# STUDENT RESOURCES

| Word or Phrase | Definition |
|---|---|
| distributive property | The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers $a$, $b$, and $c$.<br><br>$3(4 + 5) = 3(4) + 3(5)$ and $(4 + 5)8 = 4(8) + 5(8)$ |
| exponential notation | The exponential notation $b^n$ (read as " $b$ to the power $n$ ") is used to express $n$ factors of $b$. The number $b$ is the base, and the number $n$ is the exponent.<br><br>$2^3 = 2 \bullet 2 \bullet 2 = 8$. The base is 2 and the exponent is 3.<br>$3^2 = 3 \bullet 3 = 9$. The base is 3 and the exponent is 2. |
| integers | The integers are the whole numbers and their opposites. They are the numbers 0, 1, 2, 3, … and -1, -2, -3, … . |
| inverse operation | The inverse operation to a mathematical operation reverses the effect of the operation.<br><br>Addition and subtraction are inverse operations.<br>Multiplication and division are inverse operations. |
| product | A product is the result of multiplying two or more numbers or expressions. The numbers or expressions being multiplied to form the product are factors of the product.<br><br>$\underset{\text{factor}}{3} \bullet \underset{\text{factor}}{5} = \underset{\text{product}}{15}$ |
| quotient | In a division problem, the quotient is the result of the division.<br><br>$\underset{\text{dividend}}{12} \div \underset{\text{divisor}}{3} = \underset{\text{quotient}}{4}$ |
| rational number | Rational numbers are numbers expressible in the form $\frac{m}{n}$, where $m$ and $n$ are integers, and $n \neq 0$.<br><br>$\frac{3}{5}$ is rational because it is a quotient of integers.<br><br>$2\frac{1}{3}$ and 0.7 are rational numbers because they **can be** expressed as quotients of integers, namely $\frac{7}{3}$ and $\frac{7}{10}$, respectively.<br>$\sqrt{2}$ and $\pi$ are NOT rational numbers. They cannot be expressed as a quotient of integers.<br>$\frac{7}{0}$ is undefined. It is NOT a rational number. |

## Symbols for Multiplication

The product of 8 and 4 can be written as:

8 times 4 $\quad 8 \times 4 \quad 8 \bullet 4 \quad (8)(4) \quad \begin{array}{r} 8 \\ \underline{\times\ 4} \end{array}$

The product of 8 and the variable $x$ is written simply as $8x$. We are cautious about using certain symbols for multiplication. The $\times$ could be misinterpreted as the variable $x$ and the $\bullet$ could be misinterpreted as a decimal point.

## Symbols for Division

The quotient of 8 and 4 can be written as:

8 divided by 4 $\quad 8 \div 4 \quad 4\overline{)8} \quad \dfrac{8}{4} \quad$ 8/4

In algebra, the preferred way to show division is with fraction notation.

## Mr. Mortimer's Magic Hot and Cold Cubes for Multiplication

Mr. Mortimer discovered an amazing way to control the temperature of liquid. He invented magic hot and cold cubes to change the liquid's temperature. These magic cubes never melt or change in any way. For example, ice cubes melt, but magic cold cubes do not.

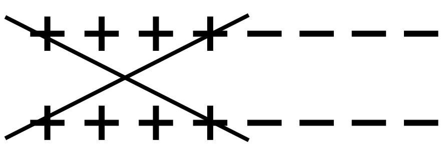
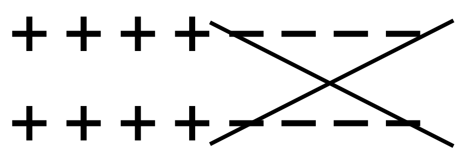
**Hot Cubes** (the basics):
- If you add 1 hot cube to a liquid, the liquid heats up by 1 degree.
- If you remove 1 hot cube from the liquid, the liquid cools down by 1 degree.

For multiplication:
- If you put in packs of hot cubes to a liquid, the liquid heats up.
  For example, adding 2 packs of 10 hot cubes is like adding 2 • 10 = 20 hot cubes.
  The liquid heats up by 20 degrees.
- If you take out packs of hot cubes from a liquid, the liquid cools down.
  For example, subtracting 2 packs of 10 hot cubes is like subtracting 2 • 10 = 20 hot cubes.
  The liquid cools down by 20 degrees.

**Cold Cubes** (the basics):
- If you add 1 cold cube to the liquid, the liquid cools down by 1 degree.
- If you remove 1 cold cube from the liquid, the liquid heats up by 1 degree.

