Shape-Up

Hi! I'm Know-a Lot. Do you get the point?

Hi! I'm Think Hard. I always measure up!

Greenes & Wolfram

Also produced by the PRIME Group

Alge-Grid: What's the a?

Pattern Grid-unLocks

Play It Positively or Negatively!

Factor Max

Make It Proper

Practice Research Innovation in Mathematics Education (PRIME) Group

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https://mathandteaching.org/

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Author Bios

Carole Greenes, Ed.D. is Professor Emerita, Mathematics Education at Arizona State University. While at ASU, she served as Associate Vice President for STEM Education, Dean of the School of Educational Innovation and Teacher Preparation, Director of the Practice Research and Innovation in Mathematics Education (PRIME) Center, Director of the Vertically Integrated Projects program that provides research experiences for undergraduate students, and Professor of Mathematics Education in the Ira A. Fulton Schools of Engineering, the College of Liberal Arts and Sciences, and the Mary Lou Fulton Teachers College. Currently, she directs the PRIME Group that develops books of challenge problems and games for students, grades K – 12, and authors Carole's Corner for the Center for Mathematics and Teaching, Inc. in California. Carole is author of more than 350 books for students, PreK-12 and college, and teachers; 81 articles; six mathematical musicals; and two histories of mathematics in story and song. She served as editor of the Arizona Association of Teachers of Mathematics journal, *OnCore*, and author of the online monthly free *MATHgazine Senior* (grades 8-12), *MATHgazine Junior* (grades 5-8), *MATHgazine Elementary* (grades 3-5) and *MATHgazine Primary* (grades K-2). In 2003, Greenes was inducted into the Massachusetts Mathematics Educators' Hall of Fame. In 2011, she received the NCSM Ross Taylor/Glenn Gilbert National Leadership Award in Mathematics Education. In 2016, she received the Copper Apple Award for Leadership in Mathematics in Arizona, and in 2018 she received the National Council of Teachers of Mathematics Lifetime Achievement Award. Her 2021 and 2022 books/games include: *Alge-Grid: What'the a?*, *Pattern Grid-unLocks, Play It Positively or Negatively?*! *Factor Max!*, *Make It Proper!,* and *Shape-Up.* She is author of *Carole's Corner* and *Carole's Commentary* for the Center for Mathematics and Teaching.

Tanner Wolfram is a Fall 2019 graduate, Summa cum Laude, of Barrett, The Honors College at Arizona State University. He holds a major in Physics and minors in both Spanish and Chinese. Tanner is co-author of *Make It Proper, Solve It Positively and Negatively!*, *Pattern Grid-unLocks, Factor Max, Alge-Grid: What's the a?, Make It Proper*, and *Shape-Up* puzzle books distributed by the Center for Mathematics and Teaching, and senior author of the *Facasumi Puzzle Book* for the Arizona Association of Teachers of Mathematics. From Spring 2016 to Fall 2020, Tanner served as Senior Project Assistant in the Practice, Research, and Innovation in Mathematics Education (PRIME) Center at ASU, and is now co-Director of the PRIME Group. During his time with the PRIME Center, Tanner assisted with the NSF-funded App Maker Pro (AMP) project, contributed to and edited eight *MATHadazzle Puzzle Books*, co-authored six articles, and co-edited two free monthly online *MATHgazines.*

Shape-Up: Draw One Line

Goal: Partition equilateral polygons to produce two specified shapes, some congruent and some non-congruent.

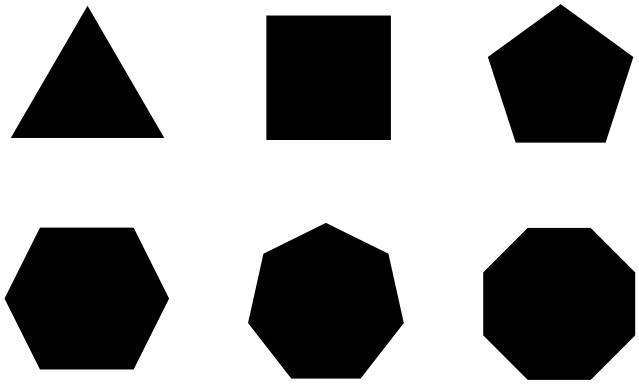
Description: Each problem presents three pictures of the same equilateral shape: triangle, square, pentagon, hexagon, heptagon, or octagon. Directions are to draw one line in each of the shapes to create two other shapes that meet the specified criteria. The three drawings must differ. Shapes created by the one line may be congruent or non-congruent.

If there is a finite number of ways to draw one line to make the shapes, the directions will read "Show all ways." If there is an infinite number of solutions, the directions will read "Show three of the ways."

In the Shape-Up Solutions at the end of this book, all possibilities for congruent shapes are shown. In the case of non-congruent shapes in shapes, three possibilities are presented.

Objectives:

- Visualize shapes in shapes.
- Recognize these geometric figures: triangles, quadrilaterals (including rectangles, squares, trapezoids, and irregular quadrilaterals), pentagons, heptagons, and octagons.
- Know properties of polygons: number of sides; shapes that are congruent or non-congruent.
- Identify alternative constructions.
- Recognize constructions that produce finite versus infinite solutions.



Definitions

Congruent: Same shape and size.

Two polygons are congruent when they are the same shape, and their sides are the same length. Congruent shapes can be flipped or turned, so that flipped or turned shapes will fit on top of the original shapes. They are mirror images.

Non-Congruent: Different size or shape.

Two polygons are non-congruent when either the shape or the lengths of sides differ.

Triangle: Polygon with 3 sides and 3 angles.

Right Triangle: A triangle with one right (90-degree) angle.

Quadrilateral: Polygon with 4 sides and 4 angles.

Square: 4 sides that are the same length; they are congruent.
All angles are right angles.

Rectangle: 4 sides. Opposite sides are the same length. They are congruent.
Adjacent sides differ in length. All angles are right angles.

Note: In this book, the term <u>rectangle</u> excludes <u>squares</u>.

Trapezoid: 4 sides with only one pair of parallel sides.

Pentagon: Polygon with 5 sides and 5 angles.

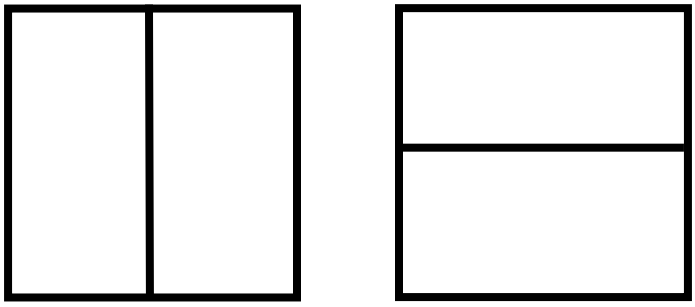
Hexagon: Polygon with 6 sides and 6 angles.

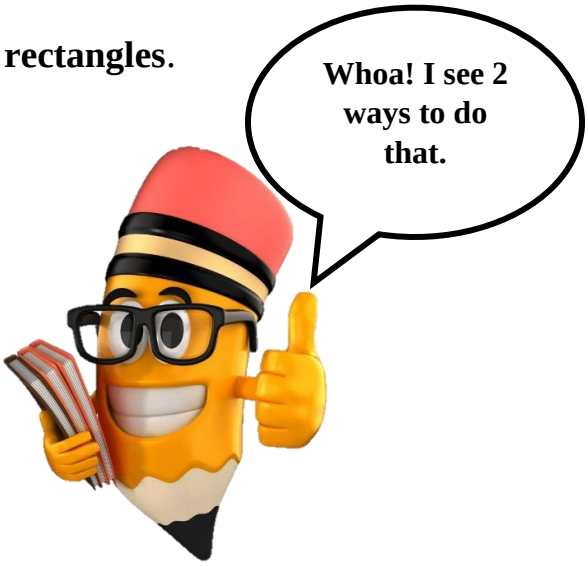
Heptagon: Polygon with 7 sides and 7 angles.

Octagon: Polygon with 8 sides and 8 angles.

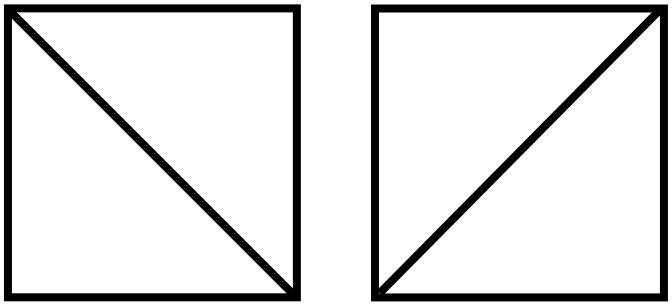
Congruent Examples

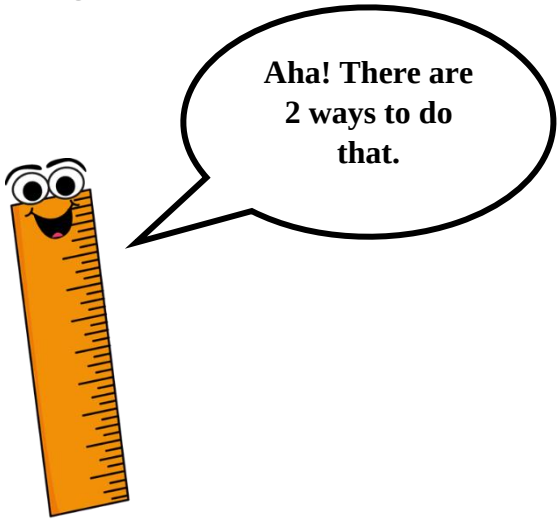
Draw one line in the square to make 2 congruent rectangles.





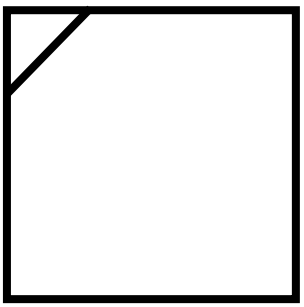
Draw one line in the square to make 2 congruent triangles.

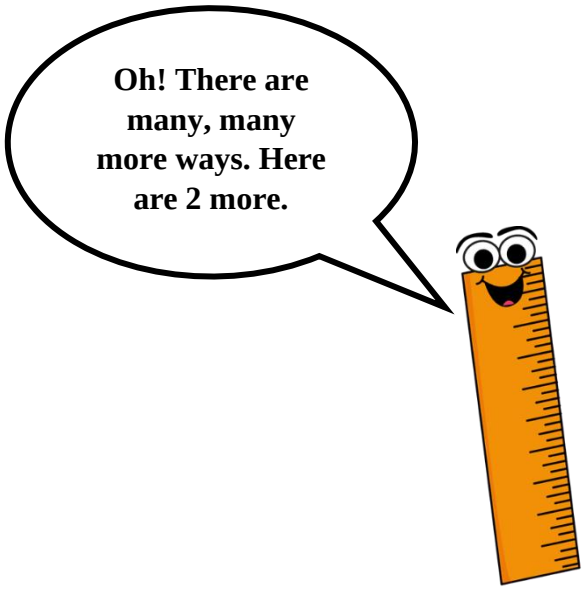


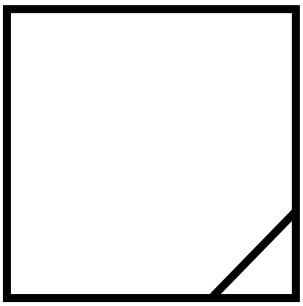


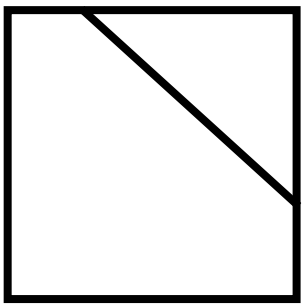
Non-congruent Examples

Draw one line in the square to make 1 triangle and 1 pentagon.







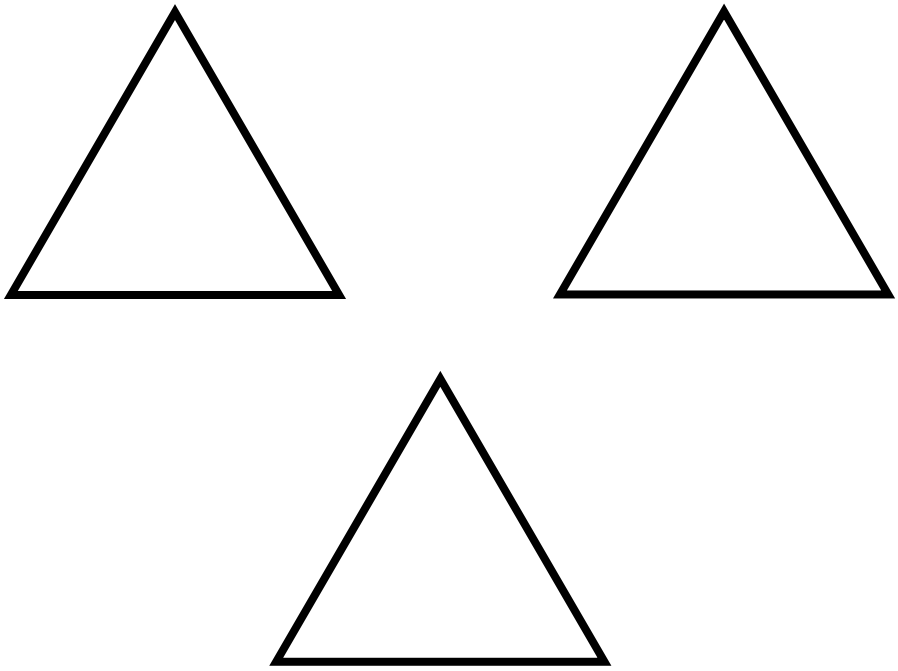


Triangles

Triangle Shape-Ups

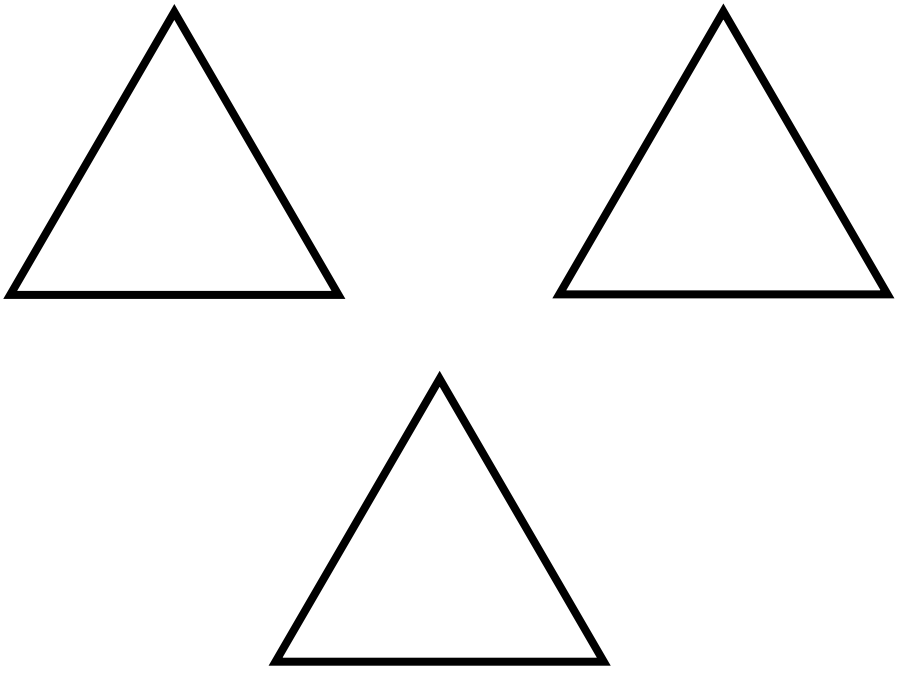
Draw one line to make 2 congruent triangles.

Show all three ways.



Draw one line to make 2 non-congruent triangles.

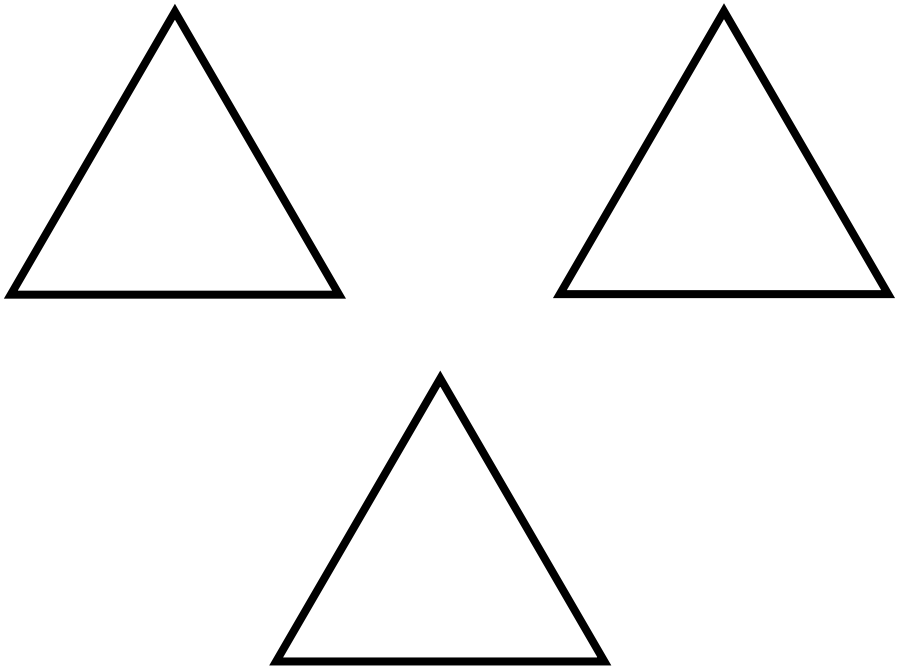
Show three of the ways.



