STUDENT RESOURCES

Word or Phrase	Definition
equivalent fractions	The fractions $\frac{a}{b}$ and $\frac{c}{d}$ are <u>equivalent</u> if they represent the same point on the number line. This occurs if the results of the division problems $a \div b$ and $c \div d$ are equal. Since $\frac{1}{2} = 1 \div 2 = 0.5$ and $\frac{2}{4} = 2 \div 4 = 0.5$, the fractions $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent.
multiplication property of 1	The <u>multiplication property of 1</u> states that $a \bullet 1 = 1 \bullet a = a$ for all numbers a. In other words, 1 is a <u>multiplicative identity</u>. The multiplication property of 1 is sometimes called the <u>multiplicative identity property</u>. $4 \bullet 1 = 4$, $1 \bullet 25 = 25$, $\frac{1}{2} \bullet \frac{4}{4} = \frac{4}{8}$ In the third equation above, since we are multiplying by 1 in the form of $\frac{4}{4}$, we refer to it as the Big 1.
percent	A <u>percent</u> is a number expressed in terms of the unit $1\% = \frac{1}{100} = 0.01$. Similarly, $p\% = \frac{p}{100} = p(0.01)$. One way to convert a number to a percent is to multiply the number by 1 in the form of 100%. $4 = 4 \times 100\% = 400\%$; $0.6 = 0.6 \times 100\% = 60\%$ One way to convert a percent to a number is to express $p\%$ as p hundredths. The fraction may be converted to a decimal by dividing. $15\% = \frac{15}{100} = 0.15$; $40\% = \frac{40}{100} = 0.40 = 0.4$.
percent of a number	A <u>percent of a number</u> is the product of the percent and the number. It represents the number of parts per 100 parts. 15% of 300 is $\frac{15}{100} \bullet 300 = 45$, or $(0.15)(300) = 45$. If 45 out of 300 students are boys, then 15 out of every 100 students are boys, and 15% of the students are boys.
ratio	A <u>ratio</u> is a pair of positive numbers in a specific order. The ratio of a to b is denoted by $a : b$ (read "*a* to *b*," or "*a* for every *b*").

Equivalent Fractions: The Big 1

The number 1 is called the multiplicative identity. Multiplying a fraction by any form of 1 does not change its value.

The Big 1 is a notation for 1 in the form of a fraction $\frac{n}{n}$ ($n \neq 0$). For example,

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \ldots$$

We can use the following picture to help remind us that these fractions are equivalent to 1:

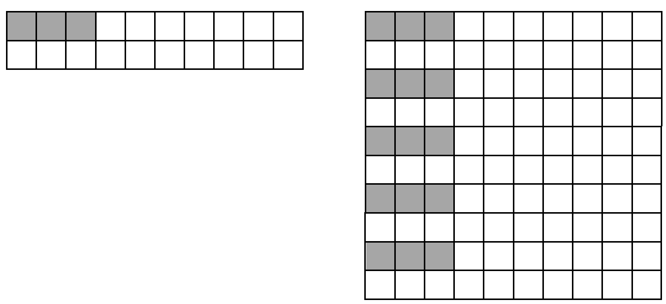
$$1 = \frac{8}{8}$$

The Big 1 can be used to show equivalence of fractions. For example,

$$\frac{2}{5} \times \frac{10}{10} = \frac{20}{50} \quad \text{or} \quad \frac{20}{50} \div \frac{10}{10} = \frac{2}{5}.$$

Equivalent Fractions

The diagrams below illustrate that $\frac{3}{20} = \frac{15}{100}$. In the second diagram, the pattern is repeated five times. The fractional part remains the same as the size of the whole changes.



$$\frac{3}{20} = \frac{15}{100}$$

Using the Big 1, this equivalence can be written:

$$\frac{3}{20} \bullet \frac{5}{5} = \frac{15}{100}.$$

Visually, multiplying the numerator by 5 represents repeating the shaded parts five times, and multiplying the denominator by 5 represents repeating the total number of parts in the denominator five times.

With this process, the size of the part does not change.