For multiplication:
- If you put in packs of cold cubes to a liquid, the liquid cools down.
  For example, adding 2 packs of 10 cold cubes is like adding 2 • 10 = 20 cold cubes.
  The liquid cools down by 20 degrees.
- If you take out packs of cold cubes from a liquid, the liquid heats up.
  For example, subtracting 2 packs of 10 cold cubes is like subtracting 2 • 10 = 20 cold cubes.
  The liquid heats up by 20 degrees.

| Counter Multiplication Sentence Frames |
|---|
| • Begin with a workspace that has a value equal to 0.<br>• **If the first factor is positive**, we will place _____ groups on the workspace.<br>**If the first factor is negative**, we will remove _____ groups on the workspace.<br>• The second factor is ____________ (positive/negative), so each group will contain _____ ____________ (positive/negative) counter(s).<br>• Introduce _____ zero pairs to remove these groups (if needed).<br>• The result is _____ ____________ (positive/negative) counter(s). |

| Integer Multiplication Using Counters | |
|---|---|
| 2(4) = 8<br>+ + + +<br>+ + + +<br>• Start with a work space equal to zero.<br>• The first factor is positive.<br>We will put 2 groups on the workspace.<br>• The second factor is positive.<br>Each group will contain 4 positive counters.<br>• [No zero pairs needed.]<br>• The result is 8 positive counters. | 2(- 4) = - 8<br>− − − −<br>− − − −<br>• Start with a work space equal to zero.<br>• The first factor is positive.<br>We will put 2 groups on the workspace.<br>• The second factor is negative.<br>Each group will contain 4 negative counters.<br>• [No zero pairs needed.]<br>• The result is 8 negative counters |
| - 2(4) = -8<br><br>• Start with a work space equal to zero.<br>• The first factor is negative.<br>We will remove 2 groups from the workspace.<br>• The second factor is positive. Each group will contain 4 positive counters.<br>• Introduce at least 8 zero pairs.<br>• The result is 8 negative counters. | - 2(- 4) = 8<br><br>• Start with a work space equal to zero.<br>• The first factor is negative.<br>We will remove 2 groups from the workspace.<br>• The second factor is negative. Each group will contain 4 negative counters.<br>• Introduce at least 8 zero pairs.<br>• The result is 8 positive counters. |

## Rules for Multiplication of Integers

Rule 1: The product of two numbers with the same sign is a positive number.

Think: (+)(+) = (+) and (–)(–) = (+)

Rule 2: The product of two numbers with opposite signs is a negative number.

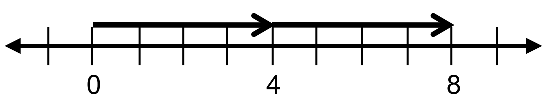
Think: (+)(–) = (–) and (–)(+) = (–)

## Multiplication on a Number Line

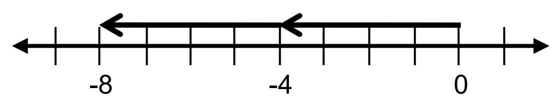
We can use arrows to represent multiplication on a number line. One interpretation for **multiplying** any two numbers is:

- The first factor tells us the number of arrows. The second factor tells us the length of each arrow.
- If the length of the arrow (second factor) is a positive number, then the arrow goes to the right. If the length of the arrow is a negative number, then the arrow goes to the left.
- If the number of arrows (first factor) is positive, then the number line diagram is complete. If the number of arrows is negative, then the entire diagram is reflected!

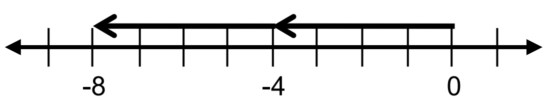
1. $2(4) = 8$



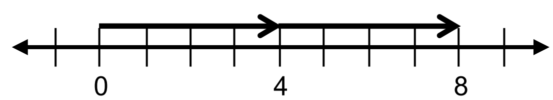

2. $2(-4) = -8$



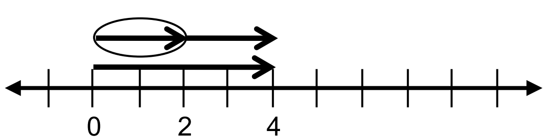

3. $-2(4) = -8$
(Example 1 reflected)



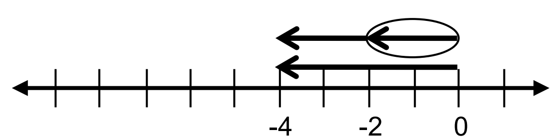

4. $-2(-4) = 8$
(Example 2 reflected)