Triangle Shape-Ups

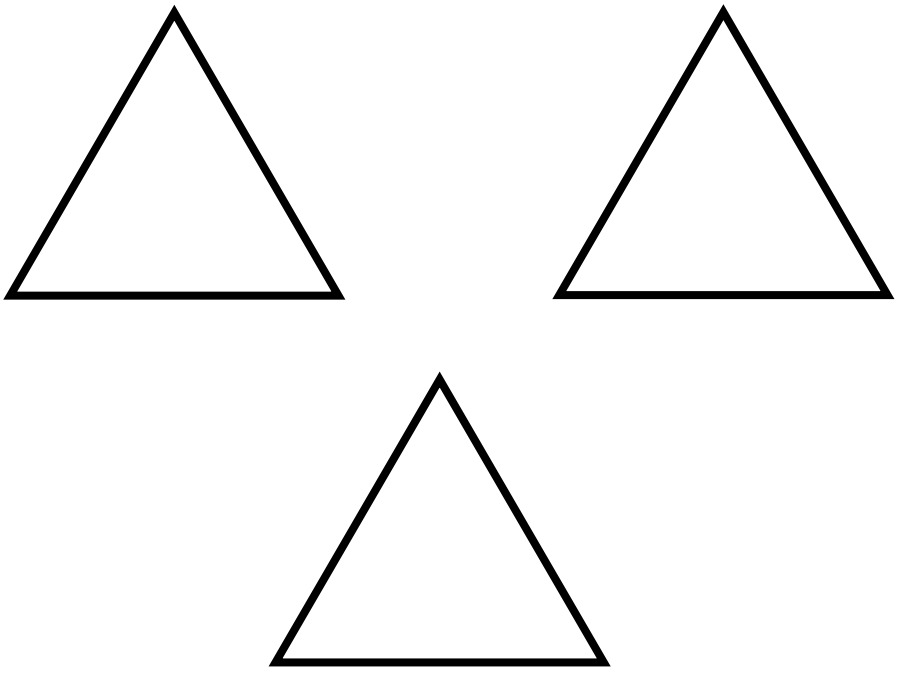
Draw one line to make 1 triangle and 1 trapezoid.

Show three of the ways.

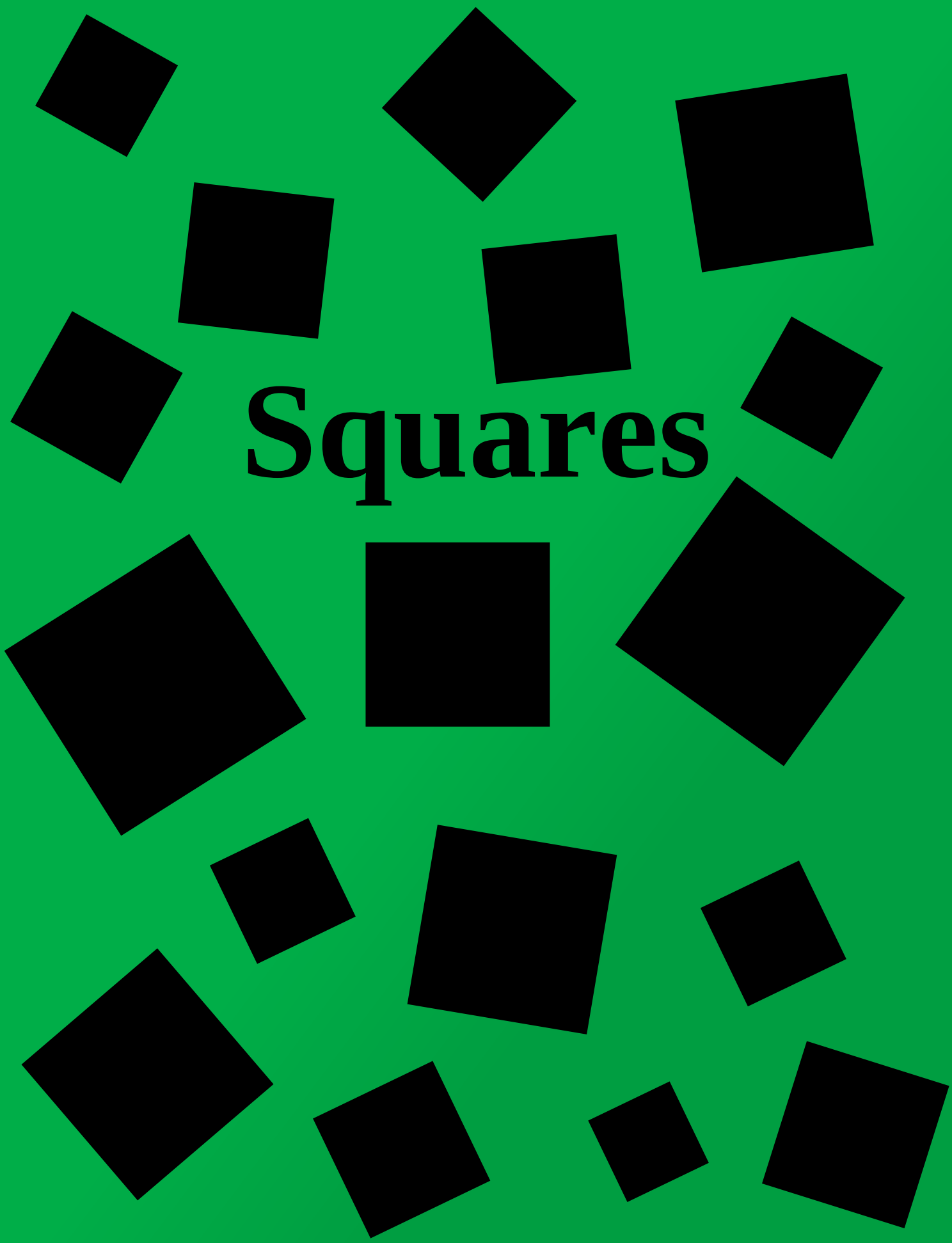


Draw one line to make 1 right triangle and 1 quadrilateral.

Show three of the ways.



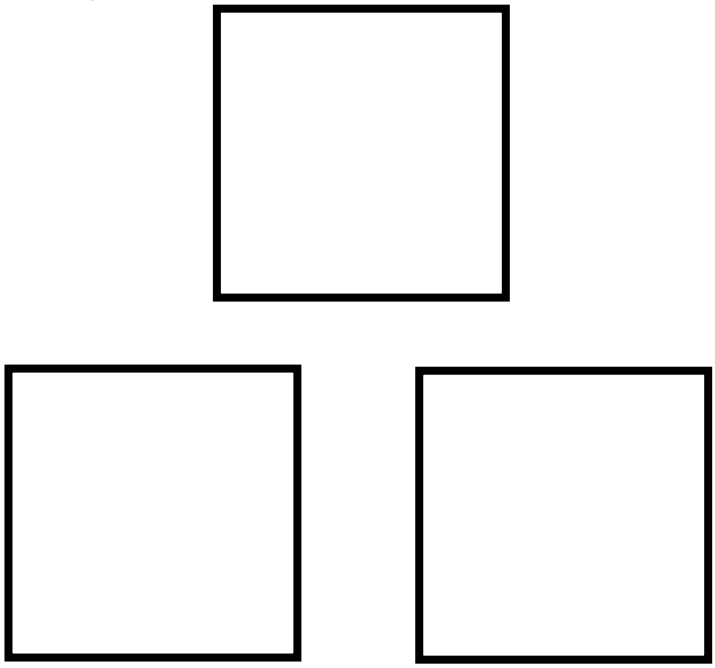
Squares



Square Shape-Ups

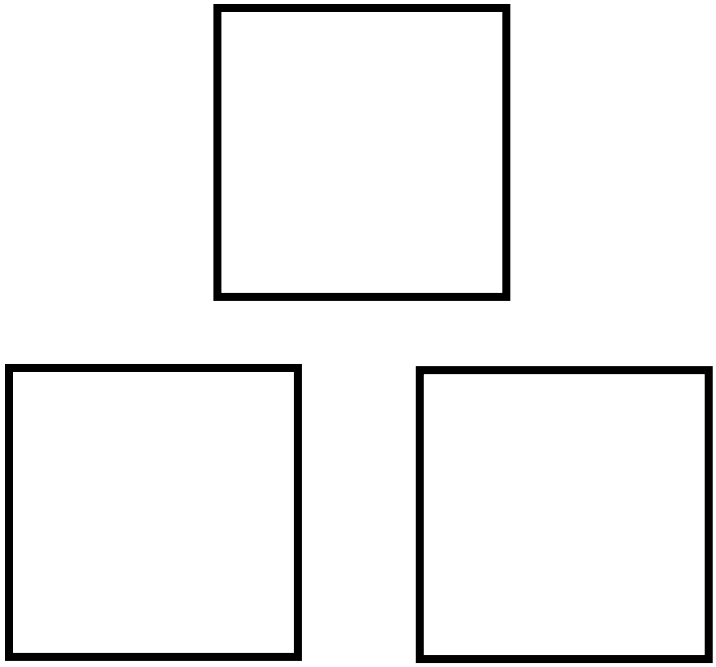
Draw one line to make 2 non-congruent rectangles.

Show three of the ways.



Draw one line to make 1 right triangle and 1 trapezoid.

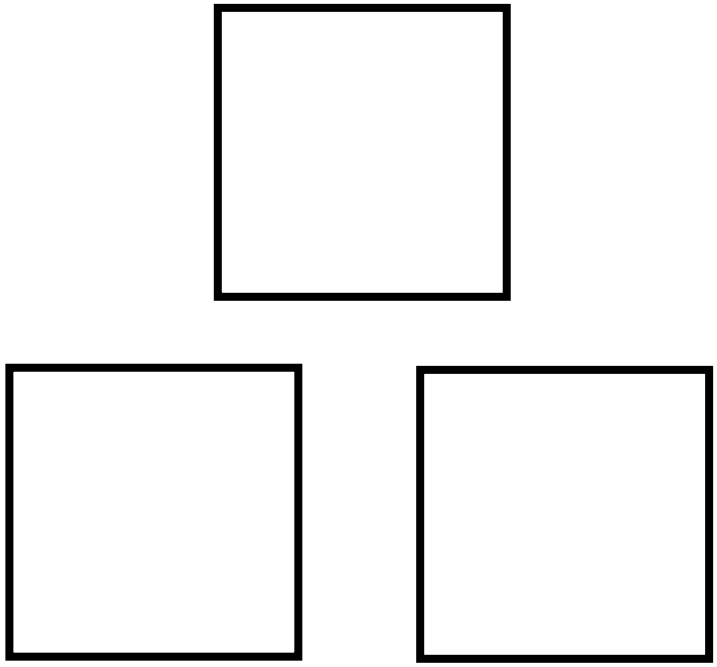
Show three of the ways.



Square Shape-Ups

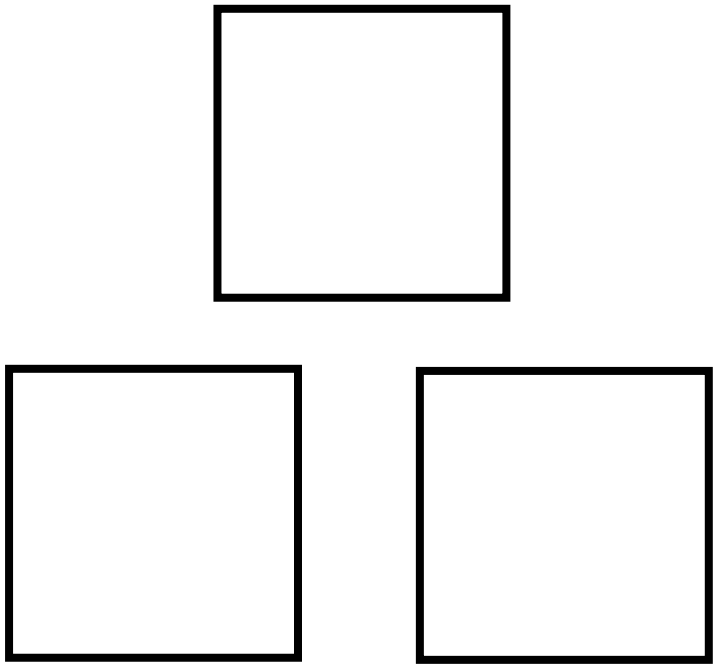
Draw one line to make 2 congruent trapezoids.

Show three of the ways.



Draw one line to make 2 non-congruent trapezoids.

Show three of the ways.

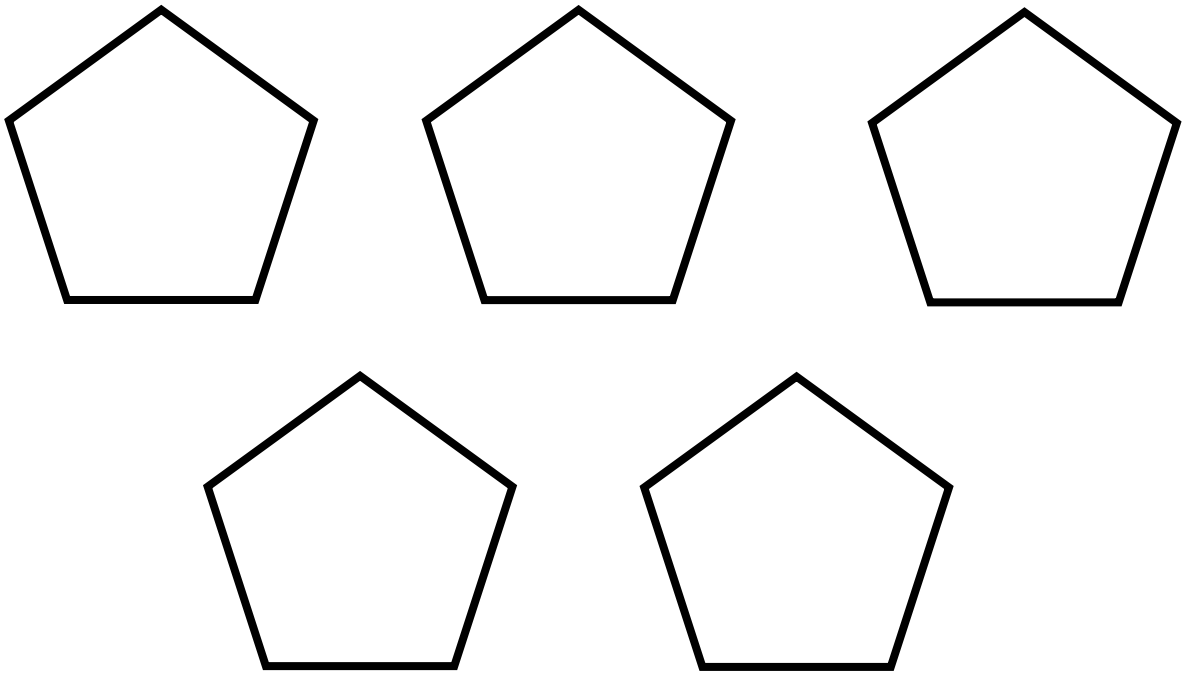



Pentagons

Pentagon Shape-Ups

Draw one line to make 2 congruent quadrilaterals.

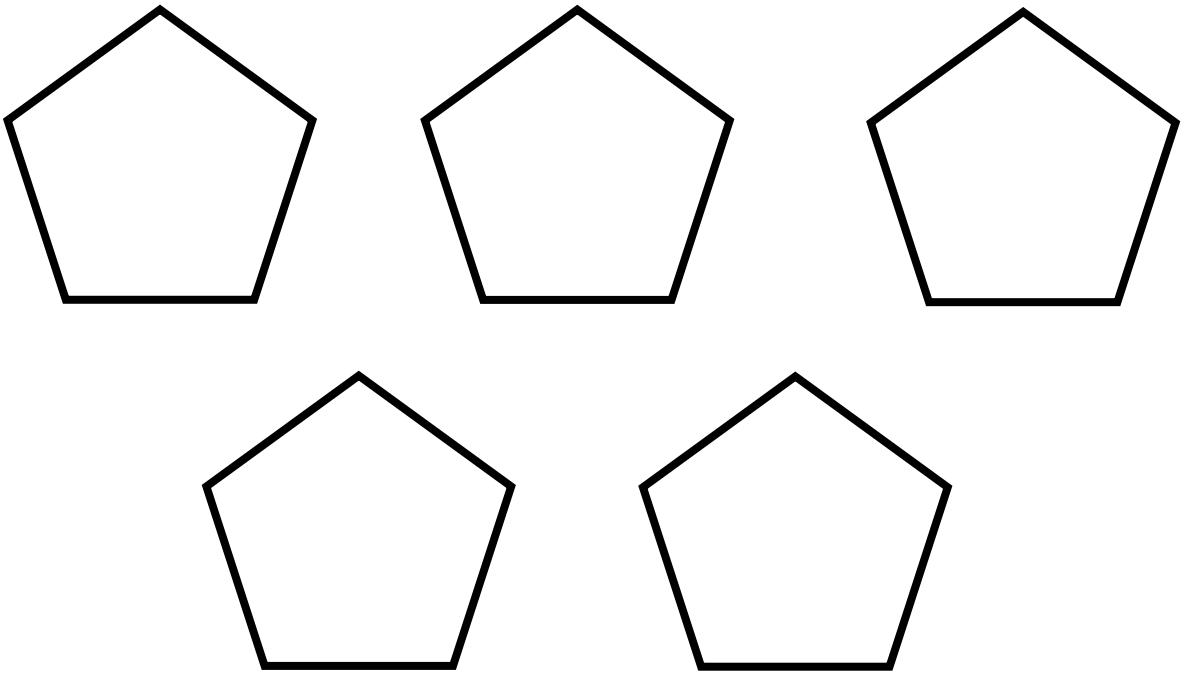
Show all five ways.



Pentagon Shape-Ups

Draw one line to make 1 triangle and 1 trapezoid.

