STUDENT RESOURCES

Word or Phrase	Definition
conjecture	A conjecture is a statement that is proposed to be true, but has neither been proven to be true nor to be false.
dividend	In a division problem, the dividend is the number being divided. In $12 \div 3 = 4$, the dividend is 12. dividend ÷ divisor = quotient
divisor	In a division problem, the divisor is the number by which another is divided. In $12 \div 3 = 4$, the divisor is 3. $\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$
multiplication property of 1	The multiplication property of 1 states that $a \bullet 1 = 1 \bullet a = a$ for all numbers a. In other words, 1 is a multiplicative identity. The multiplicative property of 1 is sometimes called the multiplicative identity property. $4 \bullet 1 = 4$, $1 \bullet (\frac{3}{8}) = \frac{3}{8}$, $\frac{3}{4} \bullet \frac{5}{5} = \frac{15}{20} = \frac{3}{4}$
quotient	In a division problem, the quotient is the result of the division. In $12 \div 3 = 4$, the quotient is 4. $\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$
reciprocal	For $b \neq 0$, the reciprocal of b is the number, denoted by $\frac{1}{b}$, that satisfies $b \bullet \frac{1}{b} = 1$. The reciprocal of b is also called the multiplicative inverse of b. The reciprocal of 3 is $\frac{1}{3}$. The reciprocal of $\frac{1}{6}$ is 6. The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.
unit rate	The unit rate associated to a ratio $a : b$, where a and b have units attached, is the number $\frac{a}{b}$, with the units “a-units per b-unit” attached. The ratio of 400 miles for every 8 hours corresponds to the unit rate 50 miles per hour.

Notation for Division

The quotient of 8 and 4 can be written as:

8 divided by 4 $8 \div 4$ $4\overline{)8}$ $\frac{8}{4}$ 8/4

In algebra, the preferred way to show division is with fraction notation.

A Chunking Division Procedure

This chunking division procedure keeps the dividend intact as we “close in” on the quotient. If you do not know all your multiplication facts, this procedure may be easier than the standard division algorithm because you subtract out groups of the divisor more flexibly, but still arrive at the correct quotient. If the largest amount possible is chosen to subtract at each step, this procedure is very efficient.

Divide 761 highlighters into 3 boxes.

Step 1: Rewrite problem

$$\begin{array}{r|l} 3\overline{)761} & \\ & \\ & \end{array}$$

Step 2: Make a Multiplication Bank that may be useful for this problem.

3 × 1 = 3	3 × 10 = 30	3 × 100 = 300
3 × 2 = 6	3 × 20 = 60	3 × 200 = 600
3 × 3 = 9	3 × 30 = 90	3 × 300 = 900
3 × 4 = 12	3 × 40 = 120	3 × 400 = 1200

Step 3: Select a fact from the Multiplication Bank that is less than or equal to the dividend, and record. Continue the routine until the remainder is less than the divisor.

$$\begin{array}{r|l} 3\overline{)\ 761} & \\ \underline{-600} & 200 \\ 161 & \end{array} \qquad \begin{array}{r|l} 3\overline{)\ 761} & \\ \underline{-600} & 200 \\ 161 & \\ \underline{-120} & 40 \\ 41 & \end{array} \qquad \begin{array}{r|l} 3\overline{)\ 761} & \\ \underline{-600} & 200 \\ 161 & \\ \underline{-120} & 40 \\ 41 & \\ \underline{-\ \ 30} & 10 \\ 11 & \end{array} \qquad \begin{array}{r|l} 253 & \text{R } 2 \\ 3\overline{)\ 761} & \\ \underline{-600} & 200 \\ 161 & \\ \underline{-120} & 40 \\ 41 & \\ \underline{-\ \ 30} & 10 \\ 11 & \\ \underline{-\ \ \ 9} & \underline{+\ \ 3} \\ 2 & 253 \end{array}$$

The last calculation shows that the quotient is (200 + 40 + 10 + 3) = 253, and the remainder is 2.

The Standard Division Algorithm for Whole Numbers

The standard division algorithm is an efficient process for dividing. It involves a cyclical process: divide, multiply, subtract, "bring down"… until the remainder is less than the divisor.