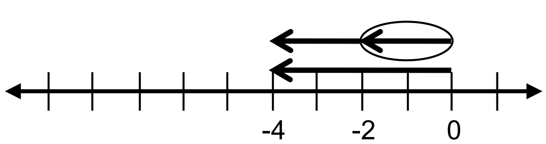

5. $\frac{1}{2}(4) = 2$



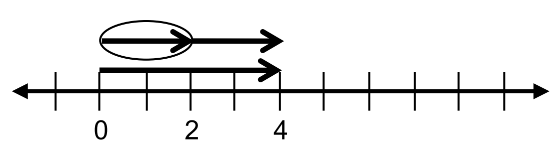

6. $\frac{1}{2}(-4) = -2$




7. $-\frac{1}{2}(4) = -2$
(Example 5 reflected)




8. $-\frac{1}{2}(-4) = 2$
(Example 6 reflected)



## Rules for Division of Integers

Rule 1: The quotient of two numbers with the same sign is a positive number.

Think: $\frac{(+)}{(+)} = (+)$ and $\frac{(-)}{(-)} = (+)$

Rule 2: The quotient of two numbers with opposite signs is a negative number.

Think: $\frac{(+)}{(-)} = (-)$ and $\frac{(-)}{(+)} = (-)$

## Mathematical Separators

Parentheses ( ) and square brackets [ ] are used in mathematical language as separators. The expression inside the parentheses or brackets is considered as a single unit. Operations are performed inside the parentheses before the expression inside the parentheses is combined with anything outside the parentheses.

$$5 - (2 + 1) = 5 - (3) = 2$$

In the example below, operate on the expression in the innermost separator first and work your way out.

$$20 \div [6 - (4 - 8)] = 20 \div [6 - (-4)] = 20 \div 10 = 2$$

The horizontal line used for a division problem is also a separator. It separates the expressions above and below the line, so the numerator and denominator must be simplified completely before dividing.

$$\frac{20 + 10}{5 \bullet 2} = \frac{30}{10} = 3$$

## Order of Operations

There are many mathematical conventions that enable us to interpret mathematical notation and to communicate efficiently about common situations. The agreed-upon rules for interpreting mathematical notation, important for simplifying arithmetic and algebraic expressions, are called the order of operations.

1. Do the operations in grouping symbols first (e.g., use rules 2 – 4 inside parentheses).
2. Calculate all the expressions with exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

$$\frac{11+(17-2\bullet 3^2)}{5} = \frac{11+(17-2\bullet 9)}{5} = \frac{11+(17-18)}{5} = \frac{11+(-1)}{5} = \frac{10}{5} = 2$$

There are many times for which these rules make complete sense and are quite natural. Take this case, for example:

You purchase 2 bottles of water for $1.50 each and 3 bags of peanuts for $1.25 each. Write an expression for this situation, and simplify the expression to find the total cost.

$$\underbrace{2\bullet(1.50)}_{3.00} + \underbrace{3\bullet(1.25)}_{3.75} = \$6.75$$

In this problem, it is natural to find the cost of the 2 bottles of water and then the cost of the 3 bags of peanuts prior to adding these amounts together. In other words, we perform the multiplication operations before the addition operation.

Note however that if we were to perform the operations in order from left to right (as we read the English language from left to right), we would obtain a different result:

**WRONG** → 2(1.50) = 3 → 3 + 3 = 6 → 6(1.25) = $7.50

| Using Order of Operations to Simplify Expressions | | |
|---|---|---|
| **Order of Operations** | **Example**<br><br>$\dfrac{40 - 2 \bullet 5^2 - (8 - 6)}{4 + 2 \bullet 10}$ | **Comments** |
| Simplify expressions within grouping symbols. | $\dfrac{40 - 2 \bullet 5^2 - 2}{4 + 2 \bullet 10}$ | Parentheses are grouping symbols:<br>(8 – 6) = 2<br><br>The fraction bar, used for division, is also a grouping symbol, so the numerator and denominator must be simplified completely prior to dividing. |
| Calculate all the expressions with exponents. | $\dfrac{40 - 2 \bullet 25 - 2}{4 + 2 \bullet 10}$ | $5^2 = 5 \bullet 5 = 25$ |
| Perform multiplication and division from left to right. | $\dfrac{40 - 50 - 2}{4 + 20}$ | In the numerator:<br>Multiply 2 • 25 = 50.<br><br>In the denominator:<br>Multiply 2 • 10 = 20. |
| Perform addition and subtraction from left to right. | $\dfrac{-12}{24}$ | In the numerator:<br>Subtract from left to right 40 – 50 – 2 = -12.<br><br>In the denominator:<br>Add 4 + 20 = 24 |
| | $\dfrac{-1}{2}$ or $-\dfrac{1}{2}$ | Now the groupings in both the numerator and denominator have been simplified, so the final division can be performed. |