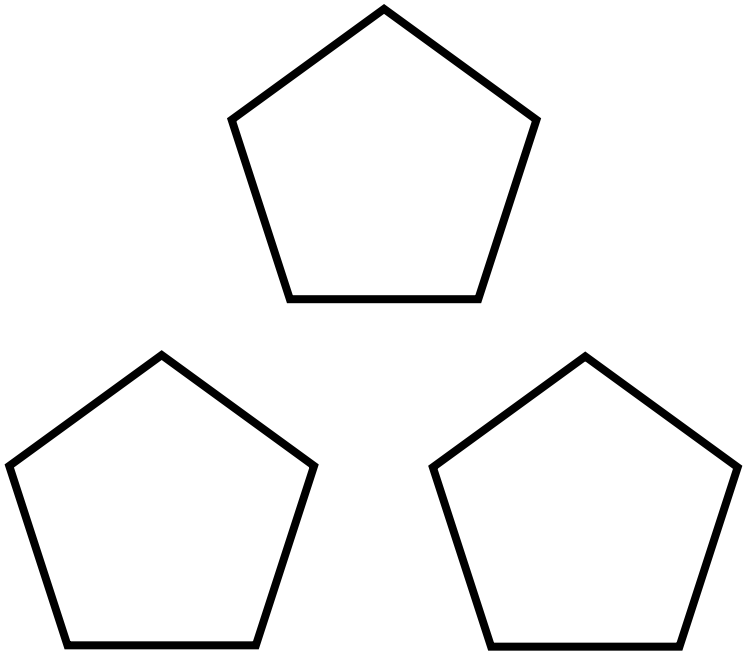
Show all five ways.



Pentagon Shape-Ups

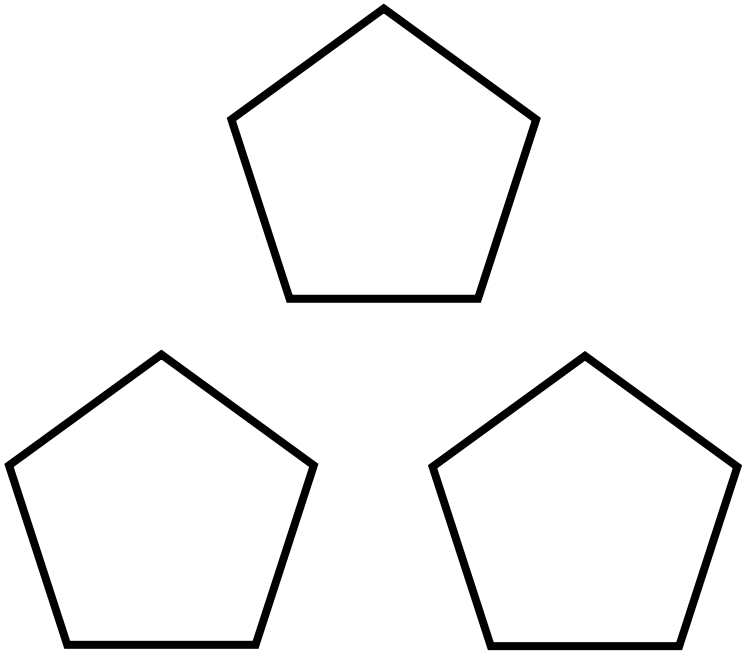
Draw one line to make 1 triangle and 1 hexagon.

Show three of the ways.



Draw one line to make 1 triangle and 1 pentagon.

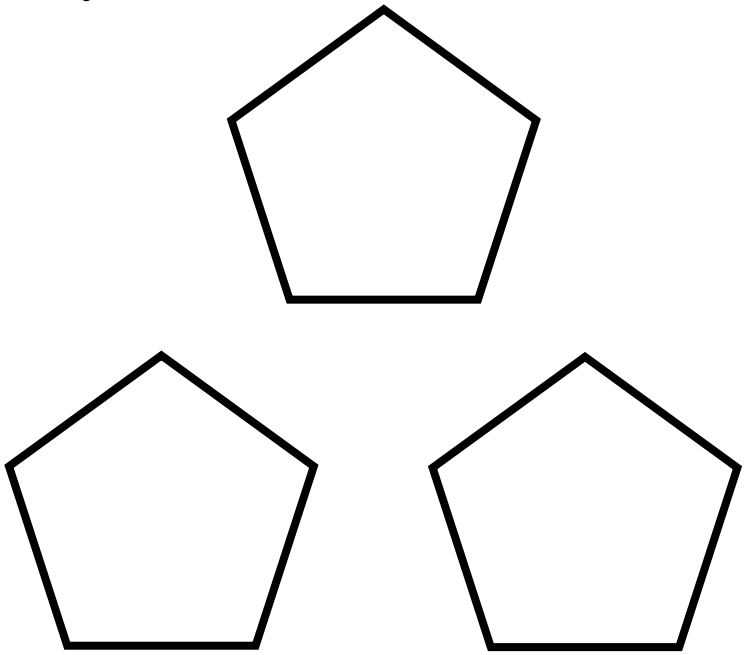
Show three of the ways.



Pentagon Shape-Ups

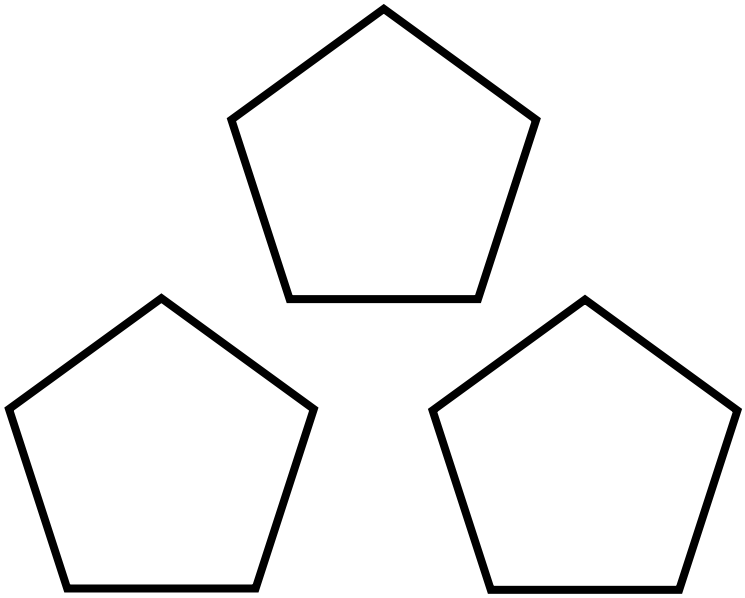
Draw one line to make 2 non-congruent quadrilaterals.

Show three of the ways.



Draw one line to make 1 trapezoid and 1 pentagon.

Show three of the ways.

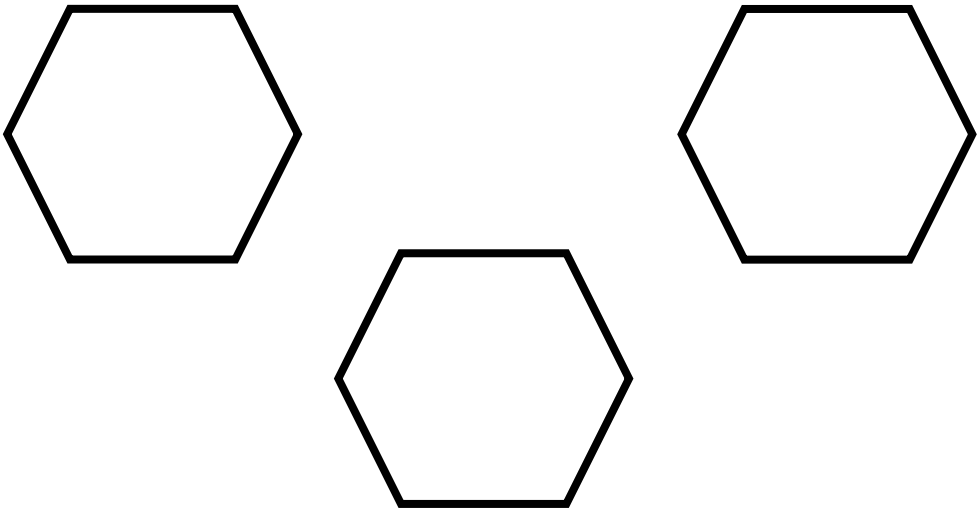



Hexagons

Hexagon Shape-Ups

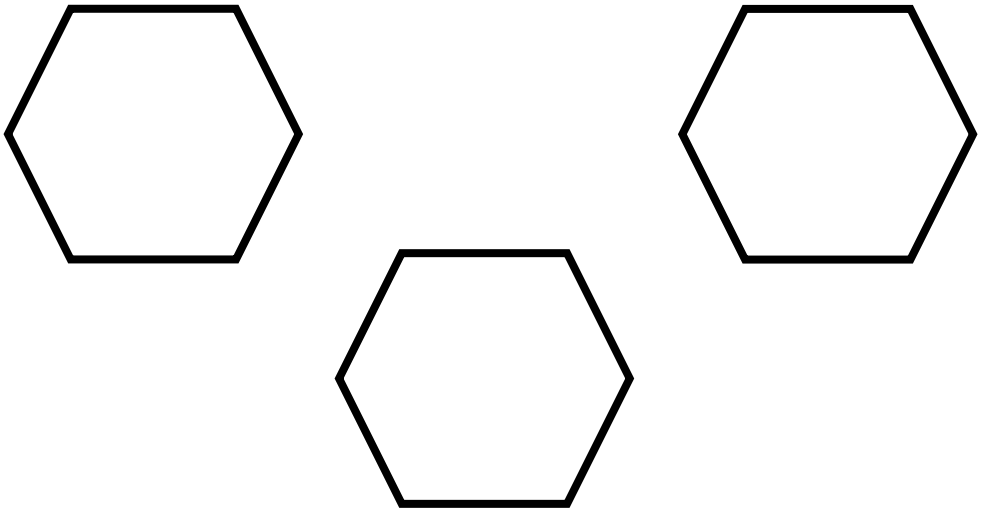
Draw one line to make 2 congruent trapezoids.

Show all three ways.



Draw one line to make 2 congruent pentagons.

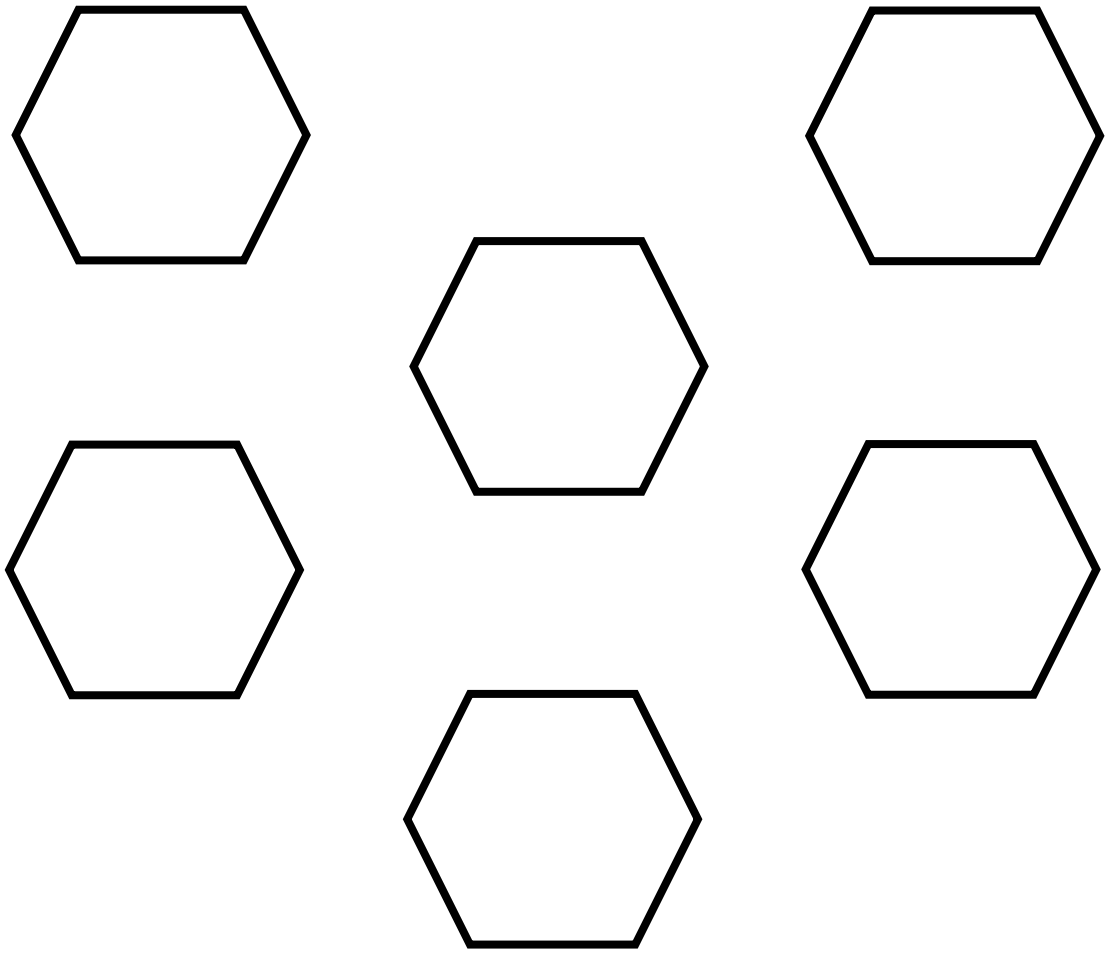
Show all three ways.



Hexagon Shape-Ups

Draw one line to make 1 triangle and 1 pentagon.

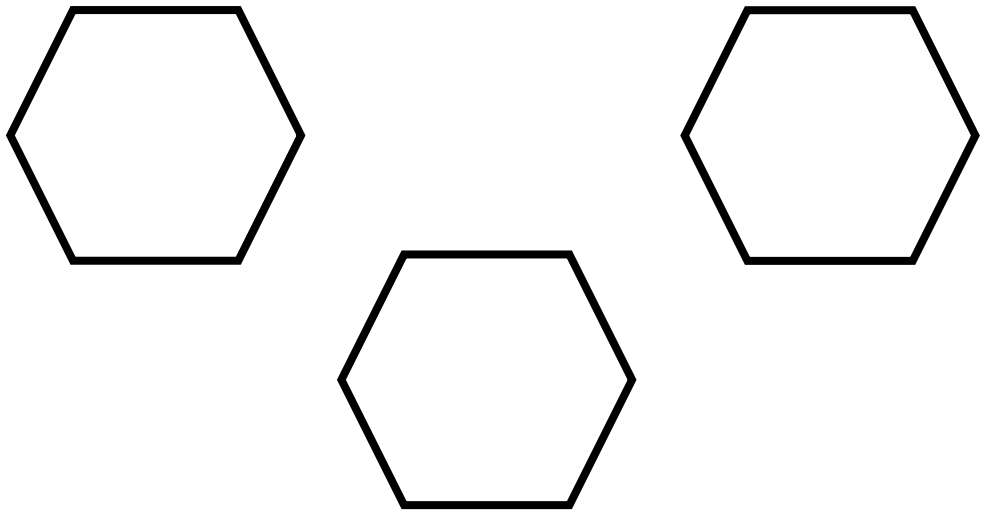
Show all six ways.



Hexagon Shape-Ups

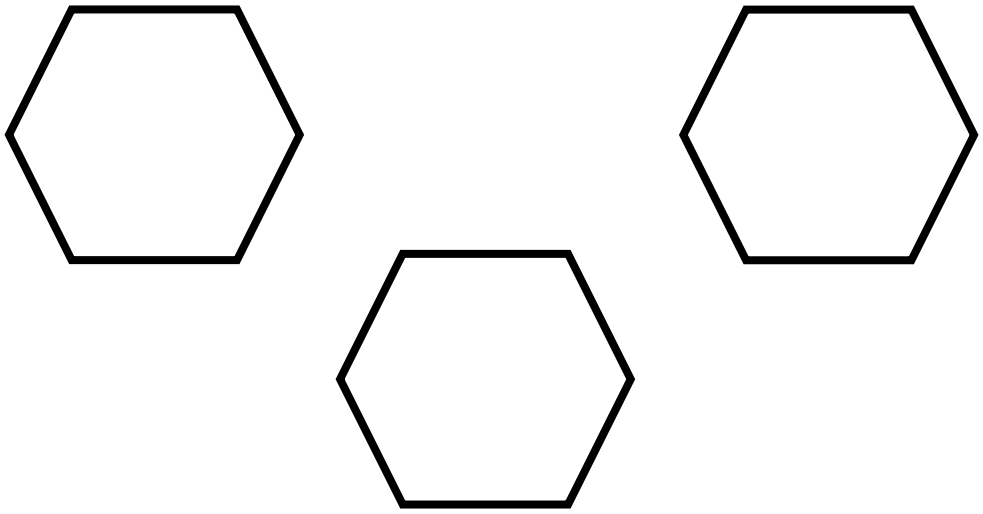
Draw one line to make 1 triangle and 1 heptagon.

Show three of the ways.



Draw one line to make 2 non-congruent pentagons.

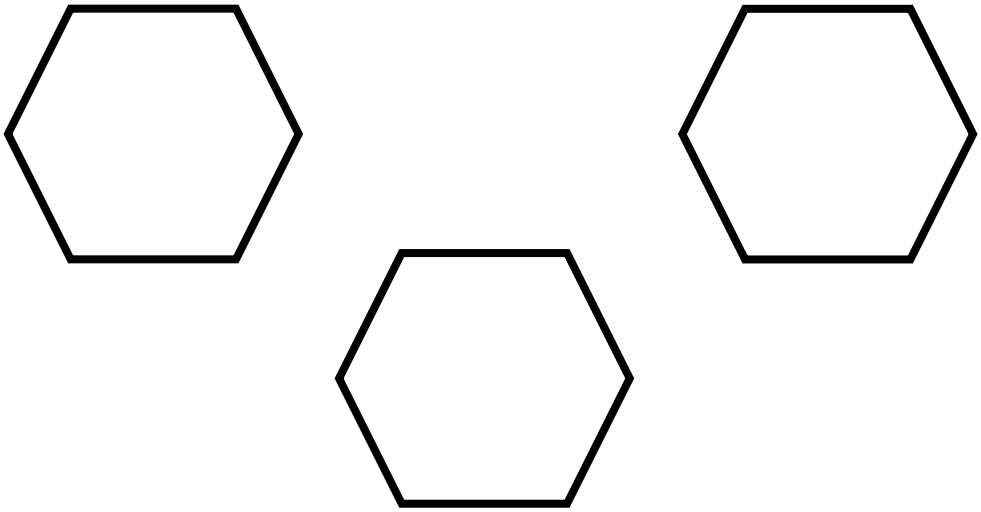
Show three of the ways.



