STUDENT RESOURCES

Word or Phrase	Definition
additive inverse property	The additive inverse property states that $a + (-a) = 0$ for any number a. In other words, the sum of a number and its opposite is 0. The number $-a$ is the additive inverse of a. $3 + (-3) = 0$, $-25 + 25 = 0$
coefficient	A coefficient is a number or constant factor in a term of an algebraic expression. In the expression $3x + 5$, 3 is the coefficient of the term $3x$, and 5 is the constant term.
constant term	A constant term in an algebraic expression is a term that has a fixed numerical value. In the expression $5 + 2x + 3$, the terms 5 and 3 are constant terms. If this expression is rewritten as $2x + 8$, the term 8 is the constant term of the new expression.
distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. $3(4 + 5) = 3(4) + 3(5)$; $(4 + 5)8 = 4(8) + 5(8)$; $6(8 – 1) = 6(8) – 6(1)$
equation	An equation is a mathematical statement that asserts the equality of two expressions. $18 = 8 + 10$ is an equation that involves only numbers. This is a numerical equation. $18 = x + 10$ is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.
equivalent expressions	Two mathematical expressions are equivalent if, for any possible substitution of values for the variables, the two resulting numbers are equal. In particular, two numerical expressions are equivalent if they represent the same number. See expression. The numerical expressions $3 + 2$ and $6 – 1$ are equivalent. Both are equal to 5. The algebraic expressions $3(x + 2)$ and $3x + 6$ are equivalent. For any value of the variable x, the expressions represent the same number.
evaluate	Evaluate refers to finding a numerical value. To evaluate an expression, replace each variable in the expression with a value and then calculate the value of the expression. To evaluate the numerical expression $3 + 4(5)$, we calculate $3 + 4(5) = 3 + 20 = 23$. To evaluate the variable expression $2x + 5$ when $x = 10$, we calculate $2x + 5 = 2(10) + 5 = 20 + 5 = 25$.
expression	A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8 + x}{10}$, and $4v – w$.

Word or Phrase	Definition
input-output rule	An input-output rule for a sequence of values is a rule that establishes explicitly an output value for each given input value. input value (x): 1, 2, 3, 4, 5, x output value (y): 1.5, 3, 4.5, 6, 7.5, $1.5x$ In the table above, the input-output rule could be $y = 1.5x$. In other words, to get the output value, multiply the input value by 1.5. If $x = 100$, then $y = 1.5(100) = 150$. The "independent variable" is typically associated with the input value, and the "dependent variable" is typically associated with the output value.
like terms	See terms.
proportional	Two variables are proportional if the values of one are the same constant multiple of the corresponding values of the other. The variables are said to be in a proportional relationship, and the constant is referred to as the constant of proportionality. If Wrigley eats 3 cups of kibble each day, then the number of cups of kibble is proportional to the number of days. If x is the number of days, and y is the number of cups of kibble, then $y = 3x$. The constant of proportionality is 3.
proportional relationship	See proportional.
simplify	Simplify refers to converting a numerical or variable expression to a simpler form. A variable expression might be simplified by combining like terms. A fraction might be simplified by dividing numerator and denominator by a common divisor. $2x + 6 + 5x + 3 = 7x + 9$ $\qquad \frac{8}{12} = \frac{2}{3}$
terms	The terms in a mathematical expression involving addition (or subtraction) are the quantities being added (or subtracted). Terms with the same variable part are called like terms. The expression $2x + 6 + 3x + 5$ has four terms: $2x$, 6, $3x$, and 5. The terms $2x$ and $3x$ are like terms, since each is a constant multiple of x. The terms 6 and 5 are like terms, since each is a constant.
variable	A variable is a quantity whose value has not been specified. Variables are used in many different ways. They may refer to quantities that vary in a relationship (as in a formula or an input-output rule). They may refer to unknown quantities in expressions, equations or inequalities. Finally, they may be used to generalize rules of arithmetic. In the equation $d = rt$, the quantities d, r, and t are variables. In the equation $2x = 10$, the variable x may be referred to as the unknown. The equation $a + b = b + a$ generalizes the commutative property of addition for all numbers a and b.

The Coordinate Plane: Quadrant I

In this packet, all graphing is done in Quadrant I, because all coordinates graphed are nonnegative.
A coordinate plane is determined by a horizontal number line (the *x*-axis) and a vertical number line (the *y*-axis) intersecting at the zero on each line. The point of intersection (0, 0) of the two lines is called the origin. Points are located using ordered pairs (*x*, *y*).

- The first number (*x*-coordinate) indicates how far the point is to the right of the *y*-axis.
- The second number (*y*-coordinate) indicates how far the point is above the *x*-axis.