$14\overline{)\underline{96}3}$	Determine where to start	Look at the divisor. Choose digits in the dividend so that the quotient using these digits is between 1 and 9.
$\begin{array}{r} 6 \\ 14\overline{)963} \end{array}$	Divide	How many 14s in 96? Write this number above the 96. Place value reminder: The 96 in the dividend represents 960. The 6 in the quotient represents 60.
$\begin{array}{r} 6 \\ 14\overline{)963} \\ \underline{-84} \end{array}$	Multiply	Find the product of 6 and 14. Write this below the 96. Place value reminder: 6 × 14 = 84 is compact notation for 60 × 14 = 840.
$\begin{array}{r} 6 \\ 14\overline{)963} \\ \underline{-84} \\ 12 \end{array}$	Subtract	Find the difference between 96 and 84. Write this below the 84. Place value reminder: 96 – 80 = 12 is compact notation for 960 – 840 = 120.
$\begin{array}{r} 6 \\ 14\overline{)963} \\ \underline{-84}\downarrow \\ 123 \end{array}$	Bring down	Bring down the next digit.
$\begin{array}{r} 68 \\ 14\overline{)963} \\ \underline{-84}\downarrow \\ 123 \\ \underline{-112} \\ 11 \end{array}$	Divide Multiply Subtract Bring down (remainder)	Repeat the divide, multiply, subtract, bring down (if necessary) process until the remainder is less than the divisor.

Some ways to represent the dividend, divisor, quotient, and remainder:

$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$ remainder

dividend = (divisor)(quotient) + remainder

$\begin{array}{r} 68 \text{ R11} \\ 14\overline{)963}\phantom{\text{ R11}} \end{array}$

$\begin{array}{r} 68\frac{11}{14} \\ 14\overline{)963}\phantom{\frac{11}{14}} \end{array}$

963 = (14)(68) + 11

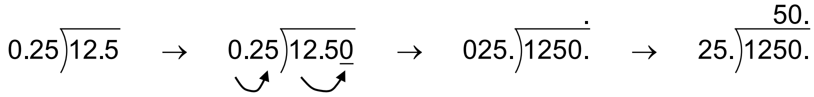
Why Do We Move the Decimal Point when Dividing Decimals?

The procedure for dividing decimals involves "moving the decimal point." The reason this is done is because we usually consider dividing by a whole number to be an easier process.

Consider $12.5 \div 0.25$, which can be written as $0.25\overline{)12.5}$ or $\frac{12.5}{0.25}$.

Since $12.5 \div 0.25$ may be multiplied by 1 in the form of $\boxed{\frac{100}{100}}$, it is equal to $\frac{12.5}{0.25} \bullet \frac{100}{100} = 1{,}250 \div 25$.

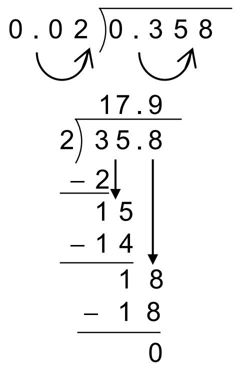
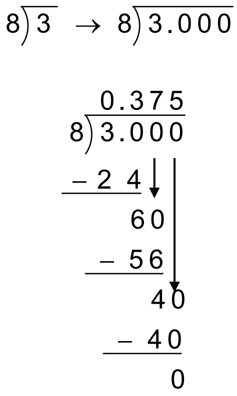
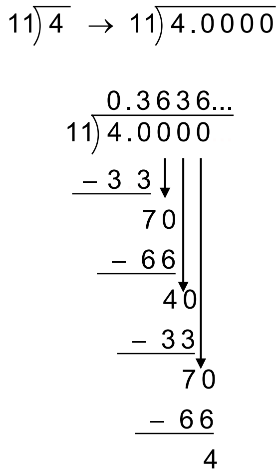
Now we can divide by a whole number. This process often is depicted this way:



Division of Decimals: Examples

- Multiply the divisor and dividend by the same power of 10 (10, 100, 1000, etc.) so that the divisor is a whole number.
- Divide as usual, lining up the digits of the quotient above the dividend so that the tens line up with tens, ones with ones, tenths with tenths, and so on. Place the decimal in the quotient in the same location as the dividend.

To obtain more decimal place accuracy, attach zeroes to the right of the final place in the decimal part and continue dividing until the remainder is zero (example 2) or the quotient pattern repeats (example 3).