Hexagon Shape-Ups

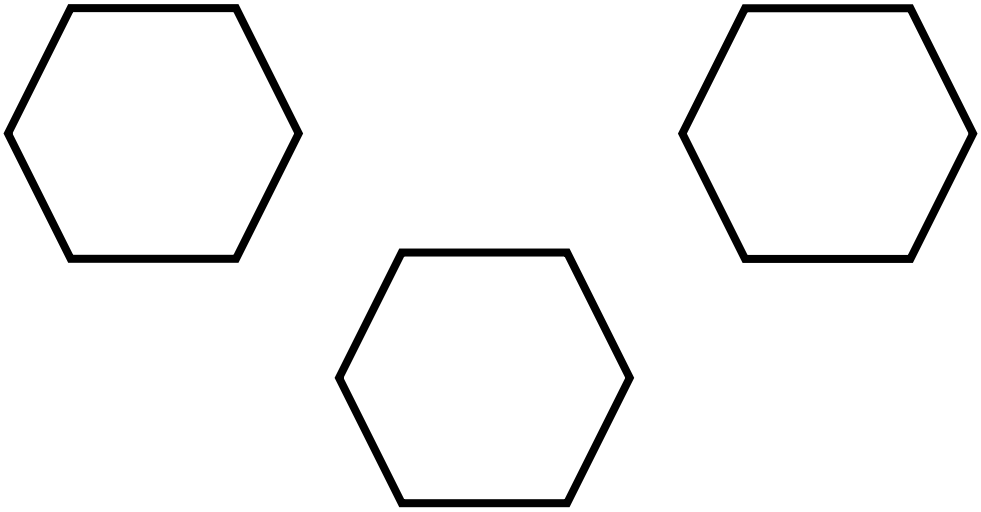
Draw one line to make 1 quadrilateral and 1 pentagon.

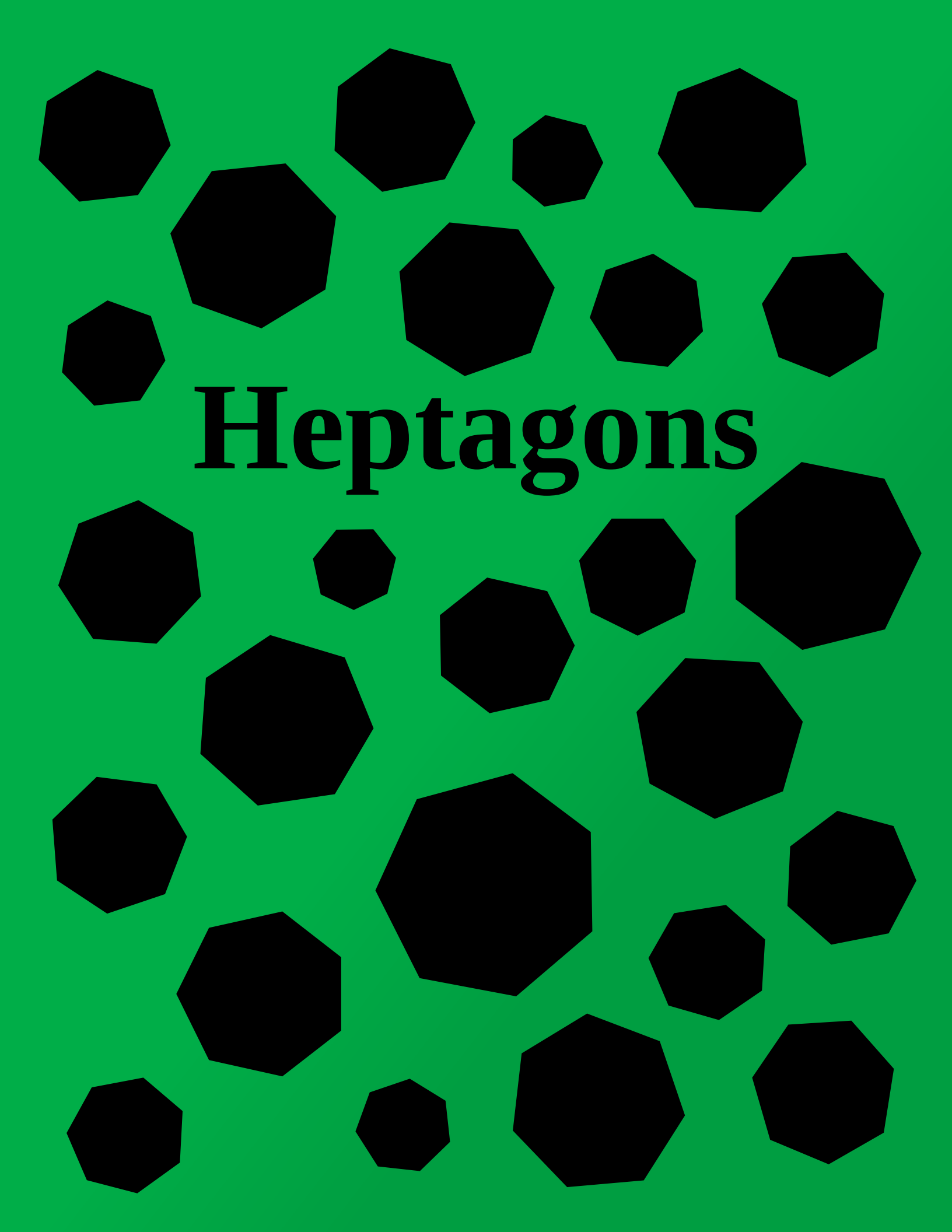
Show three of the ways.



Draw one line to make 1 trapezoid and 1 hexagon.

Show three of the ways.

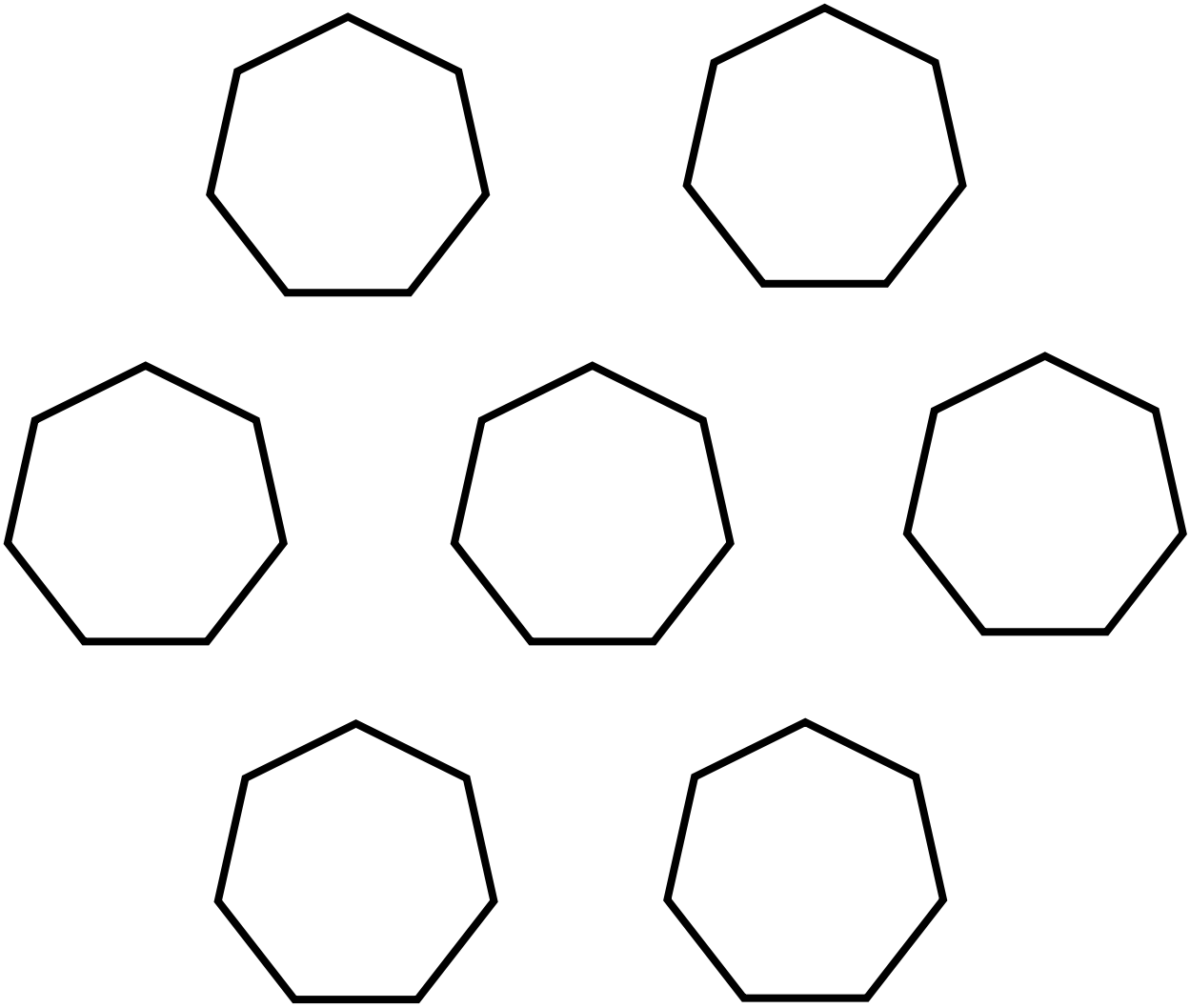



Heptagons

Heptagon Shape-Ups

Draw one line to make 2 congruent pentagons.

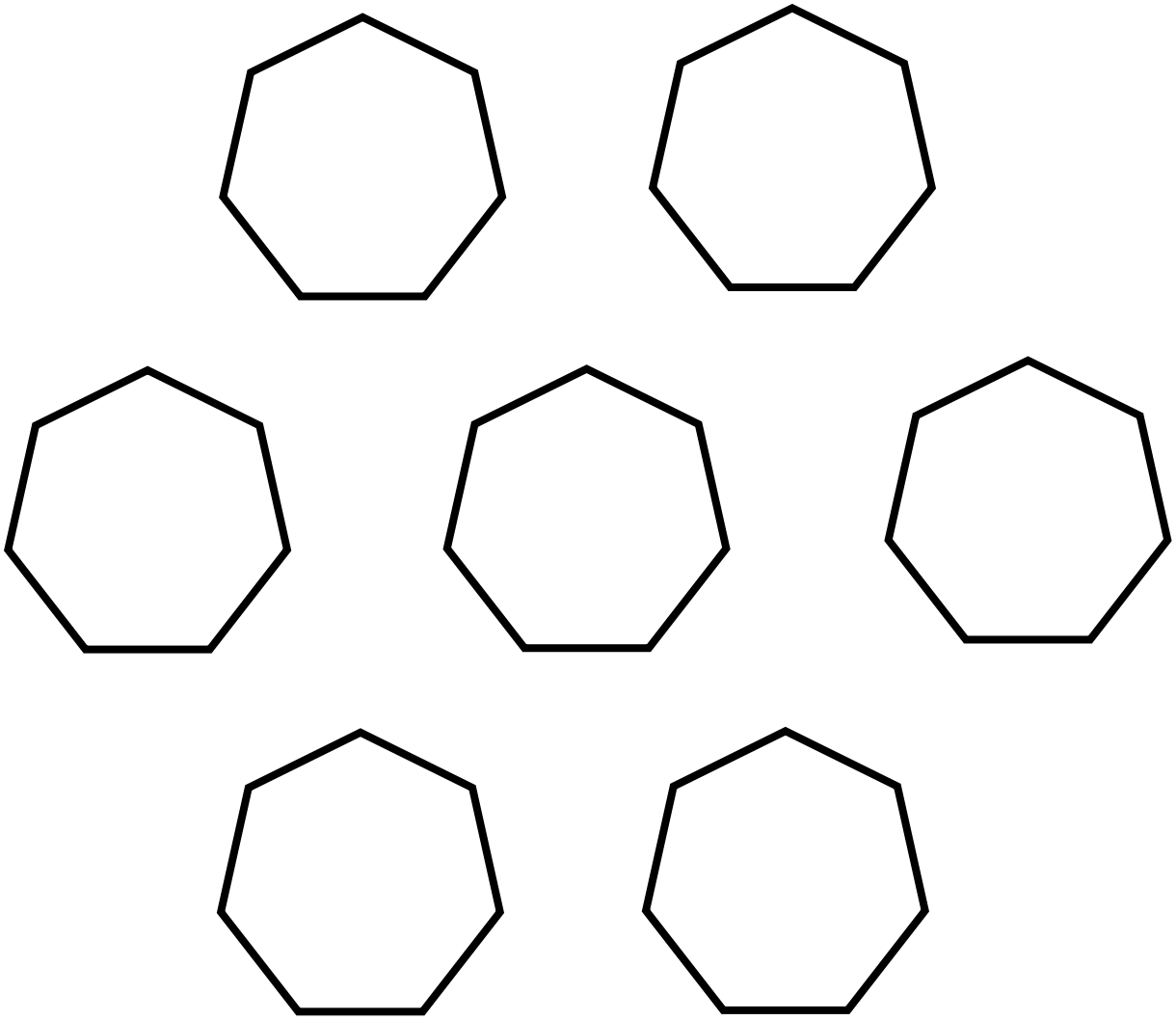
Show all seven ways.



Heptagon Shape-Ups

Draw one line to make 1 trapezoid and 1 pentagon.

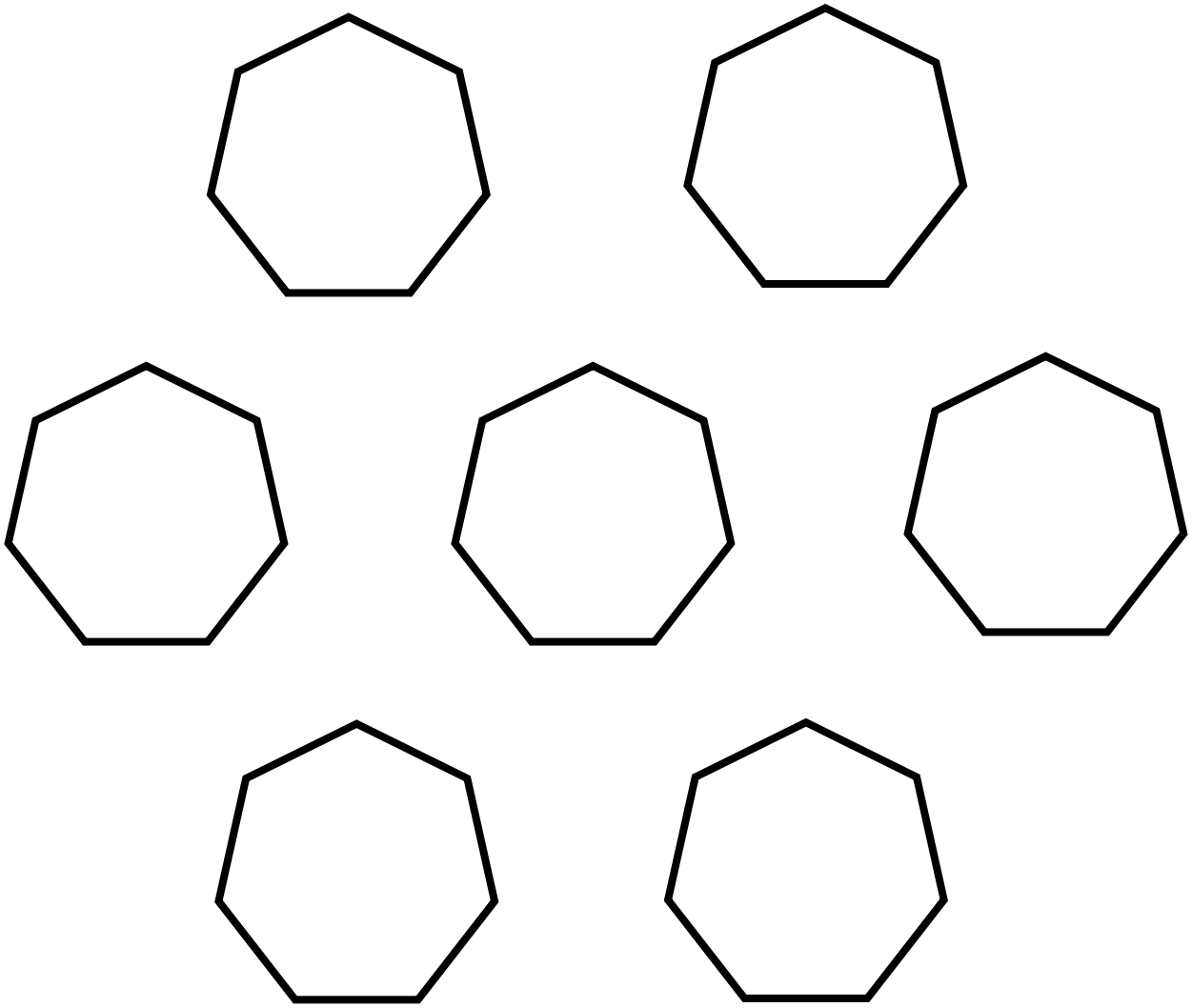
Show all seven ways.



Heptagon Shape-Ups

Draw one line to make 1 triangle and 1 hexagon.

