STUDENT RESOURCES

Word or Phrase	Definition
distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers *a*, *b*, and *c*. $3(4 + 5) = 3(4) + 3(5)$ and $(4 + 5)8 = 4(8) + 5(8)$
equation	An equation is a mathematical statement that asserts the equality of two expressions. $18 = 8 + 10$ is an equation that involves only numbers. This is a numerical equation. $18 = x + 10$ is an equation that involves numbers and a variable and $y = x + 10$ is an equation that involves a number and two variables. These are both algebraic (variable) equations.
evaluate	Evaluate refers to finding a numerical value. To evaluate an expression, replace each variable in the expression with a value and then calculate the value of the expression. To evaluate the numerical expression $3 + 4(5)$, we calculate $3 + 4(5) = 3 + 20 = 23$. To evaluate the variable expression $2x + 5$ when $x = 10$, we calculate $2x + 5 = 2(10) + 5 = 20 + 5 = 25$.
equivalent expressions	Two mathematical expressions are equivalent if, for any possible substitution of values for the variables, the two resulting numbers are equal. In particular, two numerical expressions are equivalent if they represent the same number. See expression. The numerical expressions $3 + 2$ and $9 – 4$ are equivalent, since both are equal to 5. The algebraic expressions $3(x + 4)$ and $3x + 12$ are equivalent. For any value of the variable *x*, the expressions represent the same number.
expression	A mathematical expression is a combination of numbers, variables, and operation symbols. When values are assigned to the variables, an expression represents a number. Some mathematical expressions are 19, $7x$, $a + b$, $\frac{8 + x}{10}$, and $4v – w$.
inequality	An inequality is a mathematical statement that asserts the relative size or order of two objects. When the expressions involve variables, a solution to the inequality consists of values for the variables which, when substituted, make the inequality true. $5 > 3$ is an inequality. $x + 3 > 4$ is an inequality. All values for *x* that are greater than 1 are solutions to this inequality.

Word or Phrase	Definition
simplify	Simplify refers to converting a numerical or variable expression to a simpler form. A variable expression might be simplified by combining like terms. A fraction might be simplified by dividing numerator and denominator by a common divisor. $\frac{8}{12}$ may be simplified to the equivalent numerical expression $\frac{2}{3}$. $2x + 6 + 5x + 3$ may be simplified to the equivalent variable expression $7x + 9$.
solution to an equation	A solution to an equation involving variables consists of values for the variables which, when substituted, make the equation true. The value $x = 8$ is a solution to the equation $10 + x = 18$. If we substitute 8 for x in the equation, the equation becomes true: $10 + 8 = 18$.
solve an equation	To solve an equation refers to finding all values for the variables in the equation that, when substituted, make the equation true. Values that make an equation true are called solutions to the equation. To solve the equation $2x = 6$, one might think "two times what number is equal to 6?" Since $2(3) = 6$, the only value for x that satisfies this condition is 3. Therefore 3 is the solution.
substitution	Substitution refers to replacing a value or quantity with an equivalent value or quantity. If $x + y = 10$, and $y = 8$, then we may substitute this value for y in the equation to get $x + 8 = 10$.
tape diagram	A tape diagram is a graphical representation that uses length to represent relationships between quantities. We draw rectangles with a common width to represent quantities, and rectangles with the same length to represent equal quantities. Tape diagrams are typically used to represent quantities expressed in the same unit. This tape diagram represents a drink mixture with 3 parts grape juice for every 2 parts water. G G G W W

Variables in Algebra

Loosely speaking, variables are quantities that can vary. Variables are represented by letters or symbols. Variables have many different uses in mathematics. The use of variables, together with the rules of arithmetic, makes algebra a powerful tool. Three important ways that variables appear in algebra are the following.

Usage	**Examples**
Variables can represent an *unknown quantity* in an equation or inequality. In this case, the equation or inequality is valid only for specific value(s) of the variable.	$x + 4 = 9$ $5n = 20$ $y < 6$
Variables can represent *quantities that vary* in a relationship. In this case, there is always more than one variable in the equation.	Formula: $P = 2\ell + 2w$, $A = s^2$ Function (input-output rule): $y = 5x$, $y = x + 3$
Variables can represent *quantities in statements that generalize* rules of arithmetic. In this case, there may be one or more variables.	Commutative property of addition: $x + y = y + x$ Distributive property: $x(y + z) = xy + xz$

Using Shapes to Represent Variables

If the same shape (variable) is used more than once in an equation, it must represent the same value each place it appears. Two different shapes (variables) in an equation may represent the same value or different values.

This is allowed	This is allowed	This is NOT allowed
□ + □ = ○	□ + ⬯ = ◇	○ + ○ = △
7 + 7 = 14	6 + 6 = 12	6 + 4 = 10

Evaluate or Simplify?

We use the word "evaluate" when we want to calculate the value of an expression.