Some Fraction-Decimal-Percent Equivalents		
$\frac{1}{2} = \frac{50}{100} = 0.5 = 50\%$ $\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$ $\frac{3}{4} = \frac{75}{100} = 0.75 = 75\%$ $\frac{5}{4} = \frac{125}{100} = 1.25 = 125\%$ Conversion strategy: Think: $\frac{3}{4}\left(\frac{25}{25}\right) = \frac{75}{100} = 75\%$	$\frac{1}{10} = \frac{10}{100} = 0.1 = 10\%$ $\frac{3}{10} = \frac{30}{100} = 0.3 = 30\%$ $\frac{5}{10} = \frac{50}{100} = 0.5 = 50\%$ Conversion strategy: Think: $\frac{3}{10} = \frac{30}{100}$, so $0.3 = 0.30 = 30\%$	$\frac{1}{25} = \frac{4}{100} = 0.04 = 4\%$ $\frac{16}{25} = \frac{64}{100} = 0.64 = 64\%$ $\frac{9}{50} = \frac{18}{100} = 0.18 = 18\%$ Conversion strategy: Think: 25(4) = 100, so $\frac{16}{25}\left(\frac{4}{4}\right) = \frac{64}{100} = 64\%$
$\frac{3}{20} = \frac{15}{100} = 0.15 = 15\%$ $\frac{13}{20} = \frac{65}{100} = 0.65 = 65\%$ $\frac{19}{20} = \frac{95}{100} = 0.95 = 95\%$ Conversion strategy: Think: 20 nickels in a dollar $\frac{1}{20}$ of a dollar is \$0.05	$\frac{1}{5} = \frac{2}{10} = 0.2 = 20\%$ $\frac{2}{5} = \frac{4}{10} = 0.4 = 40\%$ $\frac{3}{5} = \frac{6}{10} = 0.6 = 60\%$ $\frac{4}{5} = \frac{8}{10} = 0.8 = 80\%$ Conversion strategy: Think: If I know tenths, I can easily convert to hundredths.	$\frac{1}{8} = \frac{12.5}{100} = 0.125 = 12.5\%$ $\frac{3}{8} = \frac{37.5}{100} = 0.375 = 37.5\%$ $\frac{5}{8} = \frac{62.5}{100} = 0.625 = 62.5\%$ $\frac{7}{8} = \frac{87.5}{100} = 0.875 = 87.5\%$ Conversion strategy: Think: $\frac{1}{4} = \frac{25}{100}$, so half of $\frac{1}{4}$ is $\frac{1}{8} = \frac{12.5}{100}$ $= 12.5\%$

Connecting Multiplication and Division to Percent of a Number	
Think	**Example**
Finding 100% of something is the same as finding all of it.	100% of \$80 = \$80 100% \$80
Finding 50% of something is the same as finding one-half of it. This is the same as multiplying by $\frac{1}{2}$ or dividing by 2.	50% of \$80 = $\frac{1}{2}$(\$80) = \$40 \$80 ÷ 2 = \$40 50% | 50% \$80
Finding 25% of something is the same as finding one-fourth of it. This is the same as multiplying by $\frac{1}{4}$ or dividing by 4.	25% of \$80 = $\frac{1}{4}$(\$80) = \$20 \$80 ÷ 4 = \$20 25% | 25% | 25% | 25% \$80
Finding 10% of something is the same as finding one-tenth of it. This is the same as multiplying by $\frac{1}{10}$ or dividing by 10.	10% of \$80 = $\frac{1}{10}$(\$80) = \$8 \$80 ÷ 10 = \$8
Finding 1% of something is the same as finding one-hundredth of it. This is the same as multiplying by $\frac{1}{100}$ or dividing by 100.	1% of \$80 = $\frac{1}{100}$(\$80) = \$0.80 \$80 ÷ 100 = \$0.80
Finding 20% of something is the same as doubling 10% of it.	20% of \$80 = 2(\$8) = \$16
Finding 5% of something is the same halving 10% of it.	5% of \$80 = $\frac{1}{2}$(\$8) = \$4
Finding 15% of something is the same as adding 10% of it and 5% of it.	15% of \$80 = \$8 + \$4 = \$12

Using Chunking to Find a Percent of a Number

We use the word “chunking” to describe a process of decomposing and composing numbers to make calculations easier, especially when done mentally. Another way to describe this is “taking numbers apart and putting them back together.” For example, if adding 17 and 26, we might decompose each number into tens and ones, adding 10 + 20 = 30, and 7 + 6 = 13, and finalizing the sum by adding 30 + 13 = 43.