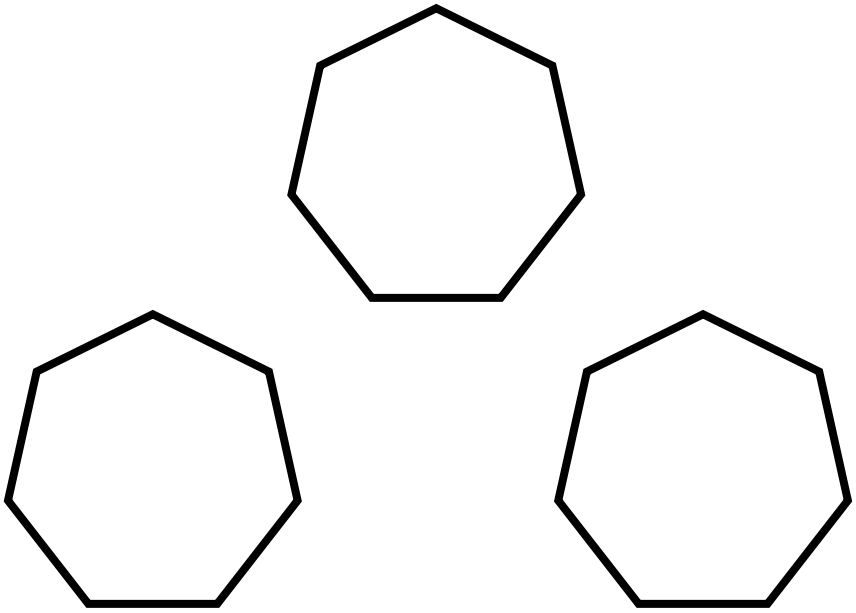
Show all seven ways.



Heptagon Shape-Ups

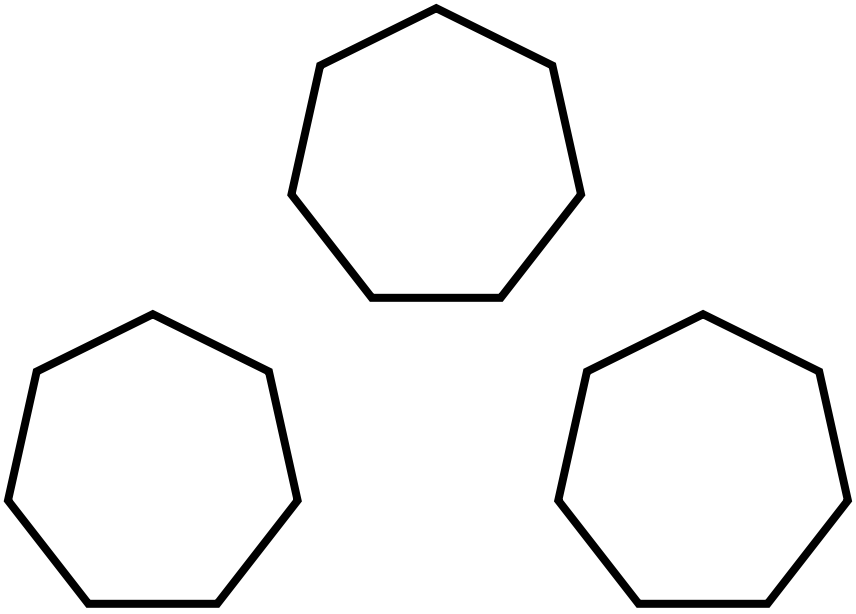
Draw one line to make 1 triangle and 1 heptagon.

Show three of the ways.



Draw one line to make 2 non-congruent pentagons.

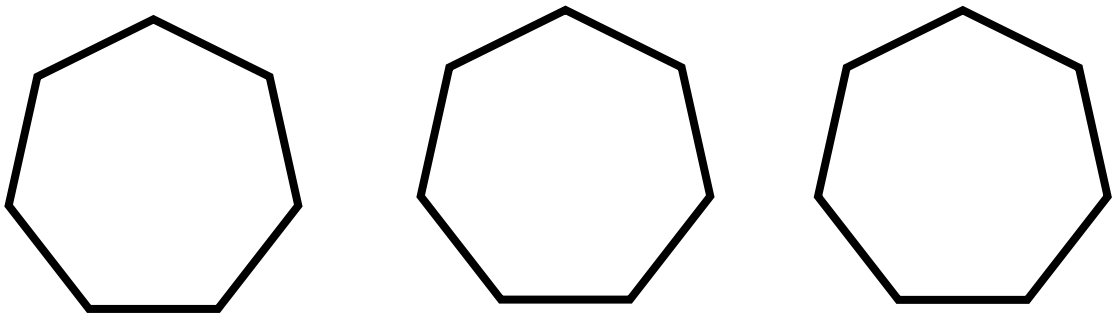
Show three of the ways.



Heptagon Shape-Ups

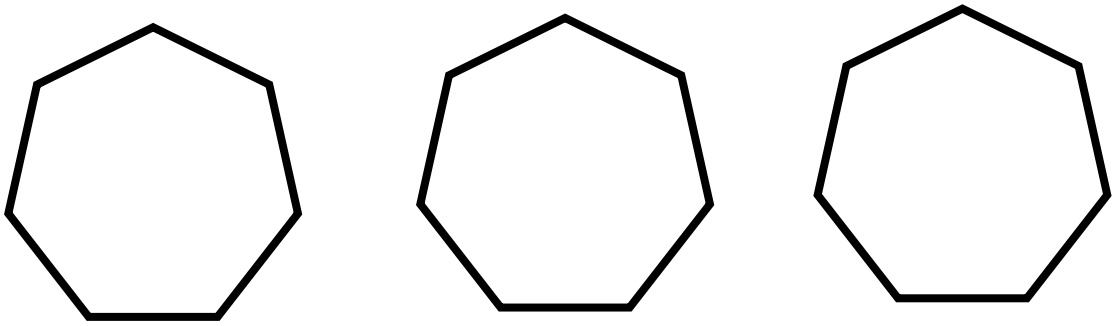
Draw one line to make 1 quadrilateral and 1 hexagon.

Show three of the ways.



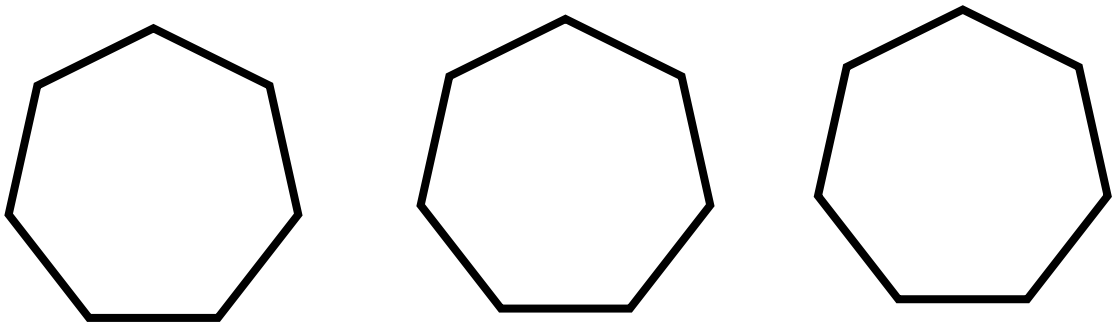
Draw one line to make 1 pentagon and 1 hexagon.

Show three of the ways.



Draw one line to make 1 trapezoid and 1 heptagon.

Show three of the ways.

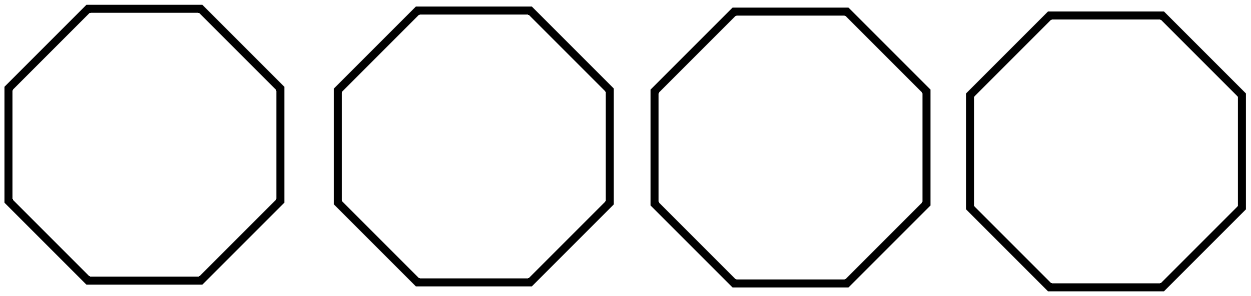



Octagons

Octagon Shape-Ups

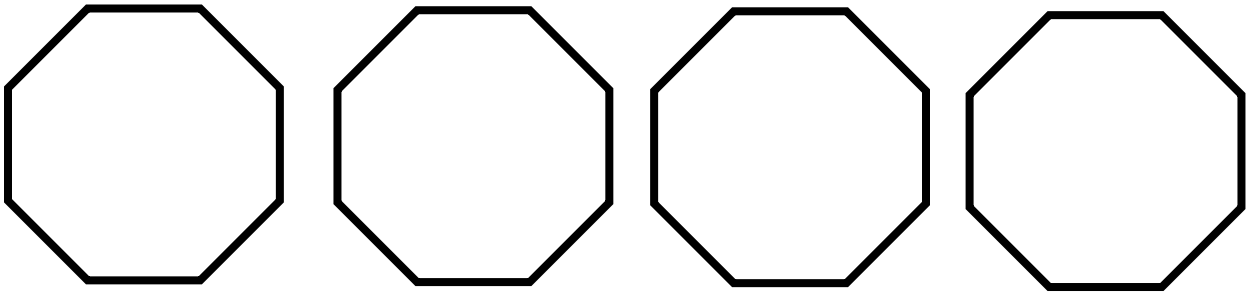
Draw one line to make 2 congruent pentagons.

Show all four ways.



Draw one line to make 2 congruent hexagons.

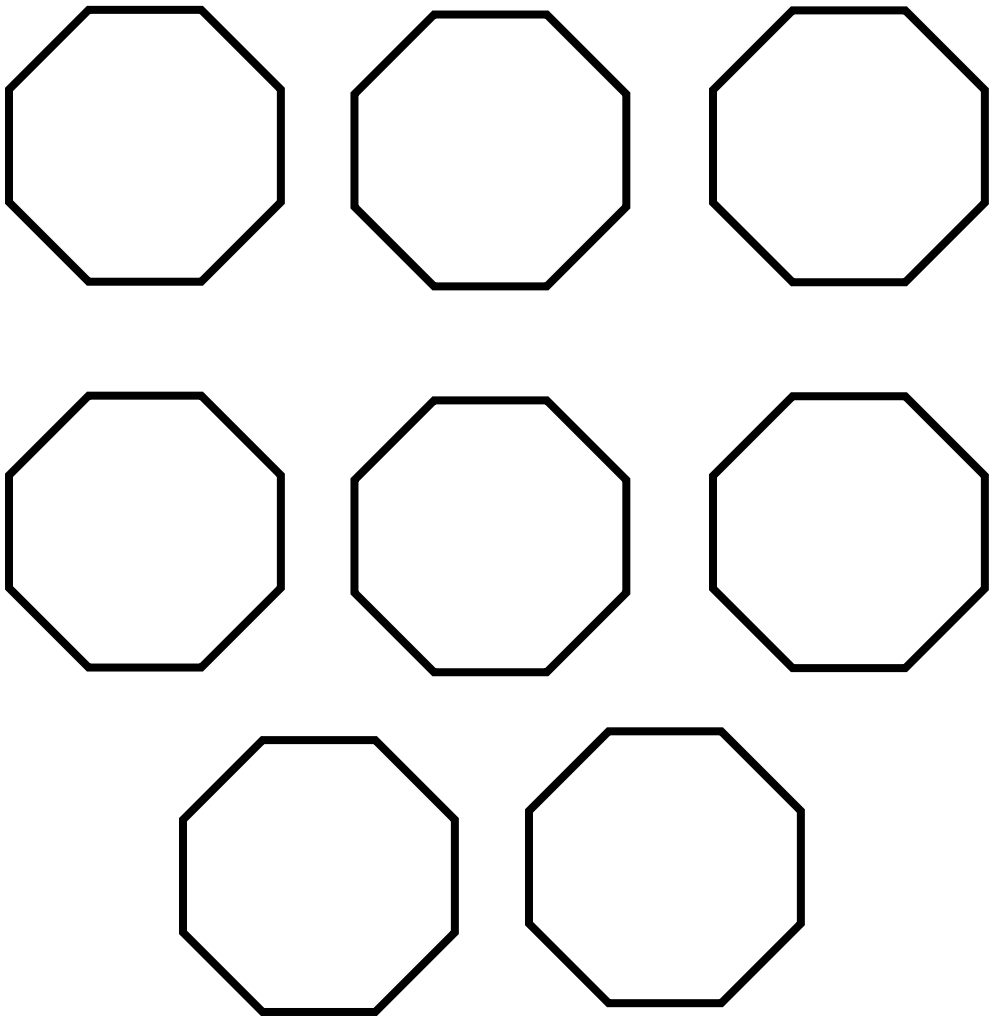
Show all four ways.



Octagon Shape-Ups

Draw one line to make 1 triangle and 1 heptagon.

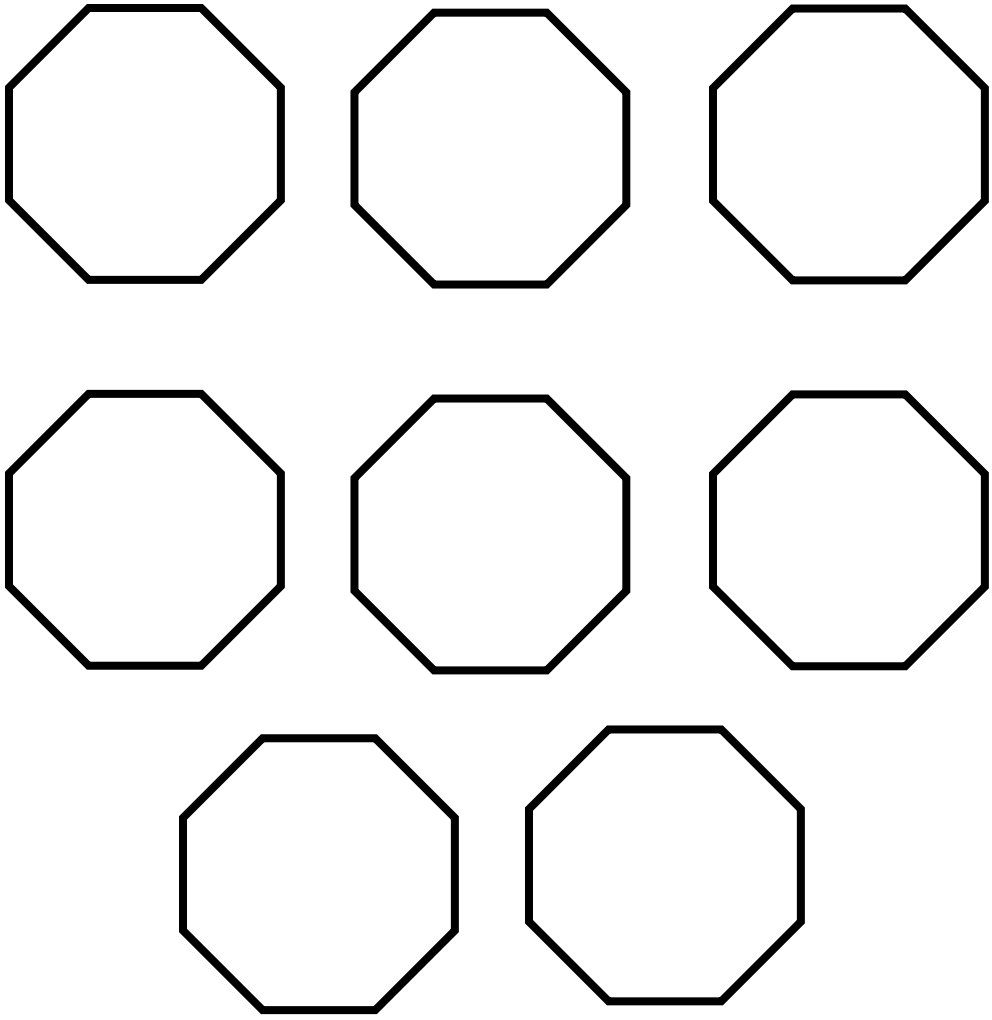
Show all eight ways.



Octagon Shape-Ups

Draw one line to make 1 trapezoid and 1 hexagon.

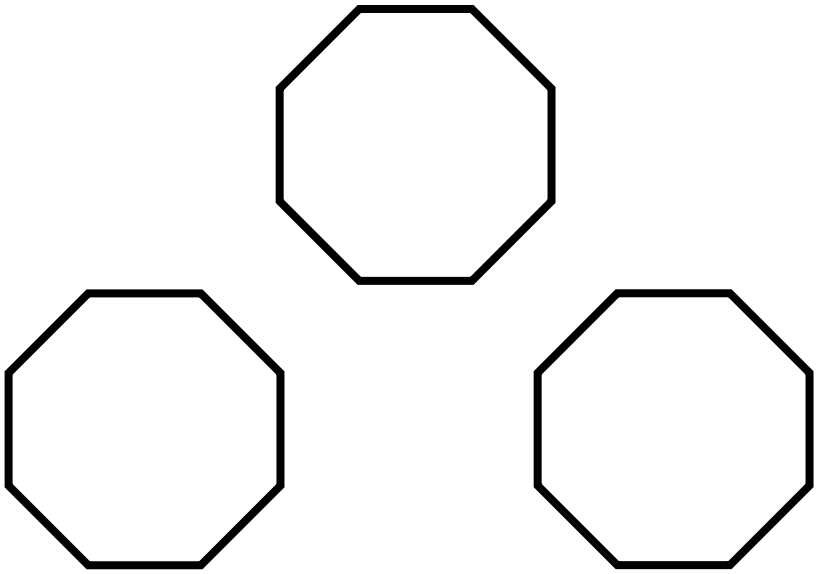
Show all eight ways.