Example 1	Example 2	Example 3
		

Standard Algorithms for Decimal Operations	
Addition • Set up the problem in columns, with place values lined up to add tens with tens, ones with ones, tenths with tenths, etc. When the digits are properly lined up, the decimal points will also align. • (Optional) Include trailing zeroes to the right of the decimal points as place holders if needed, as in this problem where 1 thousandth is added to 0 thousandths. • Add with regrouping as usual. Since the place values in the sum line up with the place values in the two addends, the decimal point in the sum will align with the decimal points in the addends.	$\begin{array}{r} {}^{1\,1} \\ 48.560 \\ +36.521 \\ \hline 85.081 \end{array}$
Subtraction • Set up the problem in columns, with place values lined up to subtract tens from tens, ones from ones, tenths from tenths, etc. When the digits are properly lined up, the decimal points will also align. • Include trailing zeroes to the right of the decimal point as place holders in the minuend (top number) as needed to line up with any trailing nonzero digit in the subtrahend (bottom number). • Subtract as though the decimal points are not there. When done calculating, place the decimal point in the difference directly below the decimal points in the problem.	$\begin{array}{r} {}^{6\ \ 13\ 10} \\ 7.40 \\ -3.51 \\ \hline 3.89 \end{array}$
Multiplication • Set up the problem in columns, with digits right justified. • Ignore decimal placement and multiply. • Place decimal in the product. The number of digits to the right of the decimal point in the product is equal to the *sum* of the number of digits to the right of the decimal point of each factor.	$\begin{array}{rl} 30.5 & \text{(1 decimal place)} \\ \times\ \underline{0.003} & \text{(3 decimal places)} \\ 0.0915 & \text{(4 decimal places)} \end{array}$
Division • Multiply the divisor and dividend by the same power of 10 (10, 100, 1000, etc.) so that the divisor is a whole number. • Divide as usual, lining up the digits of the quotient above the dividend so that the tens line up with tens, ones with ones, tenths with tenths, and so on. Place the decimal in the quotient in the same location as the dividend. To obtain more decimal place accuracy, attach zeroes to the right of the final place in the decimal part and continue dividing until the remainder is zero or the quotient pattern repeats.	$0.25\overline{)12.5}\ \rightarrow$ $0.25\overline{)12.5\underline{0}}\ \rightarrow$ $\begin{array}{r} . \\ 025.\overline{)1250.} \end{array}\ \rightarrow$ $\begin{array}{r} 50. \\ 25.\overline{)1250.} \end{array}$

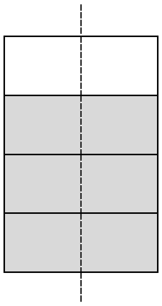
Visualizing Fraction Division as “Divvy Up”

A “divvie up” division problem poses the question:

“How can we divide ___ into ___ equal groups?

Suppose we want to divide $\frac{3}{4}$ cups of grape juice equally among two people. This division problem $\frac{3}{4} \div 2$, can be interpreted as “how can we divide $\frac{3}{4}$ into 2 equal parts?

Let the rectangle represent 1 full cup. It is filled with $\frac{3}{4}$ cups of grape juice.



From the diagram we see that each person will get $\frac{3}{8}$ cup of juice.

Therefore, $\frac{3}{4} \div 2 = \frac{3}{8}$.

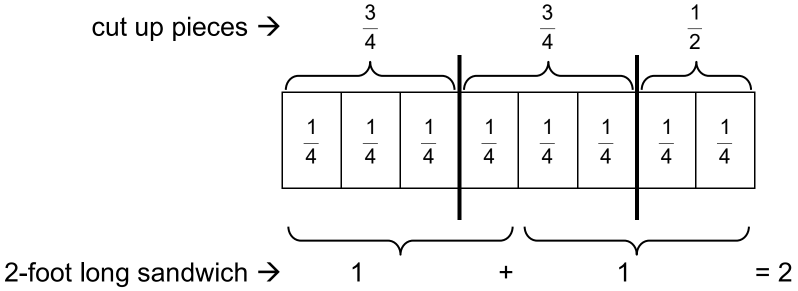
Visualizing Fraction Division as “Measure Out”

A “measure out” division problem poses the question:

“How many ___ are in ___?”

Suppose a two-foot sandwich is cut into pieces that are $\frac{3}{4}$ foot long each. This division problem $2 \div \frac{3}{4}$ can be interpreted as “how many $\frac{3}{4}$ ft. are in 2 ft.?” The unit of measure is $\frac{3}{4}$ ft. From the diagram, we see that there are TWO $\frac{3}{4}$ ft. sandwiches in the 2 ft. sandwich. We see further that there is $\frac{1}{2}$ ft. of sandwich leftover. Since $\frac{1}{2} = \frac{2}{3}$ of $\frac{3}{4}$, the leftover represents $\frac{2}{3}$ of the unit of measure.

