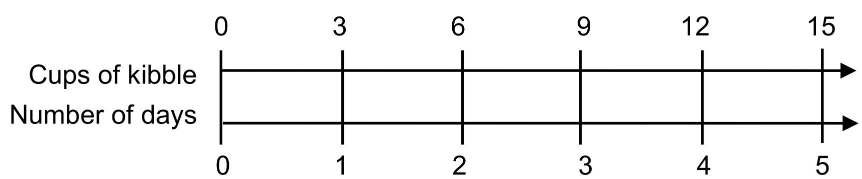
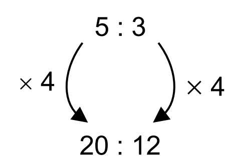
STUDENT RESOURCES

Word or Phrase	Definition
conversion rate	A conversion rate is a unit rate expressing the number of units of one measure equal to one unit of another. Two conversion rates are 1.3 dollars per euro and 60 minutes per hour.
customary units	In the United States, customary units are a system of units of measurement that includes ounces, pounds, and tons to measure weight; inches, feet, yards, and miles to measure length; pints, quarts, and gallons to measure capacity; and degrees Fahrenheit to measure temperature.
double number line	A double number line is a diagram made up of two parallel number lines that visually depict the relative sizes of two quantities. Double number lines are often used when the two quantities have different units, such as miles and hours. The proportional relationship “Wrigley eats 3 cups of kibble per day” can be represented in the following double number line diagram. 
equivalent ratios	Two ratios are equivalent if each number in one ratio is obtained by multiplying the corresponding numbers in the other ratio by the same positive number. 5 : 3 and 20 : 12 are equivalent ratios because both numbers in the ratio 5 : 3 are multiplied by 4 to get to the ratio 20 : 12. An arrow diagram can be used to show equivalent ratios. 
metric units	Metric units are a system of units of measurement that includes grams and kilograms to measure weight; millimeters, centimeters, meters, and kilometers to measure length; milliliters and liters to measure capacity; and degrees Celsius to measure temperature.
ratio	A ratio is a pair of positive numbers in a specific order. The ratio of a to b is denoted by $a : b$ (read “*a* to *b*,” or “*a* for every *b*”). The ratio of 3 to 2 is denoted by 3 : 2. The ratio of dogs to cats is 3 to 2. There are 3 cups of water for every 2 cups of juice. The fraction $\frac{3}{2}$ does not represent this ratio, but it does represent the *value of the ratio* (or the unit rate).

Word or Phrase	Definition
tape diagram	A tape diagram is a graphical representation that uses length to represent relationships between quantities. We draw rectangles with a common width to represent quantities, and rectangles with the same length to represent equal quantities. Tape diagrams are typically used to represent quantities expressed in the same unit. This tape diagram represents a drink mixture with 3 parts grape juice for every 2 parts water. | *G* | *G* | *G* | *W* | *W* |
unit price	A unit price is a price for one unit of measure. If 4 apples cost \$1.00, then the unit price is $\frac{\$1.00}{4}$ = \$0.25 for one apple, or 0.25 dollars per apple or 25 cents per apple.
unit rate	The unit rate associated with a ratio a : b of two quantities a and b, $b \neq 0$, is the number $\frac{a}{b}$, to which units may be attached. This is sometimes referred to as the value of the ratio. The ratio of 40 miles for every 5 hours has a unit rate of $\frac{40}{5}$ = 8 miles per hour.
value of a ratio	See unit rate.

Ratios: Language and Notation

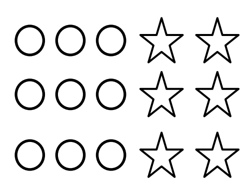
The ratio of a to b is denoted by $a : b$ (read " a to b," or "a for every b").

Note that the ratio of a to b is not the same as the ratio of b to a unless $a = b$.

We can identify several ratios for the objects in the picture to the right.

- There are 3 circles for every 2 stars.
- The ratio of stars to circles is 2 to 3.
- The ratio of circles to total shapes is 3 : 5
- The ratio of total shapes to stars is 5 : 2.

