

PRIME, EMIRP, and Armstrong Numbers

Project for the Curious

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Center for Mathematics and Teaching

This excerpt explores two Goldbach Conjectures

I'm zero. I'm the
first of the
Armstrong
numbers because
 $0^1 = 0$



I'm unique. I'm
the only prime
number that is an
even number!



PRIME, EMIRP, and Armstrong Numbers

Throughout history, mathematicians have explored a variety of types of numbers and relationships among those numbers.

Explorations took place as early as in the 200BCs.

More recently, an exploration took place in 1985.

Check out these explorations.

You will need:

- **Pencil**
- **Calculator**
- **Partner**

II-A. Goldbach Strong Conjecture

In 1742, Christian Goldbach, a Russian mathematician, proposed the relationship (to his friend Leonard Euler) that every number greater than 2 can be formed by adding two PRIME numbers.

Test Goldbach's Strong Conjecture.

For each of these even numbers, identify all sets of two PRIME Numbers that add to the even number.

$$4 = \underline{\quad} + \underline{\quad} \qquad 18 = \underline{\quad} + \underline{\quad}$$

$$6 = \underline{\quad} + \underline{\quad} \qquad 20 = \underline{\quad} + \underline{\quad}$$

$$8 = \underline{\quad} + \underline{\quad} \qquad 30 = \underline{\quad} + \underline{\quad}$$

$$10 = \underline{\quad} + \underline{\quad} \qquad 38 = \underline{\quad} + \underline{\quad}$$

$$12 = \underline{\quad} + \underline{\quad} \qquad 40 = \underline{\quad} + \underline{\quad}$$

$$14 = \underline{\quad} + \underline{\quad} \qquad 46 = \underline{\quad} + \underline{\quad}$$

$$16 = \underline{\quad} + \underline{\quad} \qquad 50 = \underline{\quad} + \underline{\quad}$$

II-B. The Goldbach Weak Conjecture

In 1742, Christian Goldbach, also proposed the relationship that every odd number greater than 5 can be formed by adding three PRIME numbers.

Test Goldbach's Weak Conjecture.

For each of these odd numbers, identify all sets of three PRIME Numbers that add to the odd number.

$$7 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$17 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$9 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$19 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$11 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$23 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$13 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$29 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$15 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

$$31 = \underline{\quad} + \underline{\quad} + \underline{\quad}$$

Solutions

II-A. Goldbach Strong Conjecture

For each of these even numbers, identify all sets of two PRIME Numbers that add to the even number.

$$\underline{4} = 2 + 2$$

$$\begin{aligned}\underline{18} &= 5 + 13 \\ &= 7 + 11\end{aligned}$$

$$\underline{6} = 3 + 3$$

$$\begin{aligned}\underline{20} &= 3 + 17 \\ &= 7 + 13\end{aligned}$$

$$\underline{8} = 3 + 5$$

$$\begin{aligned}\underline{30} &= 7 + 23 \\ &= 11 + 19 \\ &= 13 + 17\end{aligned}$$

$$\begin{aligned}\underline{10} &= 3 + 7 \\ &= 5 + 5\end{aligned}$$

$$\begin{aligned}\underline{38} &= 7 + 31 \\ &= 19 + 19\end{aligned}$$

$$\underline{12} = 5 + 7$$

$$\begin{aligned}\underline{40} &= 3 + 37 \\ &= 11 + 29 \\ &= 17 + 23\end{aligned}$$

$$\begin{aligned}\underline{14} &= 3 + 11 \\ &= 7 + 7\end{aligned}$$

$$\begin{aligned}\underline{46} &= 3 + 43 \\ &= 5 + 41 \\ &= 23 + 23\end{aligned}$$

$$\begin{aligned}\underline{16} &= 3 + 13 \\ &= 5 + 11\end{aligned}$$

$$\begin{aligned}\underline{50} &= 3 + 47 \\ &= 7 + 43 \\ &= 13 + 37 \\ &= 19 + 31\end{aligned}$$

Solutions

II-B. The Goldbach Weak Conjecture

For each of these odd numbers, identify all sets of three PRIME Numbers that add to the odd number.

$$\begin{aligned}\underline{7} &= 2 + 2 + 3 & \underline{17} &= 2 + 2 + 13 \\ & & &= 3 + 3 + 11 \\ & & &= 5 + 5 + 7\end{aligned}$$

$$\begin{aligned}\underline{9} &= 2 + 2 + 5 & \underline{19} &= 3 + 3 + 13 \\ &= 3 + 3 + 3 & &= 3 + 5 + 11 \\ & & &= 5 + 7 + 7\end{aligned}$$

$$\begin{aligned}\underline{11} &= 2 + 2 + 7 & \underline{23} &= 2 + 2 + 19 \\ &= 3 + 3 + 5 & &= 3 + 3 + 17 \\ & & &= 3 + 7 + 13\end{aligned}$$

$$\begin{aligned}\underline{13} &= 3 + 3 + 7 & \underline{29} &= 3 + 13 + 13 \\ &= 3 + 5 + 5 & &= 5 + 7 + 17 \\ & & &= 7 + 11 + 11\end{aligned}$$

$$\begin{aligned}\underline{15} &= 2 + 2 + 11 & \underline{31} &= 3 + 5 + 23 \\ &= 3 + 5 + 7 & &= 3 + 11 + 17 \\ &= 5 + 5 + 5 & &= 5 + 7 + 19 \\ &= 2 + 2 + 11 & &= 7 + 11 + 13\end{aligned}$$

AUTHOR BIOS

Carole Greenes, Ed.D. is Professor Emerita, Mathematics Education at Arizona State University and was Professor at Boston University. She has authored/co-authored more than 350 books for students and teachers; 82 articles; six mathematical musicals; and two histories of mathematics in story and song. Her 2021 - 2026 books/games include: *What's My Angle? Alge-Grid: What's the a?*, *Pattern Grid-unLocks*, *Play It Positively or Negatively!*, *Factor Max*, *Make It Proper*, *Shape Up*, *Function Frenzy*, *What's My Angle?*, *READY-X?*, *SMARTY*, and *PRIME, EMIRP*, and *Armstrong Numbers*, all available from the Center for Mathematics and Teaching. (cgreenes@asu.edu)

Tanner Wolfram has a J.D. from Georgetown University Law Center, and a B.S. in physics from Arizona State University. A native of Arizona, he has worked with Professor Greenes in multiple capacities for more than 10 years. During this time, he collaborated on three research projects developing Grades 5 – 12 students' and teachers' interests in a variety of STEM fields, and he co-authored 9 mathematics puzzle books. Tanner is interested in pursuing a career in transactional law, which involves doing the legal work for deals between large corporations. He will start his legal career at a law firm in Washington D.C. (twolfram@asu.edu).