To evaluate 16 – 4(2), follow the rules for order of operations and compute: 16 – 4(2) = 16 – 8 = 8.

To evaluate $6 + 3x$ when $x = 2$, substitute 2 for x and calculate: 6 + 3(2) = 6 + 6 = 12.

We use the word "simplify" when rewriting a number or an expression in a form more easily readable or understandable.

To simplify $2x + 3 + 5x$, combine like terms: $2x + 3 + 5x = 7x + 3$.

Sometimes it may not be clear what is the simplest form of an expression. For instance, by the distributive property, $4(x + 2) = 4x + 8$. For some applications, $4(x + 2)$ may be considered simpler than $4x + 8$, but for other applications, $4x + 8$ may be considered simpler than $4(x + 2)$.

How to Determine if an Equation is True

THE PIZZA PLACE MENU
(The variable represents the cost of an item.)

Pizza		**Drinks**	
Cheese slice (c)	\$1.00	Small drink (s)	\$0.95
Pepperoni slice (p)	\$1.25	Large drink (L)	\$1.75

What value from the menu above makes this equation true?

$p + \square = 3c$

Substitute: $1.25 + \square = 3(1.00)$

1.25 + 1.00 = 3.00 ? NO

1.25 + 1.25 = 3.00 ? NO

1.25 + 0.95 = 3.00 ? NO

1.25 + 1.75 = 3.00 ? YES

The equation is true when $\square$ represents the cost of a large drink (L = 1.75).

Solving Equations Using a Mental Math and Substitution Strategy

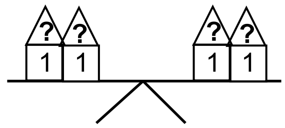
To solve an equation using mental math and substitution, apply your knowledge of arithmetic facts to find a value for the unknown that makes the equation true.

Example 1: $8x = 40$ Think: *8 times what number is 40?* Since $8(5) = 40$, $x = 5$ Check: $8(5) = 40$ $40 = 40$	Example 2: $8 + h = 20$ Think: *8 plus what number equals 20?* Since $8 + 12 = 20$, $h = 12$ Check: $8 + 12 = 20$ $20 = 20$
Example 3: $4 = 12 - k$ Think: *4 is equal to 12 minus what number?* Since $4 = 12 - 8$, $k = 8$ Check: $4 = 12 - 8$ $4 = 4$	Example 4: $\frac{n}{3} = 8$ Think: *What number divided by 3 is 8?* Since $\frac{24}{3} = 8$, $n = 24$ Check: $\frac{24}{3} = 8$ $8 = 8$

Balance Scales and Laws of Equality

Balance scales are physical representations of equations because both sides of a balanced scale must have the same weight, and both sides of an equation must have the same value.

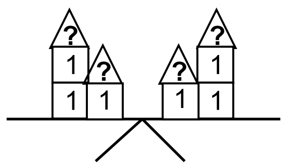
Imagine that each [1] represents one unit of weight and each [?] represents an unknown weight (not equal to zero). To represent unknowns, a popular variable is *x*.



The balanced scale above represents the equation $2x + 2 = 2x + 2$.

Example 1: Start with the balance scale above. Add the same thing to both sides, like 1.

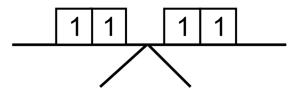
New scale:
(still balanced)



New equation: $2x + 2 + 1 = 2x + 2 + 1$
$2x + 3 = 2x + 3$

Example 2: Start with the balance scale above. Subtract the same thing from both sides, like $2x$.

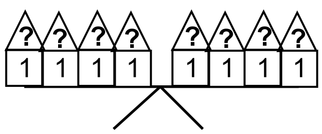
New scale:
(still balanced)



New equation: $2x + 2 - 2x = 2x + 2 - 2x$
$2 = 2$

Example 3: Multiply both sides by the same thing, like 2.

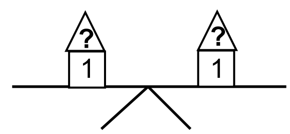
New scale:
(still balanced)



New equation: $2(2x + 2) = 2(2x + 2)$
$4x + 4 = 4x + 4$

Example 4: Divide both sides by the same thing, like 2. Here we are halving the weight on each side.

New scale:
(still balanced)



New equation: $\frac{2x+2}{2} = \frac{2x+2}{2}$
$x + 1 = x + 1$

The <u>addition property of equality</u> states that if $a = b$ and $c = d$, then $a + c = b + d$. In other words, equals added to equals are equal. (See example 1 above.)

Note that this property extends to subtraction as well. (See example 2 above.)

The <u>multiplication property of equality</u> states that if a = b and c = d, then ac = bd. In other words, equals multiplied by equals are equal. (See example 3 above.)

Note that this property extends to division as well. (See example 4 above.)