Three copies of the figure above are pictured to the right. Here the ratio of circles to stars is 9 : 6. The ratio 9 : 6 is obtained by multiplying each number in the ratio 3 : 2 by 3 (called the multiplier).



circles : stars

$$3 : 2 \xrightarrow{\times 3} 9 : 6$$

The arrow diagram to the left shows that 3 : 2 and 9 : 6 are equivalent ratios.

A fraction formed by a ratio is called the value of the ratio (or unit rate). Equivalent ratios have the same value. In our example, $\frac{3}{2} = \frac{9}{6} = 1.5$.

Tables of Number Pairs

Tables are useful for recording number pairs that have equivalent ratios. In the case of a ratio of three circles for every two stars, there are two ways that number pairs with equivalent ratios might be recorded in a table. Table 1 is aligned horizontally. Table 2 is aligned vertically. Entries may be in any order.

○ ○ ○ ☆ ☆

Table 1

Circles	3	9	6
Stars	2	6	4

Table 2

Circles	**Stars**
3	2
9	6
6	4

Tape Diagrams

A <u>tape diagram</u> is a visual model consisting of strips divided into rectangular segments whose areas represent relative sizes of quantities. Tape diagrams are typically used when quantities have the same units.

This tape diagram shows that the ratio of grape juice to water in some mixture is 2 : 4.

G	*G*	*W*	*W*	*W*	*W*

Suppose we want to know how much grape juice is needed to make a mixture that is 24 gallons. Here are two methods:

Method 1:

G	*G*	*W*	*W*	*W*	*W*
G	*G*	*W*	*W*	*W*	*W*
G	*G*	*W*	*W*	*W*	*W*
G	*G*	*W*	*W*	*W*	*W*

Replicate the tape diagram, making 24 rectangles. Each rectangle now represents 1 gallon. This shows that:

2 gallons grape : 4 gallons water (6 total gallons)

is the same ratio as

8 gallons grape : 16 gallons water (24 total gallons)

24 gallons of mixture will require 8 gallons of grape juice.

Notice here that each rectangle (piece of tape) represents 1 unit (1 gallon of liquid.)

Method 2:

G	*G*	*W*	*W*	*W*	*W*

<table>
<tr><td colspan="6">24 gallons</td></tr>
<tr><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td><td>4</td></tr>
<tr><td colspan="2">8 gal</td><td colspan="4">16 gal</td></tr>
</table>

Six rectangles in the tape diagram represent 24 gallons of mixture.

Since 24 ÷ 6 = 4, one rectangle in the tape diagram represents 4 gallons of liquid.

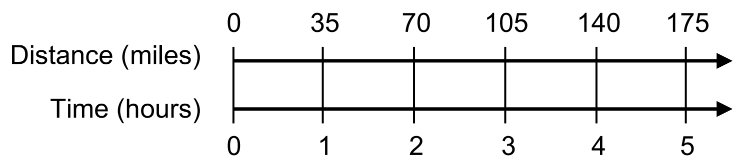
24 gallons of mixture will require 8 gallons of grape juice.

Notice here that each rectangle (piece of tape) represents more than 1 unit (4 gallons, in this case).
Pieces of tape in the diagrams do not always need to represent 1 unit.

Double Number Lines

A double number line diagram is a graphical representation of two quantities in which corresponding values are placed on two parallel number lines for easy comparison. Double number lines are often used to compare two quantities that have different units.

The double number line below shows corresponding ratios if a car travels 70 miles every 2 hours.



We can see from the double number line diagram above that at the given rate, the car goes 35 miles in 1 hour (which is the unit rate of 35 miles per hour), 105 miles in 3 hours, etc. Notice the same tick marks on the number line are used to represent different quantities, and values are scaled in numerical order.

Unit Rate and Unit Price

The unit rate associated with a ratio is the value of the ratio, to which we usually attach units for clarity. In other words, the unit rate associated with the ratio $a : b$ is the number $\frac{a}{b}$, to which we may attach units. For this to make sense, we must assume that $b \neq 0$.

Suppose a car travels 70 miles every 2 hours.