Octagon Shape-Ups

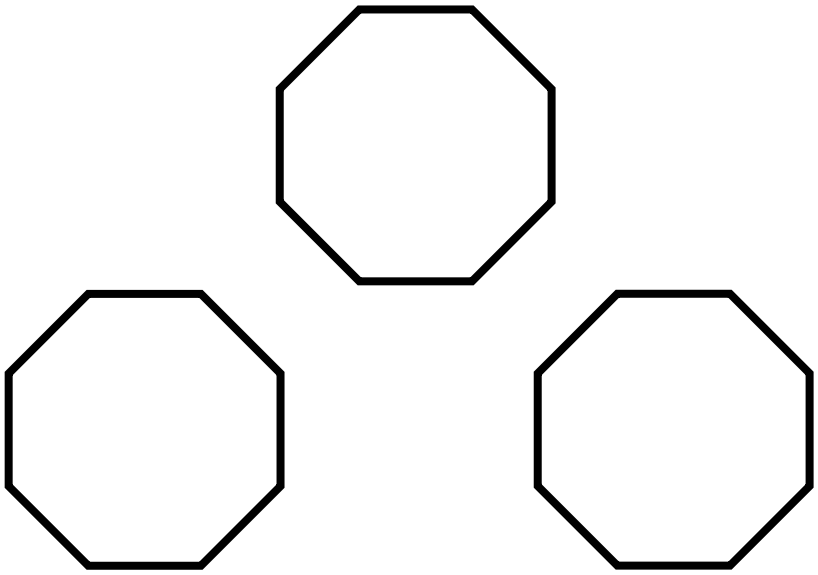
Draw one line to make 1 triangle and 1 octagon.

Show three of the ways.



Draw one line to make 1 quadrilateral and 1 heptagon.

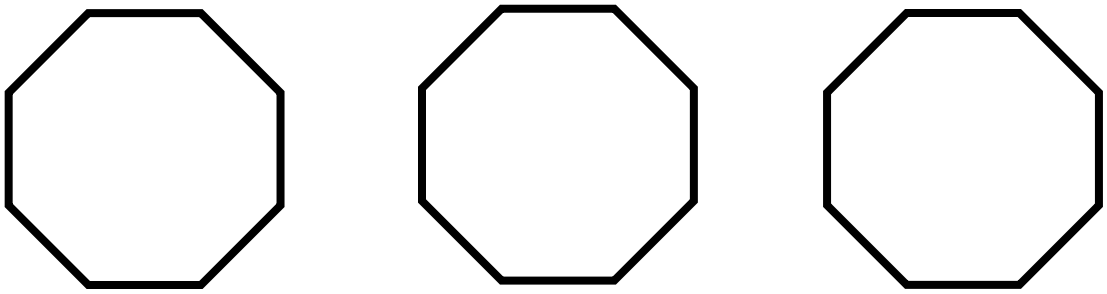
Show three of the ways.



Octagon Shape-Ups

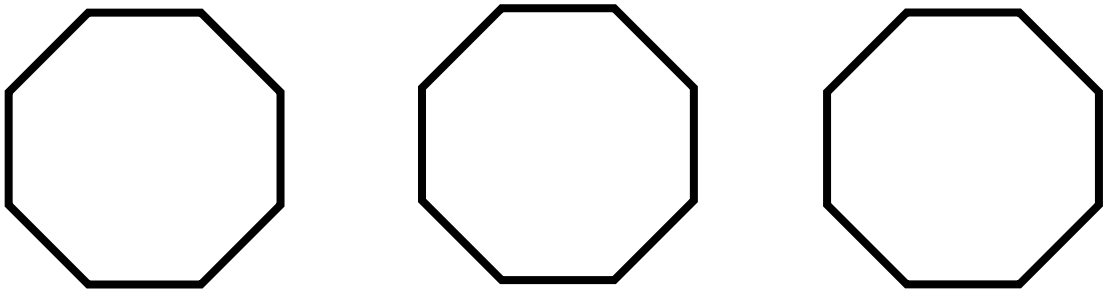
Draw one line to make 1 pentagon and 1 hexagon.

Show three of the ways.



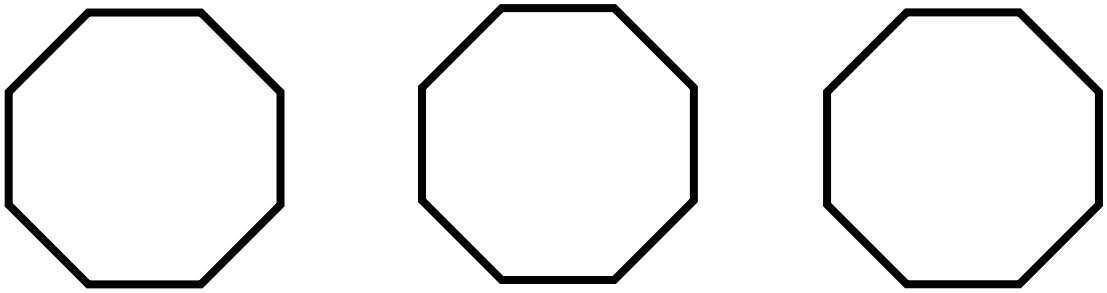
Draw one line to make 1 pentagon and 1 heptagon.

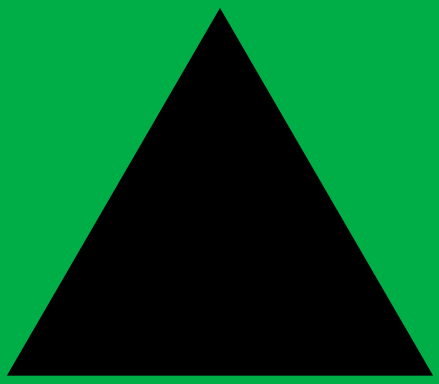
Show three of the ways.

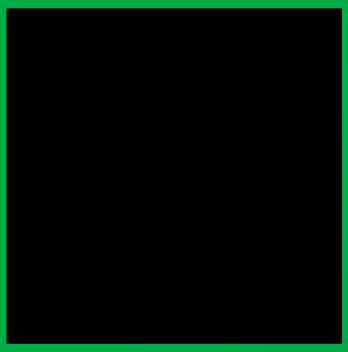


Draw one line to make 1 trapezoid and 1 octagon.

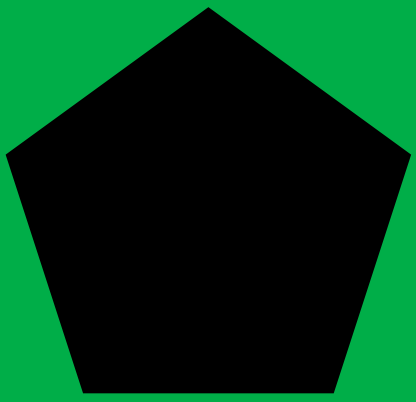
Show three of the ways.

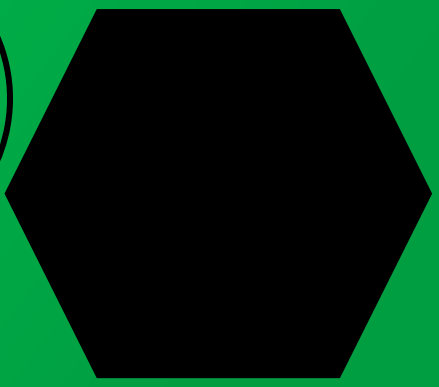


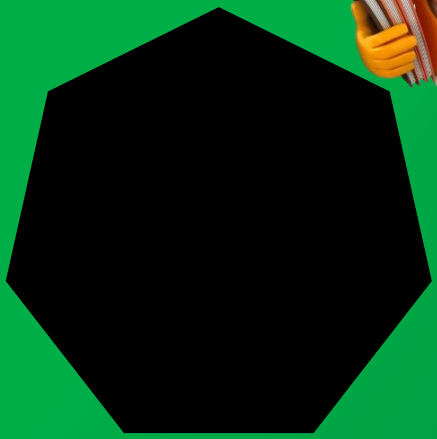


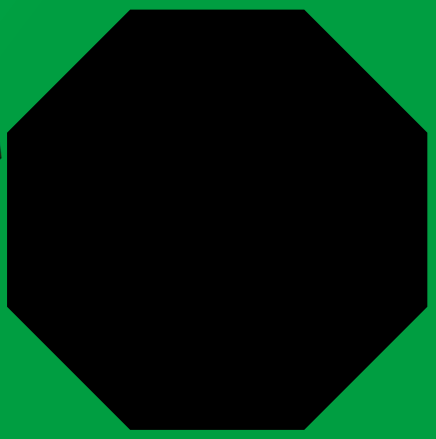


Shape-Up Solutions





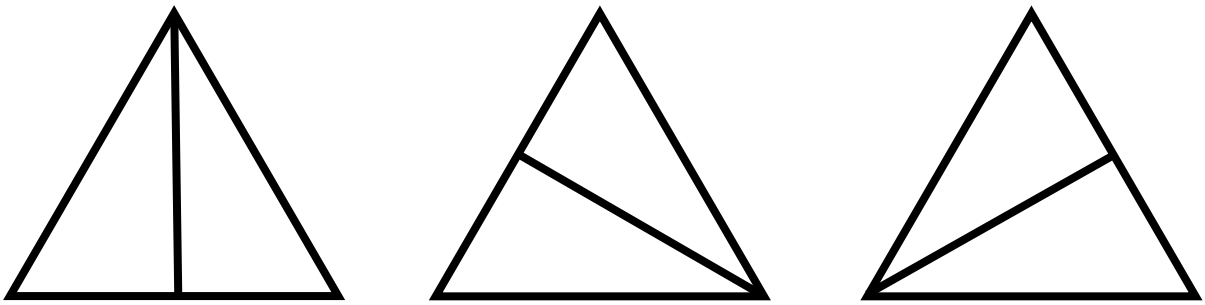




Triangle Shape-Up Solutions

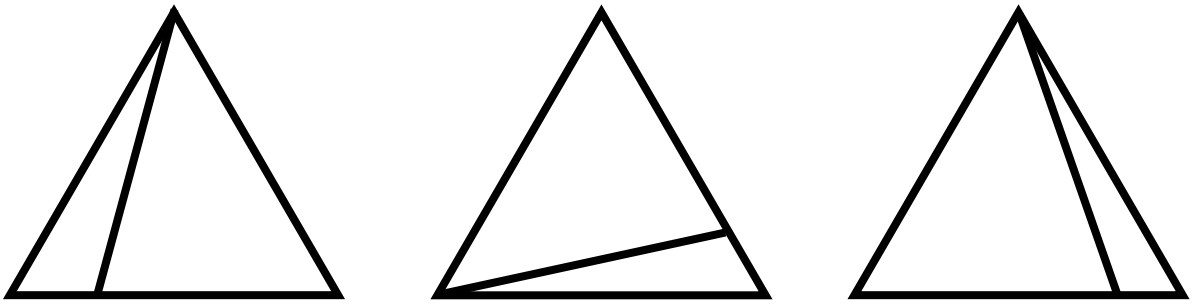
Draw one line to make 2 congruent triangles. [Page 10]

[There are exactly three solutions.]



Draw one line to make 2 non-congruent triangles. [Page 10]

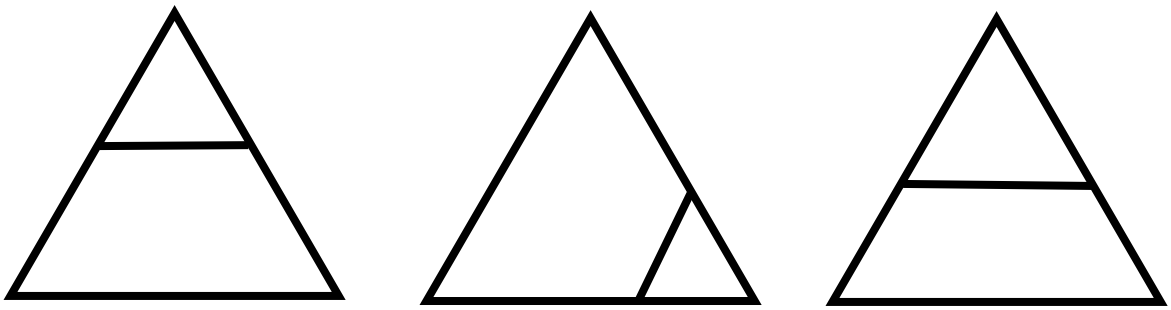
[There are an infinite number of solutions. Here are three solutions.]



Triangle Shape-Up Solutions

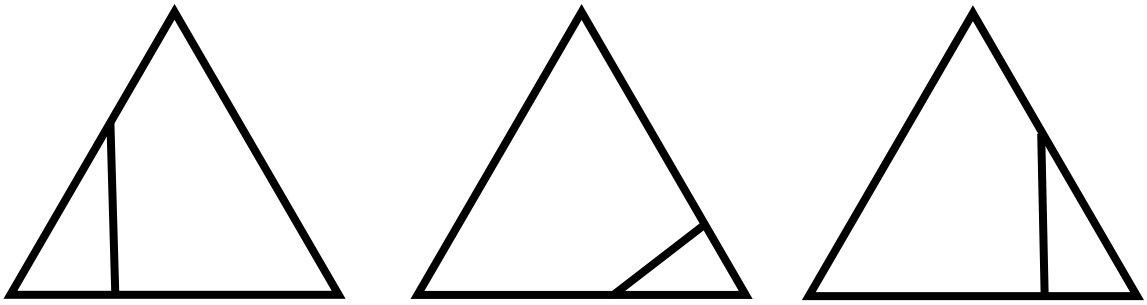
Draw one line to make 1 triangle and 1 trapezoid. [Page 11]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 1 right triangle and 1 quadrilateral. [Page 11]

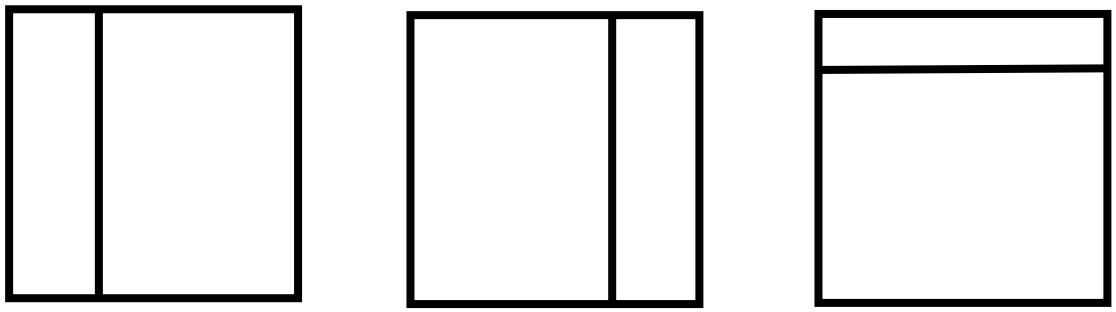
[There are an infinite number of solutions. Here are three solutions.]



Square Shape-Up Solutions

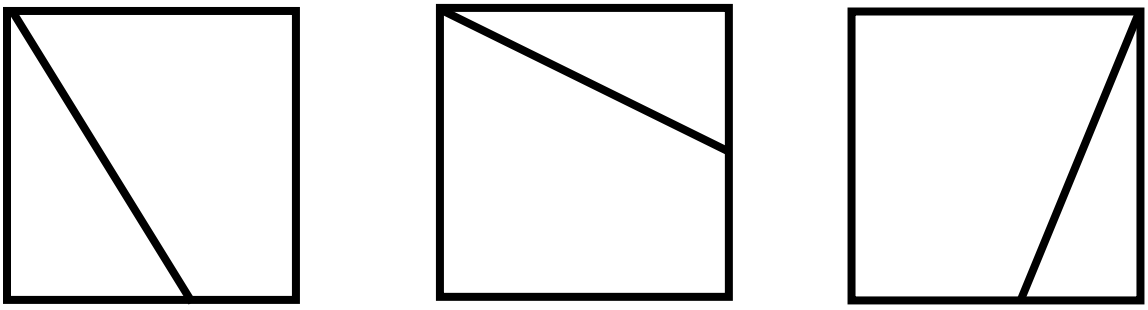
Draw one line to make 2 non-congruent rectangles. [Page 13]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 1 right triangle and 1 trapezoid. [Page 13]

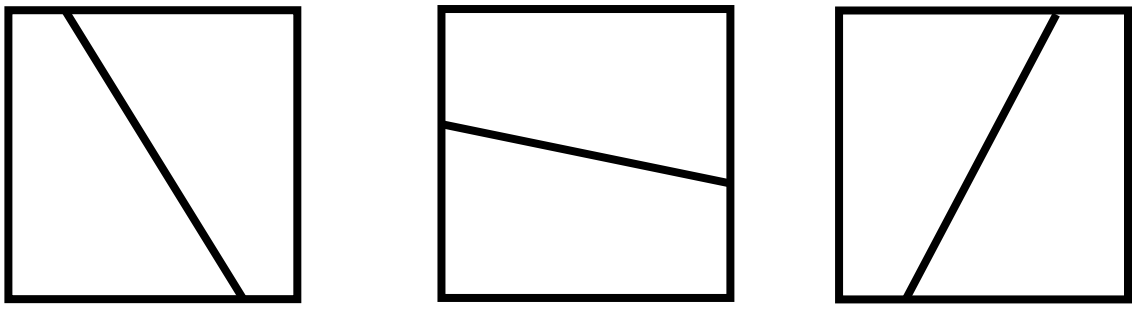
[There are an infinite number of solutions. Here are three solutions.]



Square Shape-Up Solutions

Draw one line to make 2 congruent trapezoids. [Page 14]

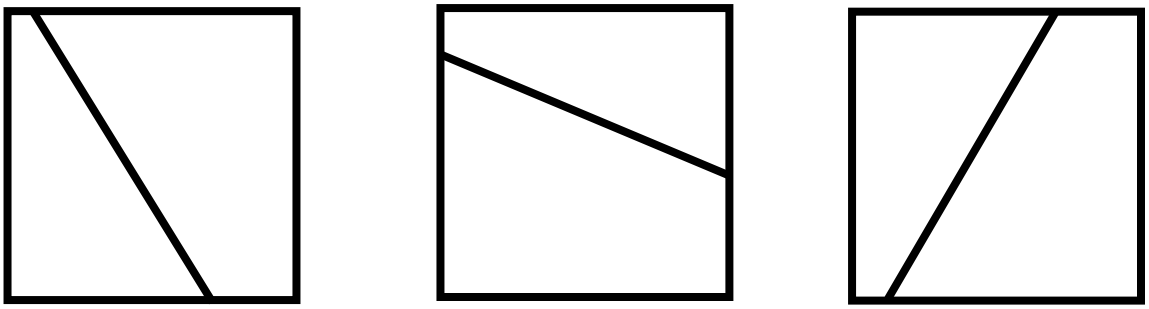
[There are an infinite number of solutions. Here are three solutions.]



