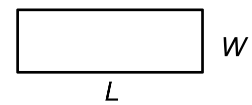
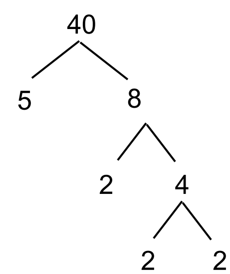
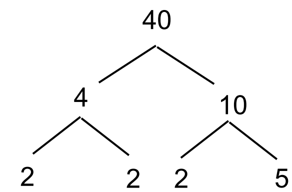
STUDENT RESOURCES

Word or Phrase	Definition
area	The area of a two-dimensional figure is a measure of the size of the figure, expressed in square units. The area of a rectangle is the product of its length and its width. If a rectangle has a length of 12 inches and a width of 5 inches, its area is (5)(12) = 60 square inches. Area = Length × Width = $L \bullet W$ 
composite number	A number is composite if it has more than two divisors or factors. 12 has six factors: 1, 2, 3, 4, 6, 12, because $12 = 1 \bullet 12$, $12 = 2 \bullet 6$, and $12 = 3 \bullet 4$. Since 12 has more than two factors, 12 is a composite number.
factor (of a number)	A factor of a number is a divisor of the number. The factors of 12 are 1, 2, 3, 4, 6, and 12.
greatest common factor	The greatest common factor (GCF) of two numbers is the greatest factor that divides the two numbers. The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 18 are 1, 2, 3, 6, 9, and 18. Therefore, the GCF of 12 and 18 is 6.
least common multiple	The least common multiple (LCM) of two numbers is the least number that is a multiple of both numbers. The multiples of 8 are 8, 16, 24, 32, 40, ... The multiples of 12 are 12, 24, 36, 48, … . Therefore, the LCM of 8 and 12 is 24.
lowest common denominator	The lowest common denominator of two fractions is the least common multiple of their denominators. The lowest common denominator of $\frac{3}{8}$ and $\frac{5}{12}$ is 24.
multiple (of a number)	A multiple of a number m is a number of the form $k \bullet m$ for any integer k. The numbers 5, 10, 15, and 20 are multiples of 5, since $1 \bullet 5 = 5$, $2 \bullet 5 = 10$, $3 \bullet 5 = 15$, and $4 \bullet 5 = 20$.
natural number	The natural numbers are the numbers 1, 2, 3, 4, ….

Word or Phrase	Definition
prime factorization	The <u>prime factorization</u> of a number is an expression of that number as a product of primes. There is a unique way to express any number as a product of primes, except for order.   40 = 5 • 2 • 2 • 2 and 40 = 2 • 2 • 2 • 5 The two <u>prime factorization trees</u> above illustrate that even though the order of the prime factors is different, the products are the same.
prime number	A <u>prime number</u> is a natural number that has exactly two factors, namely 1 and itself. The first six prime numbers are 2, 3, 5, 7, 11, and 13. 1 is *not* a prime number. It has exactly one factor.
relatively prime	Two numbers are <u>relatively prime</u> if their greatest common factor is 1. The factors of 6 are 1, 2, 3, and 6. The factors of 11 are 1 and 11. Since the greatest common factor of 6 and 11 is 1, the two numbers are relatively prime.
square number	A <u>square number</u>, or <u>perfect square</u>, is a number that is a square of a natural number. The area of a square with natural number side-length is a square number. The square numbers are $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$, … .

Symbols for Multiplication

The product of 8 and 4 can be written as:

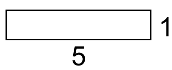
8 times 4 $\qquad 8 \times 4 \qquad 8 \bullet 4 \qquad (8)(4) \qquad \begin{array}{r} 8 \\ \times\ 4 \\ \hline \end{array}$

In algebra, we generally avoid using the × for multiplication because it could be misinterpreted as the variable *x*, and we cautiously use the symbol • for multiplication because it could be misinterpreted as a decimal point.

Using Rectangles to Visualize Prime and Composite Numbers

Building rectangles whose sides have natural number lengths is a geometric way to describe factors and multiples of numbers. If the area of the rectangle represents the product, then the side lengths of the rectangle represent the factors of the number.

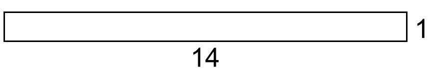
A <u>prime number</u> p corresponds to only one rectangle, since p can be factored as a product in only one way, $p = 1 \bullet p$. (Here we regard the factorization $p = 1 \bullet p$ as the same as $p = p \bullet 1$, and we regard a $1 \times p$ rectangle as being the same as a $p \times 1$ rectangle.)