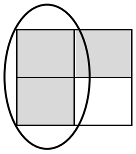
Therefore, $2 \div \frac{3}{4} = 2\frac{2}{3}$.



A Closer Look at the Unit in Fraction Measurement Division

Consider the problem: How many $\frac{1}{2}$s are in $\frac{3}{4}$?

$$\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$$



What is the whole? $\frac{3}{4}$

What is the unit of measure? $\frac{1}{2}$

Is there a full $\frac{1}{2}$ in $\frac{3}{4}$? Yes.

How much is leftover? $\frac{1}{4}$

What part of the unit is leftover? $\frac{1}{2}$ because $\frac{1}{4}$ is $\frac{1}{2}$ of $\frac{1}{2}$.

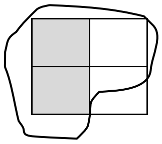
How many $\frac{1}{2}$s are in $\frac{3}{4}$? $1\frac{1}{2}$

$\frac{1}{2}$ is circled and $\frac{1}{4}$ is left over.
$\frac{1}{4}$ is $\frac{1}{2}$ of a $\frac{1}{2}$.

In this case, a larger positive number is being divided by a smaller positive number. The result is a quotient greater than 1.

Consider the problem: How many $\frac{3}{4}$s are in $\frac{1}{2}$?

$$\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$$



What is the whole? $\frac{1}{2}$

What is the unit of measure? $\frac{3}{4}$

Is there a full $\frac{3}{4}$ in $\frac{1}{2}$? No.

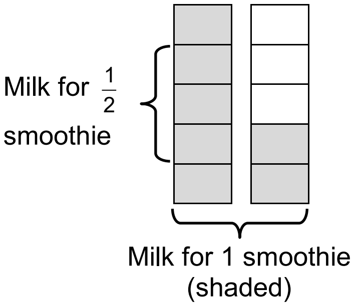
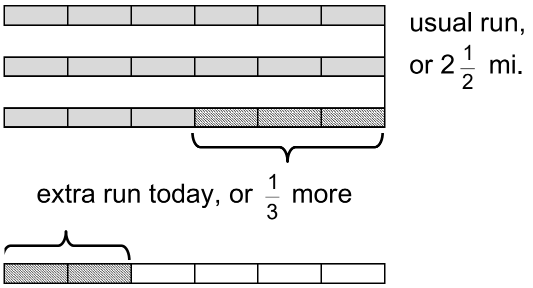
How many $\frac{3}{4}$s are in $\frac{1}{2}$? $\frac{2}{3}$

$\frac{2}{3}$ of $\frac{3}{4}$ is shaded.

In this case, a smaller positive number is being divided by a larger positive number. The result is a quotient less than 1.

Rules for Dividing Fractions

Divide Across	Multiply by the Reciprocal
$\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d}$	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$
$b \neq 0$, $d \neq 0$	$b \neq 0$, $d \neq 0$

Examples: Dividing Fractions		
Words or Diagrams	**Divide Across**	**Multiply by the Reciprocal**
Millenium needs $1\frac{1}{2}$ cups of milk to make a smoothie. How much smoothie can Millenium make with $\frac{3}{4}$ cup of milk?  	$\frac{3}{4} \div 1\frac{1}{2}$ $= \frac{3}{4} \div \frac{3}{2}$ $= \frac{3 \div 3}{4 \div 2}$ $= \frac{1}{2}$	$\frac{3}{4} \div 1\frac{1}{2}$ $= \frac{3}{4} \div \frac{3}{2}$ ` $= \frac{3}{4} \times \frac{2}{3}$ $= \frac{3 \times 2}{4 \times 3}$ $= \frac{6}{12} = \frac{1}{2}$
Helen usually runs $2\frac{1}{2}$ miles a day. Today, she ran $3\frac{1}{3}$ miles. How much of her usual run did Helen run today?  	$3\frac{1}{3} \div 2\frac{1}{2} = \frac{10}{3} \div \frac{5}{2}$ $= \frac{20}{6} \div \frac{15}{6}$ $= \frac{20 \div 15}{6 \div 6}$ $= \frac{\frac{20}{15}}{1} = \frac{20}{15}$ $= 1\frac{5}{15} = 1\frac{1}{3}$	$3\frac{1}{3} \div 2\frac{1}{2} = \frac{10}{3} \div \frac{5}{2}$ $= \frac{10}{3} \times \frac{2}{5}$ $= \frac{10 \times 2}{3 \times 5}$ $= \frac{20}{15}$ $= 1\frac{5}{15} = 1\frac{1}{3}$