STUDENT RESOURCES

Word or Phrase	Definition
decrease in a quantity	The decrease in a quantity is the original value minus the new value. The percent decrease in a quantity is the value of the ratio of the decrease to the original quantity, expressed as a percent. Last year, there were 200 students in the school. This year, there are 178 students in the school. The decrease in the number of students is 200 – 178 = 22. Since $\frac{22}{200} = \frac{11}{100}$, the percent decrease is 11%.
discount	The discount (or markdown) of an item is the decrease in the price of the item; that is, the original price of the item minus the new price. The percent discount is the percent decrease in the price of the item; that is, the value of the ratio of the decrease to the original value, expressed as a percent. Last week, the price of an MP3 player was \$200. This week, the price is \$178. The discount is 200 – 178 = 22. Since $\frac{22}{200} = \frac{11}{100}$, the percent discount is 11%.
increase in a quantity	The increase in a quantity is the new value minus the original value. The percent increase in a quantity is the value of the ratio of the increase to the original quantity, expressed as a percent. Last year there were 200 students in school. This year, there are 208 students. The increase in the number of students is 208 – 200 = 8. Since $\frac{8}{200} = \frac{4}{100}$, the percent increase is 4%.
markup	The markup on an item is the increase in the price of the item, that is, the new price of the item minus the original price. The percent markup is the percent increase in the price of the item. Last week, the price of an MP3 player was \$200. This week, the price is \$208. The markup is 208 – 200 = 8. Since $\frac{8}{200} = \frac{4}{100}$, the percent markup is 4%.
percent	A percent is a number expressed in terms of the unit $1\% = \frac{1}{100}$. To convert a positive number to a percent, multiply the number by 100. To convert a percent to a number, divide the percent by 100. $4 = 4 \times 100\% = 400\%$. Fifteen percent = $15\% = \frac{15}{100} = 0.15$.

Word or Phrase	Definition
percent decrease in a quantity	See <u>decrease in a quantity</u>.
percent increase in a quantity	See <u>increase in a quantity</u>.
percent of a number	A <u>percent of a number</u> is the product of the percent and the number. It represents the number of parts per 100 parts. 15% of 300 is $\frac{15}{100} \bullet 300 = 45$. If 45 out of 300 students are boys, then 15 out of every 100 students are boys, and 15% of the students are boys.
ratio	A <u>ratio</u> is a pair of positive numbers in a specific order. The ratio of a to b is denoted by $a : b$ (read "*a* to *b*," or "*a* for every *b*"). The ratio of 3 to 2 is denoted by 3 : 2. The ratio of dogs to cats is 3 to 2. There are 3 cups of water for every 2 cups of juice. The fraction $\frac{3}{2}$ does not represent this ratio, but it does represent the *value of the ratio* (or the <u>unit rate</u>).
scale	In a scale drawing of a figure, the <u>scale</u> is the ratio of lengths in the scale drawing to lengths in the actual figure. The blueprint of a house floorplan has a scale of 1 inch to 5 feet, or 1 in : 5 ft. Each inch on the blueprint represents 5 feet. The map has a scale of 3 centimeters to 10 kilometers, or 3 cm : 10 km. Each 3 centimeters on the map represents 10 kilometers.
scale drawing	A <u>scale drawing</u> of a geometric figure is a drawing in which all lengths have been multiplied by the same scale factor. A blueprint (drawing to scale) of a house floorplan is a scale drawing.
scale factor	A <u>scale factor</u> is a positive number which multiplies some quantity. To make a scale drawing of a figure, we multiply all lengths by the same scale factor. If the scale factor is greater than 1, the drawing is an enlargement, and if the scale factor is between 0 and 1, the drawing is a reduction.

Some Fraction-Decimal-Percent Equivalents		
$\frac{1}{2} = \frac{50}{100} = 0.5 = 50\%$ $\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$ $\frac{3}{4} = \frac{75}{100} = 0.75 = 75\%$ $\frac{5}{4} = \frac{125}{100} = 1.25 = 125\%$ Conversion strategy: Think: $\frac{3}{4}\left(\frac{25}{25}\right) = \frac{75}{100} = 75\%$	$\frac{1}{10} = \frac{10}{100} = 0.1 = 10\%$ $\frac{3}{10} = \frac{30}{100} = 0.3 = 30\%$ $\frac{5}{10} = \frac{50}{100} = 0.5 = 50\%$ Conversion strategy: Think: $\frac{3}{10} = \frac{30}{100}$, so 0.3 = 0.30 = 30%	$\frac{1}{25} = \frac{4}{100} = 0.04 = 4\%$ $\frac{16}{25} = \frac{64}{100} = 0.64 = 64\%$ $\frac{9}{50} = \frac{18}{100} = 0.18 = 18\%$ Conversion strategy: Think: 25(4) = 100, so $\frac{16}{25}\left(\frac{4}{4}\right) = \frac{64}{100} = 64\%$
$\frac{3}{20} = \frac{15}{100} = 0.15 = 15\%$ $\frac{13}{20} = \frac{65}{100} = 0.65 = 65\%$ $\frac{19}{20} = \frac{95}{100} = 0.95 = 95\%$ Conversion strategy: Think: 20 nickels in a dollar $\frac{1}{20}$ of a dollar is \$0.05	$\frac{1}{5} = \frac{2}{10} = 0.2 = 20\%$ $\frac{2}{5} = \frac{4}{10} = 0.4 = 40\%$ $\frac{3}{5} = \frac{6}{10} = 0.6 = 60\%$ $\frac{4}{5} = \frac{8}{10} = 0.8 = 80\%$ Conversion strategy: Think: If I know tenths, I can easily convert to hundredths.	$\frac{1}{8} = \frac{12.5}{100} = 0.125 = 12.5\%$ $\frac{3}{8} = \frac{37.5}{100} = 0.375 = 37.5\%$ $\frac{5}{8} = \frac{62.5}{100} = 0.625 = 62.5\%$ $\frac{7}{8} = \frac{87.5}{100} = 0.875 = 87.5\%$ Conversion strategy: Think: $\frac{1}{4} = \frac{25}{100}$, so half of $\frac{1}{4}$ is $\frac{1}{8} = \frac{12.5}{100}$ = 12.5%