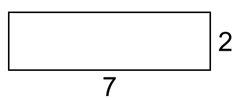


$5 = 1 \times 5$

1 and 5 are factors of 5.

A <u>composite number</u> n always corresponds to more than one rectangle.



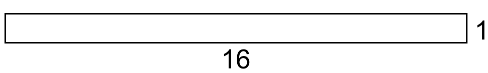


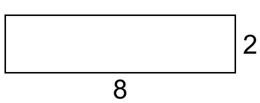
$14 = 1 \times 14$

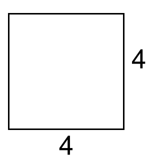
$14 = 2 \times 7$

1, 2, 7, and 14 factors of 14.

A number such as 16 is called a <u>square number</u> (or <u>perfect square</u>) because one of the rectangles it corresponds to is a square (4×4).







$16 = 1 \times 16$

$16 = 2 \times 8$

$16 = 4 \times 4$

1, 2, 4, 8, and 16 factors of 16.

Greatest Common Factor (GCF)
The greatest common factor (GCF) of two numbers is the greatest factor that divides the two numbers. Here are two ways to find the GCF of two numbers. Tensaye has 12 bottles of water and 18 granola bars. She wants to use all of the bars and bottles to make care packages for the homeless. How many care packages can Tensaye make so that there are the same number of bottles of water and granola bars in each package? Method 1: Use a list to find the GCF of 12 and 18: List all the factors of 12: 1, 2, 3, 4, 6, and 12. List all the factors of 18: 1, 2, 3, 6, 9, and 18. We can see that the factors 1, 2, 3, and 6 appear in both lists. Since 6 is the greatest factor from both lists that divides 12 and 18, the greatest common factor (GCF) of 12 and 18 is 6. Method 2: Use prime factorization to identify the GCF of 12 and 18. $12 = 2 \times 2 \times 3$ $18 = 2 \times 3 \times 3$ We see that 2 and 3 are common factors of both numbers. Therefore $2 \times 3 = 6$ is the GCF. Since the GCF of 12 and 18 is 6, Tensaye can make 6 care packages for the homeless, and each care package will contain 2 bottles of water and 3 granola bars.

Least Common Multiple (LCM)
The least common multiple (LCM) of two numbers is the least number that is a positive multiple of both numbers. Here is one way to find the LCM of two numbers. Tensaye wants buy bottles of water and granola bars to make care packages for the homeless. Bottles of water come in packages of 12, and granola bars are sold in packages of 18. How many bottles of water and how many granola bars should Tensaye buy so that she has the same number of each item? Note; She can only afford to buy the smallest amount to make this happen. Use a list to find the LCM of 12 and 18: The multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120,… . The multiples of 18 are: 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, ... The multiples that 12 and 18 have in common are 36, 72, 108, ... We can see that 36 is the least multiple the two numbers have in common. Therefore, the LCM of 12 and 18 is 36. Since the LCM of 12 and 18 is 36, Tensaye should buy 36 bottles (or 3 packages) of water and 36 granola bars (or 2 packages) so that she has the same number of each item.

The "Big One"

The "Big 1" is a notation for 1 (multiplicative identity) in the form of a fraction $\frac{n}{n}$ ($n \neq 0$).

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \ldots$$

We can use the following picture to help remind us that these fractions are equivalent to 1:

$$1 = \frac{8}{8}$$

The Big 1 can be used to help find equivalent fractions. For example,

$$\frac{2}{5} \times \frac{10}{10} = \frac{20}{50} \quad \text{or} \quad \frac{20}{50} \div \frac{10}{10} = \frac{2}{5}.$$

Diagrams that Show Equivalent Fractions

These diagrams illustrate that $\frac{1}{2} = \frac{4}{8}$. In the second diagram, each half is split into four parts, but the size of the whole does not change, nor does the amount shaded.

Using the Big 1, this equivalence can be written:

$$\frac{1}{2} \bullet \frac{4}{4} = \frac{4}{8} \quad \text{or} \quad \frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$

Fractions in "Simplest Form" with the GCF

To write a fraction in its simplest form, divide the numerator and denominator by the greatest common factor. Though it is not required to use the GCF, doing so is the most efficient way, because it only takes one step. Use the Big 1 when dividing.