- This may be represented by the ratio 70 : 2.
- The number $\frac{70}{2} = \frac{35}{1} = 35$ is the value of the ratio.
- The unit rate is then the value 35, to which we attach the units "miles per hour." Thus, the unit rate may be written:

$35\ \frac{\text{miles}}{\text{hour}}$ or 35 miles per hour or 35 miles/hour

A unit price is the price for one unit.

Suppose it costs $1.50 for 5 apples.

- This may be represented as the ratio 1.50 : 5.
- The number $\frac{1.50}{5} = 0.30$ is the value of the ratio.
- The unit price is then the value 0.30, to which we attach the units "dollars per apple." The unit price can be written in any of the forms below.

$0.30\ \frac{\text{dollars}}{\text{apple}}$ 0.30 dollars per apple $0.30 per apple

Metric Measurements	
Common metric units	**Examples (sizes approximate)**
Length	
1 millimeter (mm)	the thickness of a dime
1 centimeter (cm)	the width of a small finger
1 meter (m)	the length of a baseball bat
1 kilometer (km)	the length of 9 football fields
Capacity / Volume	
1 milliliter (mL)	an eyedropper
1 liter (L)	a juice carton
1 kiloliter (kL)	four filled bathtubs
Mass / Weight	
1 milligram (mg)	a grain of sand
1 gram (g)	a paperclip
1 kilogram (kg)	a textbook

U.S. Customary Measurements	
Common customary units	**Examples (sizes approximate)**
Length	
1 inch (in)	the length of a small paperclip
1 foot (ft)	the length of a sheet of notebook paper
1 yard (yd)	the width of a door
1 mile (mi)	the length of 15 football fields
Capacity / Volume	
1 fluid ounce (fl oz)	a serving of honey
1 cup (c)	a small cup of coffee
1 pint (pt)	a bowl of soup
1 quart (qt)	an engine oil container
1 gallon (gal)	a jug of milk
Mass / Weight	
1 ounce (oz)	a slice of bread
1 pound (lb)	a soccer ball
1 bushel (bsh)	a block of hay
1 ton (T)	a walrus

Conversion Statements		
Length	**Capacity / Volume**	**Mass / Weight**
1 foot = 12 inches	1 cup = 8 fluid ounces	1 pound = 16 ounces
1 yard = 3 feet	1 pint = 2 cups	1 bushel = 60 pounds
1 mile = 5,280 feet	1 quart = 4 cups	1 ton = 2,000 pounds
1 kilometer = 1,000 meters	1 gallon = 4 quarts	1 kilogram = 1,000 grams
1 meter = 100 centimeter		1 kilogram ≈ 2.2 pounds
	1 liter ≈ 1.06 quarts	
1 centimeter ≈ 0.4 inches		
1 meter ≈ 39 inches		
1 kilometer ≈ 0.6 mile		
Area		
1 acre = 43,560 square feet		

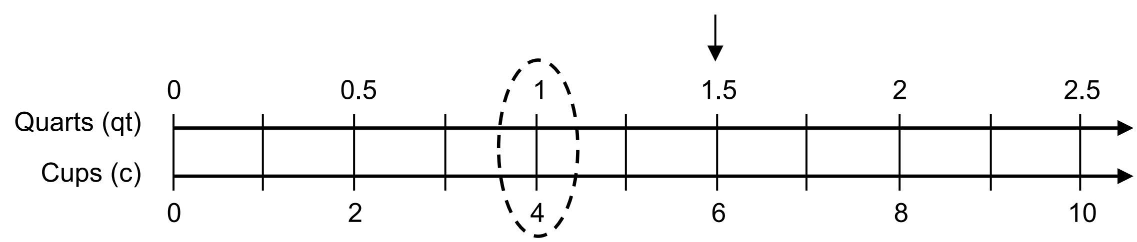
Conversions

Double number lines can be used to organize measurement conversion calculations.

How many cups are in 1.5 quarts?

Create a double number line that shows 1 quart = 4 cups.

Then fill in other numbers on the line to answer the question.



There are 6 cups in 1.5 quarts.

Information from a double number line may also be organized into a table.