Using “Chunking Strategies” to Find Percents of Numbers

We use the word “chunking” to describe a process of decomposing and composing numbers to make calculations easier, especially when done mentally. Another way to describe this is “taking numbers apart and putting them back together.” For example, if adding 17 and 26, we might decompose each number into tens and ones, adding 10 + 20 = 30, and 7 + 6 = 13, and finalizing the sum by adding 30 + 13 = 43.

Think	**Example**
Finding 100% of something is the same as finding all of it.	100% of \$80 = \$80 $\underbrace{\boxed{100\%}}_{\$80}$
Finding 50% of something is the same as finding half of it. This is the same as multiplying by $\frac{1}{2}$ or dividing by 2.	50% of \$80 = $\frac{1}{2}$(\$80) = \$40 \$80 ÷ 2 = \$40 $\underbrace{\boxed{50\%}\boxed{50\%}}_{\$80}$
Finding 25% of something is the same as finding one-fourth of it. This is the same as multiplying by $\frac{1}{4}$ or dividing by 4.	25% of \$80 = $\frac{1}{4}$(\$80) = \$20 \$80 ÷ 4 = \$20 $\underbrace{\boxed{25\%}\boxed{25\%}\boxed{25\%}\boxed{25\%}}_{\$80}$
Finding 10% of something is the same as finding one-tenth of it. This is the same as multiplying by $\frac{1}{10}$ or dividing by 10.	10% of \$80 = $\frac{1}{10}$(\$80) = \$8 \$80 ÷ 10 = \$8
Finding 1% of something is the same as finding one-hundredth of it. This is the same as multiplying by $\frac{1}{100}$ or dividing by 100.	1% of \$80 = $\frac{1}{100}$(\$80) = \$0.80 \$80 ÷ 100 = \$0.80
Finding 20% of something is the same as doubling 10% of it.	20% of \$80 = 2(\$8) = \$16
Finding 5% of something is the same as halving 10% of it.	5% of \$80 = $\frac{1}{2}$(\$8) = \$4
Finding 15% of something is the same as adding 10% of it and 5% of it.	15% of \$80 = \$8 + \$4 = \$12

Using Multiplication to Find Percents of Numbers
Some percents are hard to find mentally. For example, finding 17% of something is the same as finding $\frac{17}{100}$ = 0.17 of it. In this case, it may be easier to find the percent by using the definition of a percent of a number: A percent of a number is the product of the percent and the number.

Find 17% of \$80.

Strategy 1: **Use fractions**

$$\frac{17}{100} \bullet 80 = \frac{17 \bullet 80}{100} = \frac{1360}{100} = 13.60$$

So 17% of \$80 is \$13.60.

Strategy 2: **Use decimals**

$(0.17) \bullet (80) = 13.6$ or 13.60
So 17% of \$80 is \$13.60.

Percent Increase

Percent increases occur frequently as tips, taxes, and price markups. To find a percent increase, find the amount of the increase and add it to the original quantity.

Example	**Original amount**	**Percent increase**	**Amount of increase**	**New amount** (original + increase)
Leave a **tip** on a restaurant bill.	\$40	20%	20% of \$40 = \$8	\$40 + \$8 = \$48
Pay **tax** on a clothes purchase.	\$50	8%	8% of \$50 = \$4	\$50 + \$4 = \$54
Pay a **markup** on a video game.	\$75	10%	10% of \$75 = \$7.50	\$75 + \$7.50 = \$82.50