Point, coordinates, and interpretation

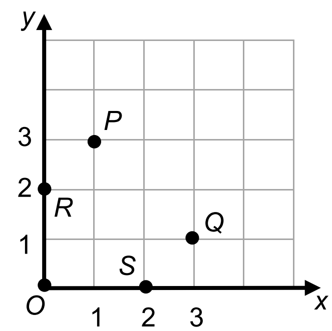
O (0, 0) → at the intersection of the axes

P (1, 3) → start at the origin, move 1 unit right, then 3 units up

Q (3, 1) → start at the origin, move 3 units right, then 1 unit up

R (0, 2) → start at the origin, move 0 units right, then 2 units up

S (2, 0) → start at the origin, move 2 units right, then 0 units up



Multiple Representations: Tables, Graphs, and Equations

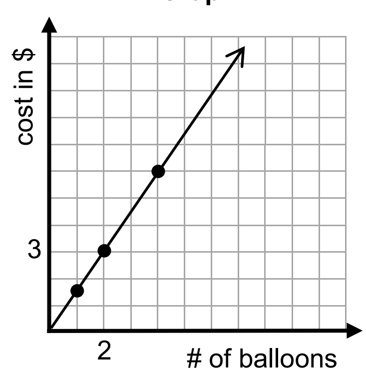
Suppose 4 balloons cost $6.00 and each balloon is the same price. Here are some representations for this relationship.

Table

Number of Balloons	Cost in $
4	6.00
2	3.00
1	1.50
8	12.00

Note that the unit price is $1.50 per balloon

Graph



Numbers of balloons must be discrete values (specifically, whole numbers). A trend line may be drawn to show a growth pattern.

Equation (input-output rule)

Let *y* = cost in dollars
Let *x* = number of balloons.

We can see from the table that the unit price is 1.50 dollars per balloon.

It appears that multiplying any input value by 1.5 yields its corresponding output value.

Therefore, $y = 1.5x$.

Variables in Algebra	
Loosely speaking, variables are quantities that can vary. Variables are represented by letters or symbols. Variables have many different uses in mathematics. The use of variables, together with the rules of arithmetic, makes algebra a powerful tool. Three important ways that variables appear in algebra:	
Usage	**Examples**
Variables can represent an *unknown quantity* in an equation or inequality. In this case, the equation is valid only for specific value(s) of the variable.	$x + 4 = 9$ $5n = 20$ $y < 6$
Variables can represent *quantities that vary* in a relationship. In this case, there is always more than one variable in the equation.	Formula: $P = 2\ell + 2w$, $A = s^2$ Function (input-output rule): $y = 5x$, $y = x + 3$
Variables can represent *quantities in statements that generalize* rules of arithmetic. In this case, there may be one or more variables.	Commutative property of addition: $x + y = y + x$ Distributive property: $x(y + z) = xy + xz$

Evaluate or Simplify?
We use the word “evaluate” when we want to calculate the value of an expression. To evaluate 16 – 4(2), follow the rules for order of operations and compute. $16 - 4(2) = 16 - 8 = 8$ To evaluate $6 + 3x$ when $x = 2$, substitute 2 for x and calculate. $6 + 3(2) = 6 + 6 = 12$ We use the word “simplify” when rewriting a number or an expression in a form more easily readable or understandable. To simplify $2x + 3 + 5x$, combine like terms: $2x + 3 + 5x = 7x + 3$. Sometimes it may not be clear what is the simplest form of an expression. For instance, by the distributive property, $4(x + 2) = 4x + 8$. For some applications, $4(x + 2)$ may be considered simpler than $4x + 8$, but for other applications, $4x + 8$ may be considered simpler than $4(x + 2)$.

Equivalent Expressions

Two numerical expressions are equivalent if they are equal.

> 2 + 4 and -2 + 8 are equivalent numerical expressions. They are both equal to 6.

Two mathematical expressions are equivalent if, for any possible substitution of values for the variables, the two resulting values are equal.

> The expressions $x + 2x$ and $4x – x$ are equivalent. For any value of the variable x, the expressions represent the same number. We see this by combining like terms.
>
> $$x + 2x = 3x \text{ and } 4x - x = 3x$$
>
> The expressions x^2 and $2x$ are NOT equivalent. While they happen to be equal if $x = 0$ or $x = 2$, they are not equal for all possible values of x. For instance, if $x = 1$, then $x^2 = 1$ and $2x = 2$.

Properties of arithmetic, such as the distributive property, can be used to write expressions in different, equivalent ways.

$$4x + 6x = (4 + 6)x \qquad 24x + 9x = 3(8x + 3x) = 3x(8 + 3)$$

Simplifying Expressions Using a Model

In mathematics, we simplify a numerical or algebraic expression by rewriting it in a less complicated form.

We can illustrate simplifying expressions using a cups and counters model.

Positive Counter	Negative Counter	Cup	Upside-down Cup
draw as: **+** value: +1	draw as: **–** value: -1	draw as: **V** value: unknown (x)	draw as: **Λ** value: unknown ($-x$)

Expressions	Pictures	Descriptions
$2(x + 3)$ $= 2x + 6$	**V + + +** **V + + +**	Build the expression. Think: 2 groups of $x + 3$, which is an application of the distributive property.
$-2(x + 3)$ $= -2x – 6$	**Λ – – –** **Λ – – –**	Build the expression. Think: 2 groups of $x + 3$ from above -> then build the opposite (distributive property).
$-2(x – 3)$ $= -2x + 6$	**Λ + + +** **Λ + + +**	Build the expression. Think: 2 groups of $x – 3$ -> then build the opposite (distributive property).
$4x – 2(x – 3)$ $= 4x – 2x + 6$ $= 2x + 6$	**V V Λ + + +** **V V Λ + + +**	Build the expression. Think: 4x AND 2 groups of $x – 3$ -> then build the opposite of the groups (distributive property). Then combine like terms (think zero pairs).