Longer method (applying the distributive property)	**Shorter method** (mostly done mentally)
20% of 60 = (10% + 10%) of 60 = 10% of 60 + 10% of 60 = 6 + 6 = 12	20% of 60 10% → 6 10% → 6 20% → 12 Note that given a number representing the whole, we use an arrow to efficiently show the percent of the number using mental math and chunking.
15% of 60 = (10% + 5%) of 60 = 10% of 60 + 5% of 60 = 6 + 3 = 9	15% of 60 10% → 6 5% → 3 15% → 9

Using Multiplication to Find a Percent of a Number

Some percent values are hard to find mentally. For example, finding 17% of something is the same as finding $\frac{17}{100}$ = 0.17 of it. In this case, it may be easier to find the percent by using the definition of a percent of a number:

> A <u>percent of a number</u> is the product of the percent and the number.

Find 17% of $80.

Strategy 1: **Use fractions**

$$\frac{17}{100} \bullet 80 = \frac{17 \bullet 80}{100} = \frac{1360}{100} = 13.60$$

So 17% of $80 is $13.60.

Strategy 2: **Use decimals**

(0.17) • (80) = 13.6 or 13.60

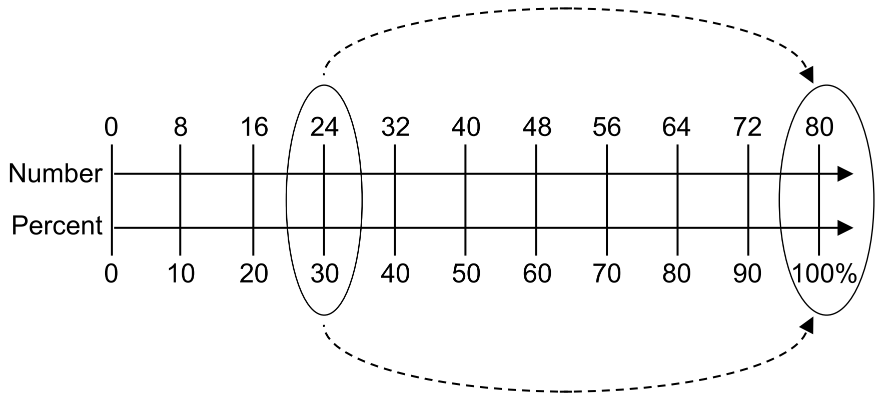
So 17% of $80 is $13.60.

Using Double Number Lines to Solve Percent Problems

Strategy 1: Solve on the double number line

30% of 80 is what amount?

Create a double number line with percents represented in increments of 10% on the bottom line, and the whole number represented in equal increments on the top. Since the whole is 80 (in this case), count by 8s for the equal increments (80 ÷ 10 = 8).



Since 30% corresponds to 24 on the double number line, 30% of 80 is 24.

Strategy 2: Identify equivalent ratios on the double number line and create equivalent fractions.

Create equivalent fractions based on the part-to-whole relationships.

$$\frac{\text{part}_{number}}{\text{whole}_{number}} = \frac{\text{part}_{percent}}{\text{whole}_{percent}}$$

$$\frac{\boxed{24}}{80} = \frac{30}{100}$$

This equivalence is based on the dotted arrows above.

Create equivalent fractions based on the part-to-part relationships.

$$\frac{\text{part}_{number}}{\text{part}_{percent}} = \frac{\text{whole}_{number}}{\text{whole}_{percent}}$$

$$\frac{\boxed{24}}{30} = \frac{80}{100}$$

This equivalence is based on the circles above.

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT	
6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations:
c.	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
6.SP.A	**Develop understanding of statistical variability.**
6.SP.2	Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
6.SP.B	**Summarize and describe distributions.**
6.SP.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
6.SP.5	Summarize numerical data sets in relation to their context, such as by:
c.	giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d.	relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
6.NS.B	**Compute fluently with multi-digit numbers and find common factors and multiples.**
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

STANDARDS FOR MATHEMATICAL PRACTICE	
SMP1	Make sense of problems and persevere in solving them.
SMP2	Reason abstractly and quantitatively.
SMP3	Construct viable arguments and critique the reasoning of others.
SMP4	Model with mathematics.
SMP5	Use appropriate tools strategically.
SMP6	Attend to precision.
SMP7	Look for and make use of structure.
SMP8	Look for and express regularity in repeated reasoning.

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