Percent Decrease

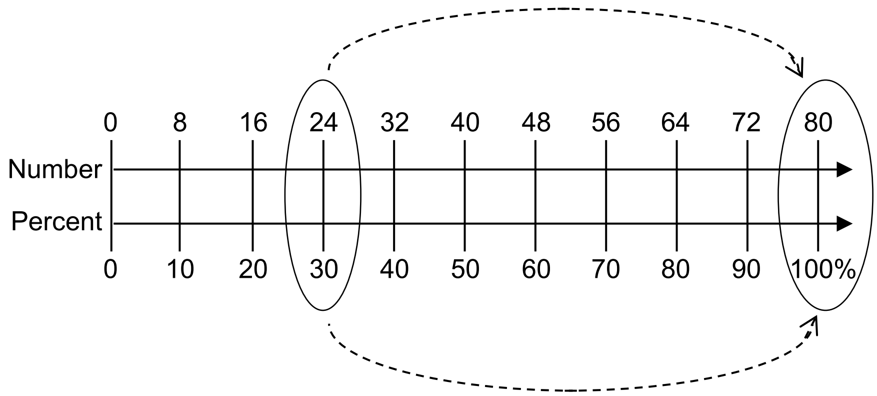
Percent decreases occur frequently as sales and discounts. To find a percent decrease, find the amount of the decrease and subtract it from the original quantity.

Example	**Original amount**	**Percent decrease**	**Amount of decrease**	**New amount** (original – decrease)
Sale on shoes purchase	\$50	25%	25% of \$50 = \$12.50	\$50 – \$12.50 = \$37.50
Discount on a dress	\$90	40%	40% of 90 = \$36.00	\$90 – \$36 = \$54

Using Double Number Lines to Solve a Percent Problem:
30% of 80 is what amount?

Strategy 1: **Solve on the double number line**

Create a double number line with percents represented in increments of 10% on the bottom line, and the whole number represented in increments on the top. Since the whole is 80 (in this case), count by 8s for the increments (80 ÷ 10 = 8).



Since 30% corresponds to 24 on the double number line, 30% of 80 is 24.

Strategy 2: **Identify equivalent ratios on the double number line.**

Create equations based on the part to whole ratio relationships.

$$\frac{\text{part}_{number}}{\text{whole}_{number}} = \frac{\text{part}_{percent}}{\text{whole}_{percent}}$$

$$\frac{\boxed{24}}{80} = \frac{30}{100}$$

This equivalence is based on the dotted arrows above.

Create equations based on the part to part ratio relationships.

$$\frac{\text{part}_{number}}{\text{part}_{percent}} = \frac{\text{whole}_{number}}{\text{whole}_{percent}}$$

$$\frac{\boxed{24}}{30} = \frac{80}{100}$$

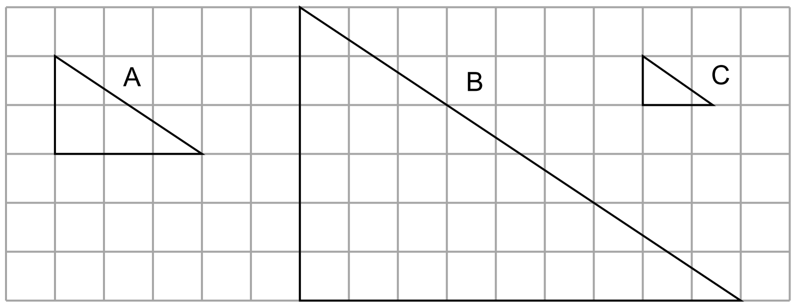
This equivalence is based on the circles above.

Scale Factors

Consider triangle A as the original figure.

To make Triangle B below, multiply each dimension of Triangle A by a scale factor of 3. Triangle B is a 300% enlargement of Triangle A. An enlargement is created when multiplying by a scale factor greater than 1.

To make Triangle C below, multiply each dimension of Triangle A by a scale factor of $\frac{1}{2}$. Triangle C is a 50% reduction of Triangle A. A reduction is created when multiplying by a scale factor between 0 and 1.



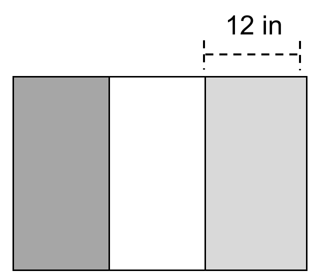
Scale Drawings

The flag of Italy is composed of three stripes (green, white, and red) that divide the flag into thirds. Pictured below is a scale drawing of the flag.

Suppose the original flag is 3 feet by 2 feet, and the scale drawing is 1.5 inches by 1 inch.

This scale may be represented as a ratio:

$$\begin{array}{lclcl} 1.5 \text{ in} & : & 3 \text{ ft} & \rightarrow & 1.5 \text{ in} : 36 \text{ in} \\ 1 \text{ in} & : & 2 \text{ ft} & \rightarrow & 1 \text{ in} : 24 \text{ in} \\ & & & & 1 : 24 \end{array}$$



The scale drawing is a reduction of the flag. The scale factor (value of the ratio) that produces this reduction is $\frac{1}{24}$. In other words, to obtain lengths for the drawing, multiplying the corresponding actual lengths by $\frac{1}{24}$.