Square Shape-Up Solutions

Draw one line to make 2 non-congruent trapezoids. [Page 14]

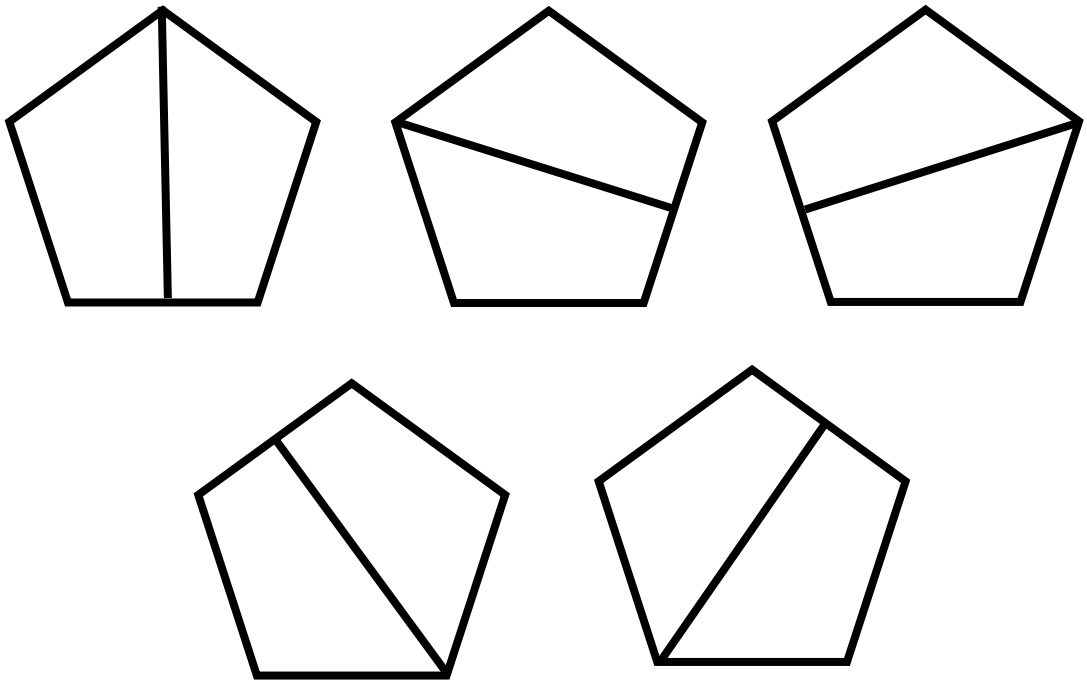
[There are an infinite number of solutions. Here are three solutions.]



Pentagon Shape-Up Solutions

Draw one line to make 2 congruent quadrilaterals. [Page 16]

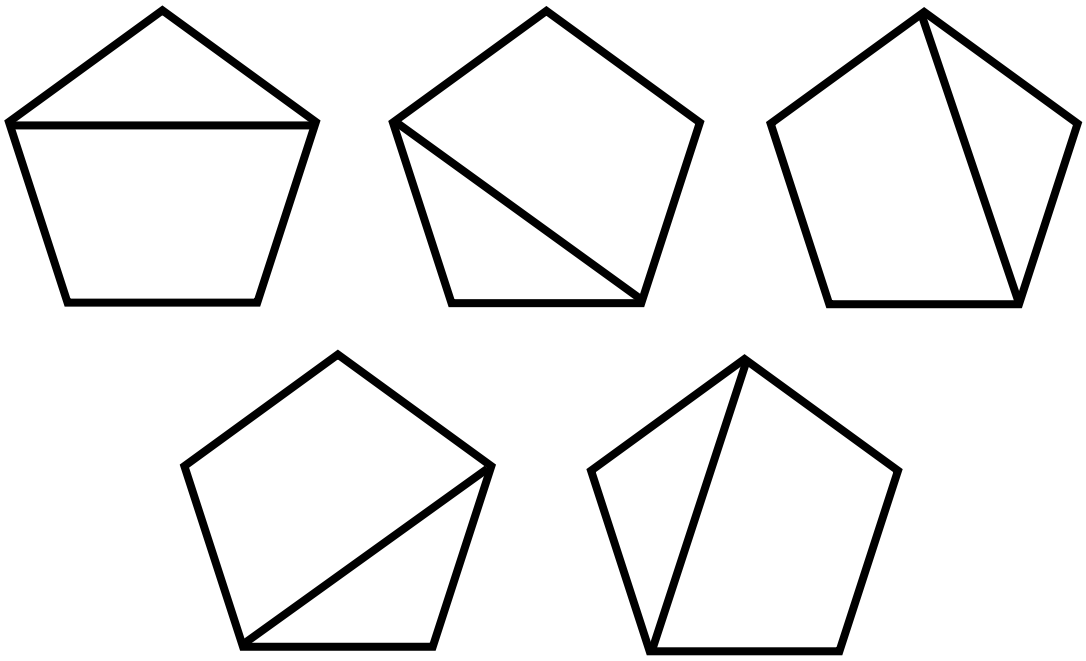
[There are exactly five solutions.]



Pentagon Shape-Up Solutions

Draw one line to make 1 triangle and 1 trapezoid. [Page 17]

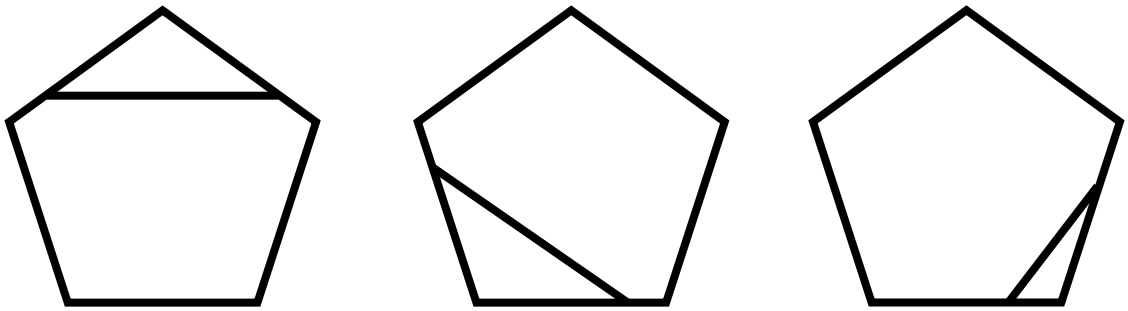
[There are exactly five solutions.]



Pentagon Shape-Up Solutions

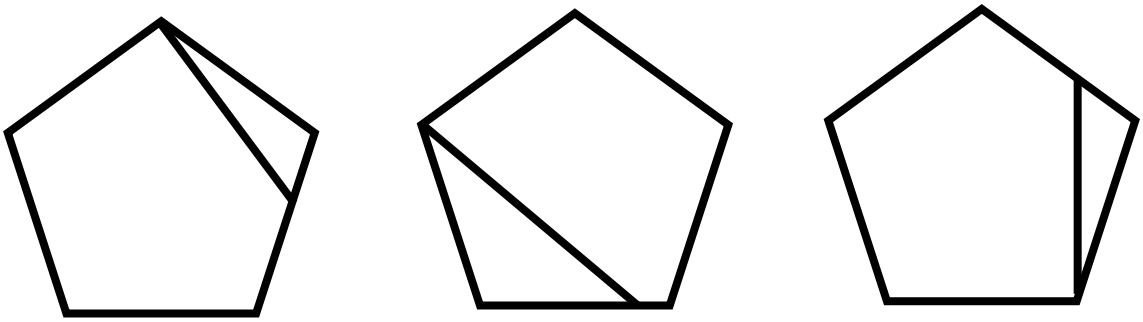
Draw one line to make 1 triangle and 1 hexagon. [Page 18]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 1 triangle and 1 pentagon. [Page 18]

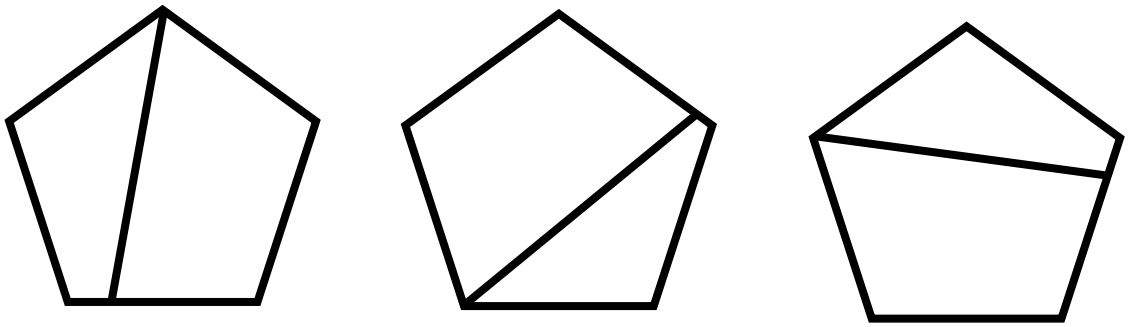
[There are an infinite number of solutions. Here are three solutions.]



Pentagon Shape-Up Solutions

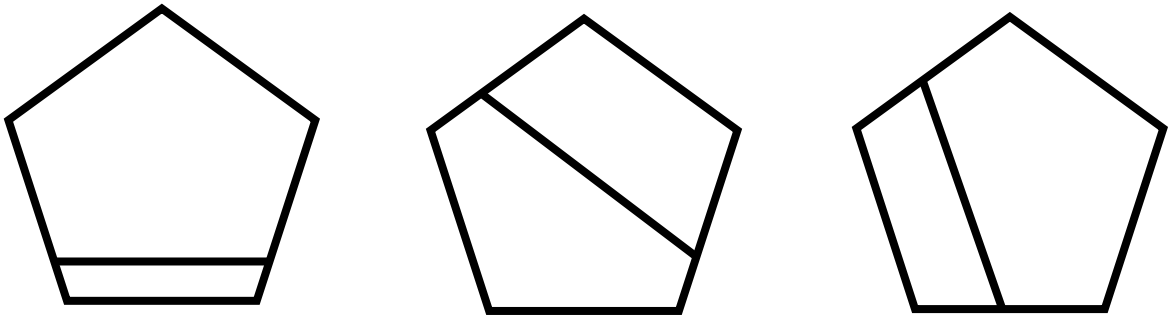
Draw one line to make 2 non-congruent quadrilaterals. [Page 19]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 1 trapezoid and 1 pentagon. [Page 19]

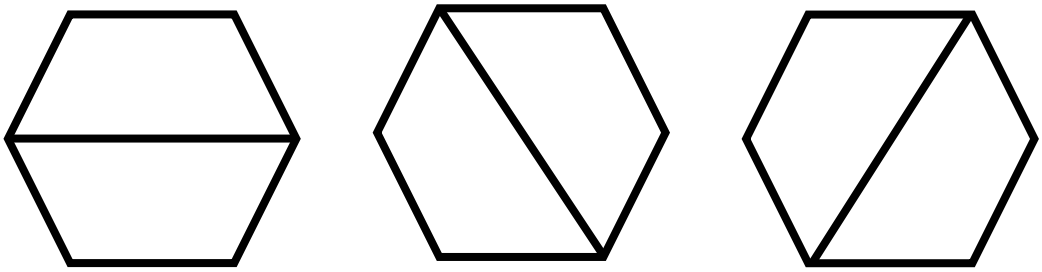
[There are an infinite number of solutions. Here are three solutions.]



Hexagon Shape-Up Solutions

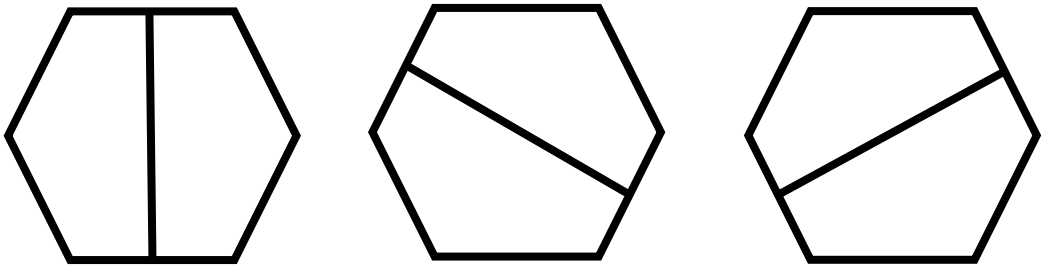
Draw one line to make 2 congruent trapezoids. [Page 21]

[There are exactly three solutions.]



Draw one line to make 2 congruent pentagons. [Page 21]

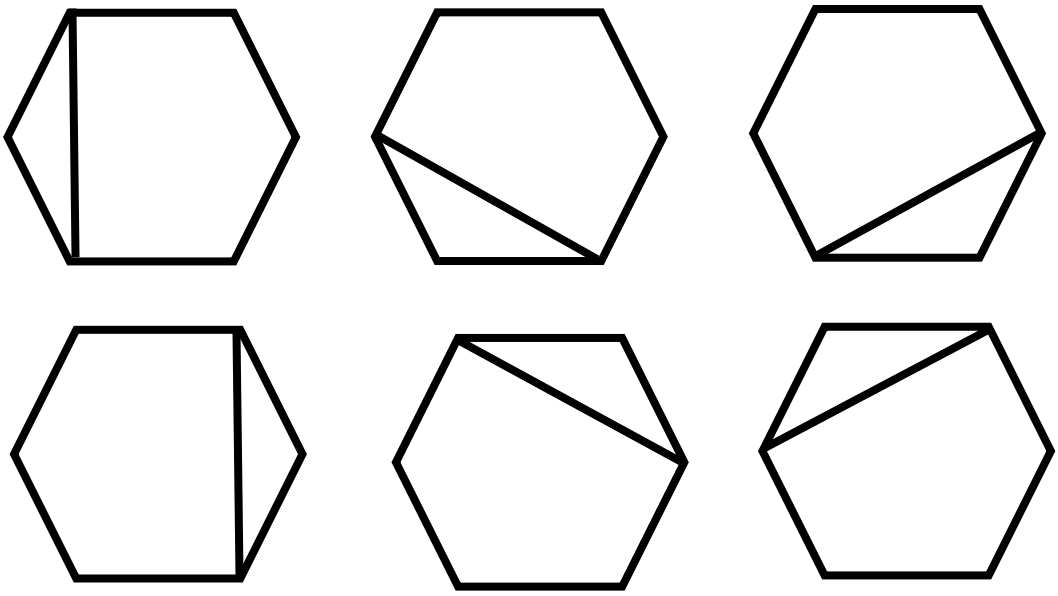
[There are exactly three solutions.]



Hexagon Shape-Up Solutions

Draw one line to make 1 triangle and 1 pentagon. [Page 22]

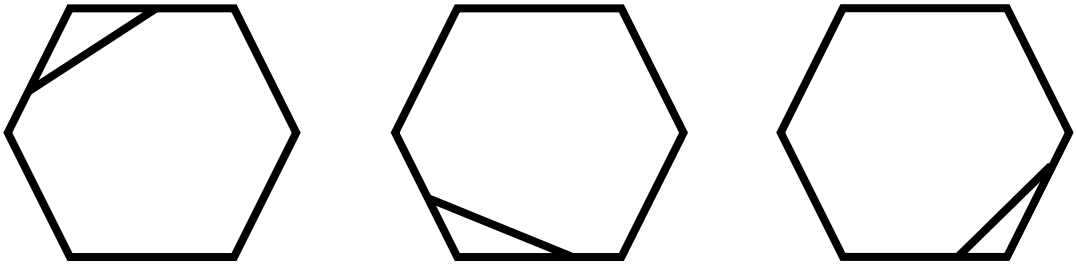
[There are exactly six solutions.]



Hexagon Shape-Up Solutions

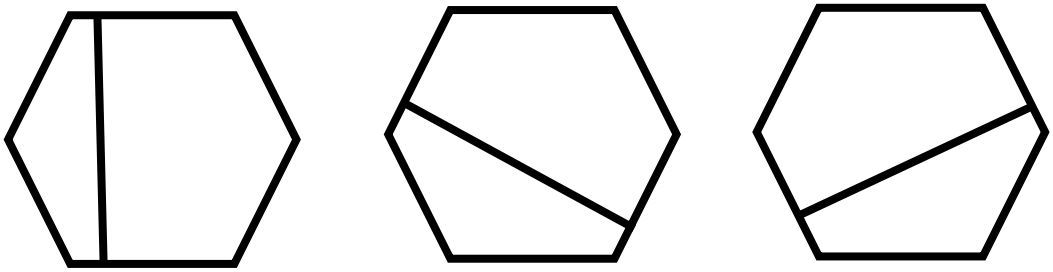
Draw one line to make 1 triangle and 1 heptagon. [Page 23]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 2 non-congruent pentagons. [Page 23]

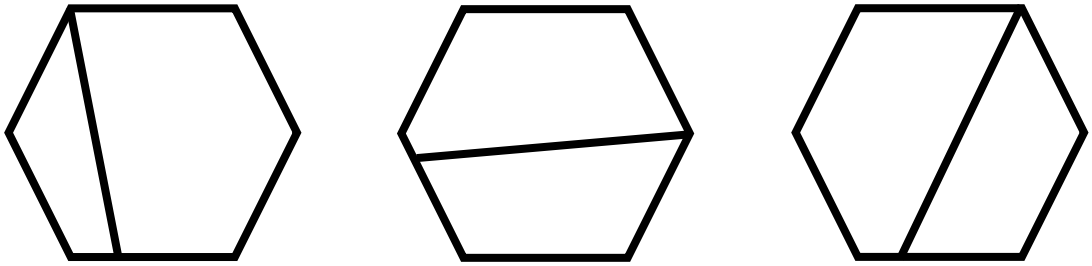
[There are an infinite number of solutions. Here are three solutions.]