To simplify $\frac{12}{30}$, first use any method to determine that the GCF of 12 and 30 is 6. Then divide the numerator and denominator by 6, in the form of the Big 1.

$$\frac{12}{30} \div \frac{6}{6} = \frac{2}{5}$$

Renaming Fractions with their Lowest Common Denominator (LCD)

To rename fractions with their LCD, first find the least common multiple (LCM) of the denominators. Then change each fraction to an equivalent fraction by multiplying each of them by the appropriate forms of the Big 1.

To write two fractions, $\frac{3}{4}$ and $\frac{5}{6}$, with their LCD, first find the LCM of the denominators. After using any method to determine that the LCD of 4 and 6 is 12, rename the fractions so that they both have a denominator of 12 using the Big 1.

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \qquad\qquad \frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$$

This computation results in lesser numerators and denominators in the fractions because 12 is the least mutiple that 4 and 6 share in common. In other words, 12 is the LCM of 4 and 6, or the LCD of the fractions.

Using "Factor Ladders" to Find the GCF and LCM of Two Numbers

Factor ladders are a useful tool for finding the GCF and LCM of two numbers.

Use repeated division to find the GCF and LCM of 12 and 18:

Divide each number by any common factor greater than 1. In this case we have choices, so let's begin by dividing both numbers by 2. The resulting quotients are 6 and 9.

$$\begin{array}{c|cc} 2 & 12 & 18 \\ \hline 3 & 6 & 9 \\ \hline & 2 & 3 \end{array}$$

Keep dividing until both resulting quotients have no factors in common greater than 1. In this case, we can still divide by 3. The resulting quotients are now 2 and 3, and they have no common factors greater than 1. They are relatively prime.

The GCF is the product of the factors along the side. Therefore, the GCF of 12 and 18 is $2 \bullet 3 = 6$.

The LCM is the product of the factors along the side and the bottom. Therefore, the LCM of 12 and 18 is the GCF multiplied by 2 and 3, or $6 \bullet 2 \bullet 3 = 36$.

Fraction Addition with Diagrams

The standard procedure for adding fractions requires that the fractions have common denominators. An area model supports why this is reasonable.

Example 1: $\frac{1}{5} + \frac{3}{5}$

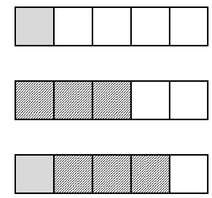
The fractions already have a common denominator.

$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$

$\frac{1}{5}$

$\frac{3}{5}$

$\frac{1}{5} + \frac{3}{5}$



Example 2: $\frac{1}{2} + \frac{1}{3}$

Find a common denominator.

$$\frac{1}{2} + \frac{1}{3} = \frac{1}{2} \bullet \frac{3}{3} + \frac{1}{3} \bullet \frac{2}{2}$$

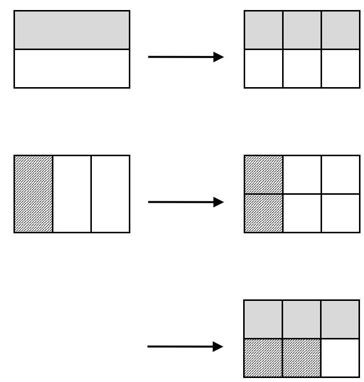
$$= \frac{3}{6} + \frac{2}{6}$$

$$= \frac{5}{6}$$

$\frac{1}{2} = \frac{3}{6}$

$\frac{1}{3} = \frac{2}{6}$

$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

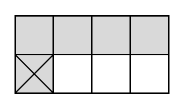


Fraction Subtraction with Diagrams

The standard procedure for subtracting fractions requires that the fractions have common denominators. An area model supports why this is reasonable.

Example 1: $\frac{5}{8} - \frac{1}{8}$

The fractions already have a common denominator.



$\frac{5}{8} - \frac{1}{8} = \frac{4}{8}$.

Example 2: $\frac{1}{2} - \frac{1}{3}$

Find a common denominator.

$$\frac{1}{2} - \frac{1}{3} = \frac{1}{2} \bullet \frac{3}{3} - \frac{1}{3} \bullet \frac{2}{2}$$

$$= \frac{3}{6} - \frac{2}{6}$$

$$= \frac{1}{6}$$

$\frac{1}{2} = \frac{3}{6}$

$\frac{1}{3} = \frac{2}{6}$

$\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$