition property of equality additive inverse/identity properties distributive property $3x + x = (3 + 1)x = 4x$
$\frac{8}{4} = \frac{4x}{4}$ $2 = x$	multiply both sides by $\frac{1}{4}$ (or divide both sides by 4) arithmetic	multiplication(division) property of equality multiplicative inverse/identity properties

Check by substituting the solution into the original equation:

$3 - x + 3$	=	$5x - 2x - 2$	
$3 - 2 + 3$	=	$5(2) - 2(2) - 2$	?
4	=	$10 - 4 - 2$	?
4	=	4	true