Hexagon Shape-Up Solutions

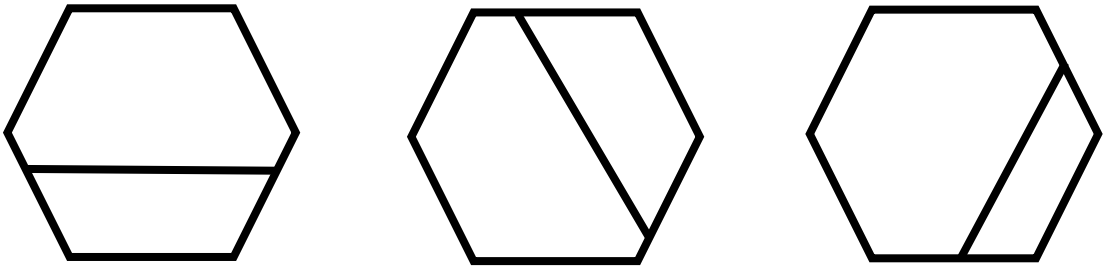
Draw one line to make 1 quadrilateral and 1 pentagon. [Page 24]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 1 trapezoid and 1 hexagon. [Page 24]

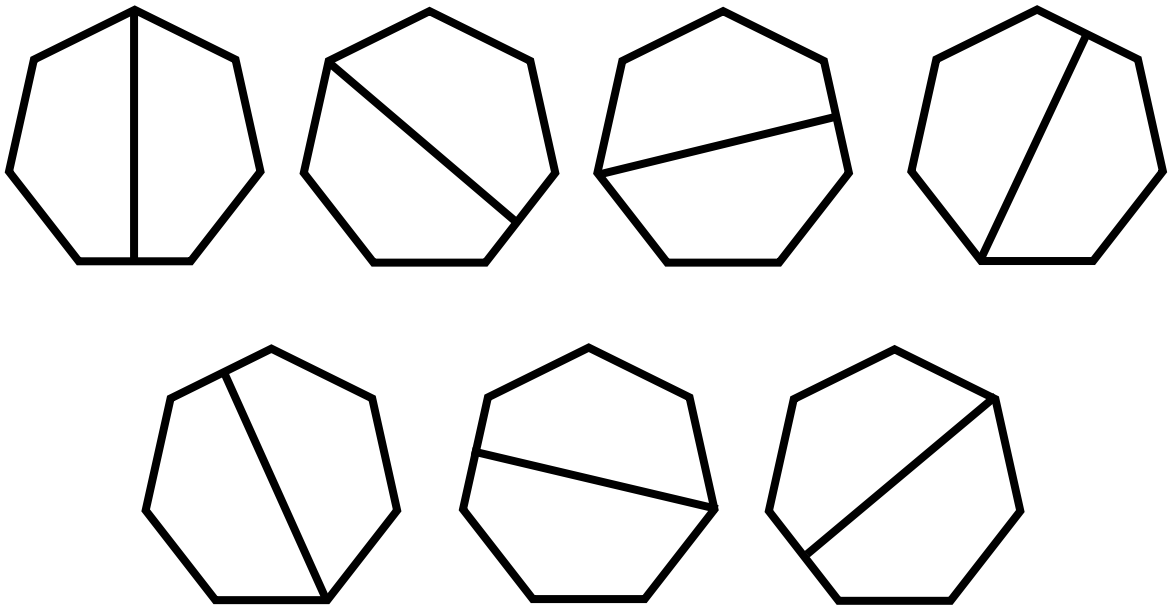
[There are an infinite number of solutions. Here are three solutions.]



Heptagon Shape-Up Solutions

Draw one line to make 2 congruent pentagons. [Page 26]

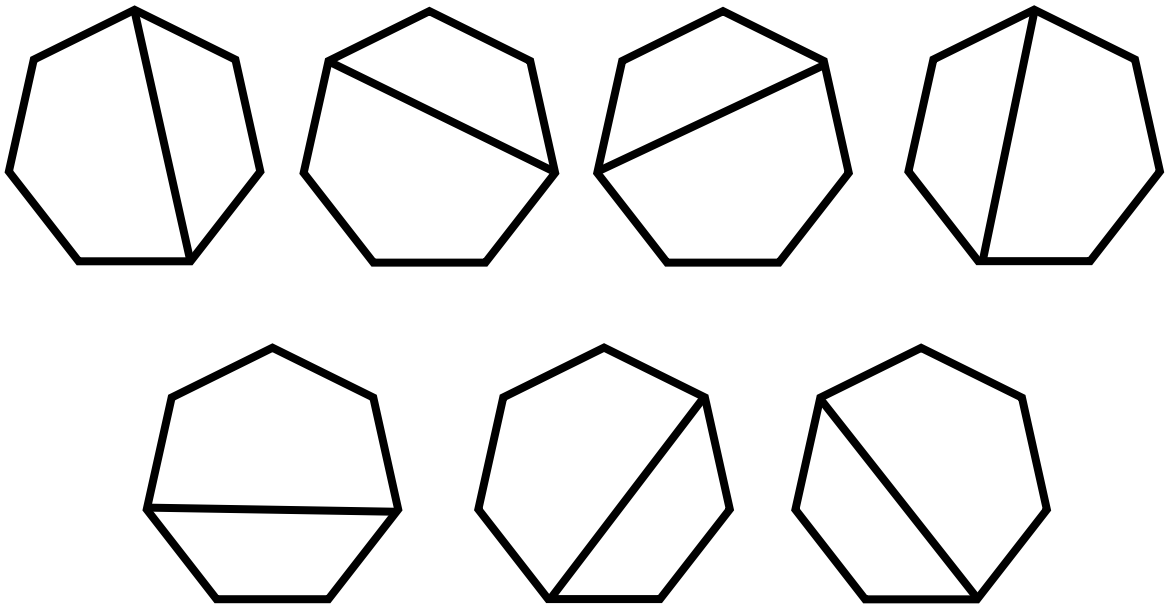
[There are exactly 7 solutions.]



Heptagon Shape-Up Solutions

Draw one line to make 1 trapezoid and 1 pentagon. [Page 27]

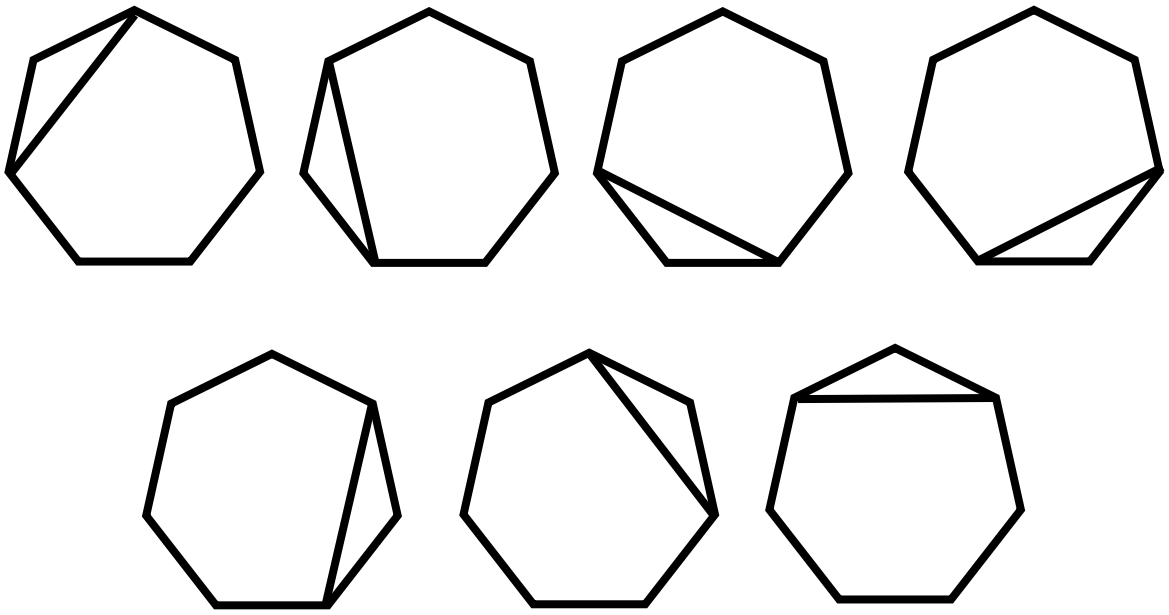
[There are exactly 7 solutions.]



Heptagon Shape-Up Solutions

Draw one line to make 1 triangle and 1 hexagon. [Page 28]

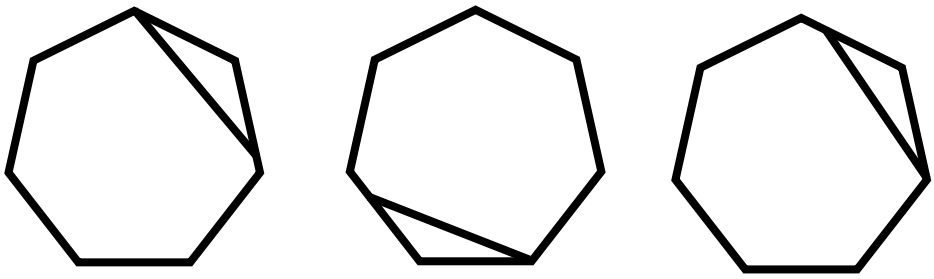
[There are exactly 7 solutions.]



Heptagon Shape-Up Solutions

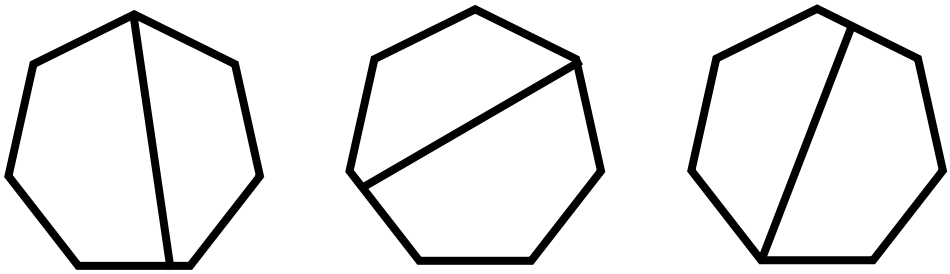
Draw one line to make 1 triangle and 1 heptagon. [Page 29]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 2 non-congruent pentagons. [Page 29]

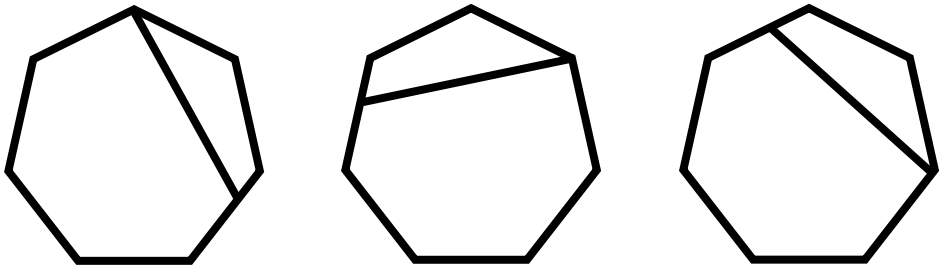
[There are an infinite number of solutions**.** Here are three solutions.]



Heptagon Shape-Up Solutions

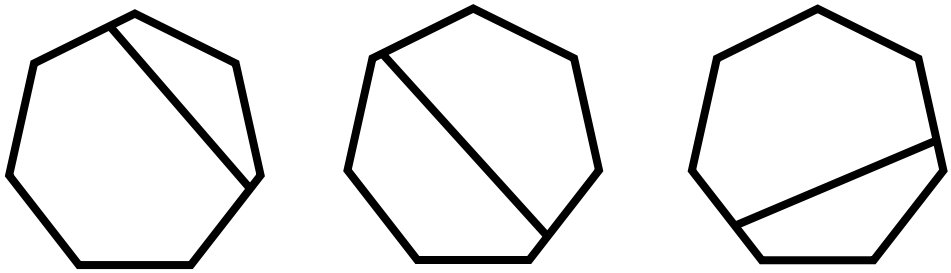
Draw one line to make 1 quadrilateral and 1 hexagon. [Page 30]

[There are an infinite number of solutions. Here are three solutions.]



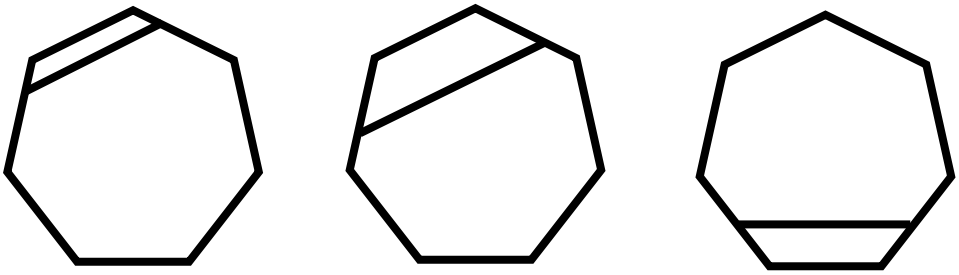
Draw one line to make 1 pentagon and 1 hexagon. [Page 30]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 1 trapezoid and 1 heptagon. [Page 30]

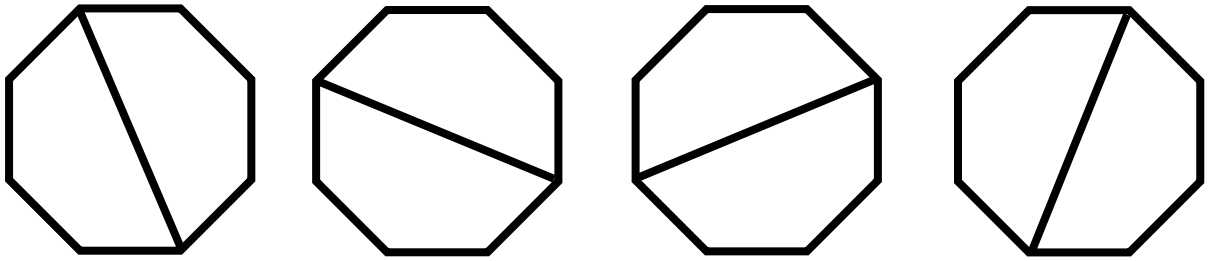
[There are an infinite number of solutions. Here are three solutions.]



Octagon Shape-Up Solutions

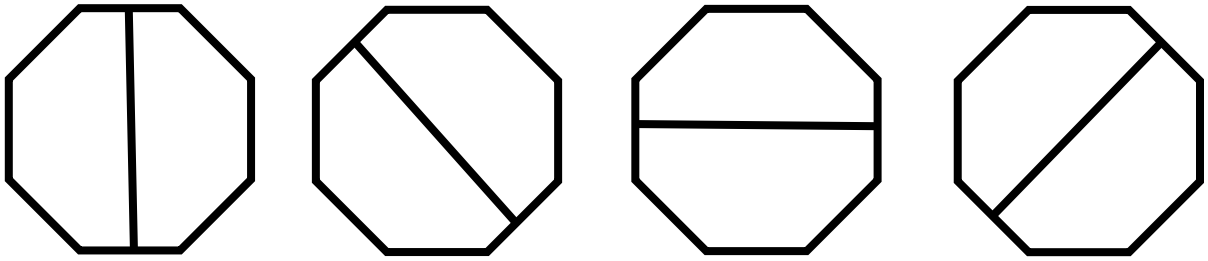
Draw one line to make 2 congruent pentagons. [Page 32]

[There are exactly 4 solutions.]



Draw one line to make 2 congruent hexagons. [Page 32]

[There are exactly 4 solutions.]

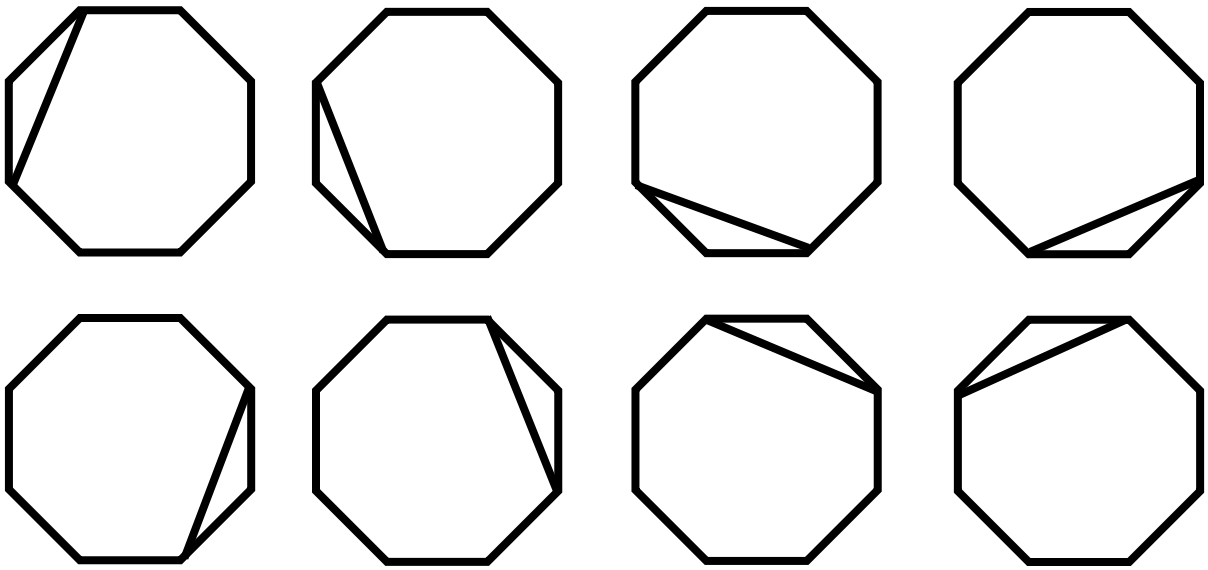


Octagon Shape-Up Solutions

Draw one line to make 1 triangle and 1 heptagon. [Page 33]

[There are exactly 8 solutions.]

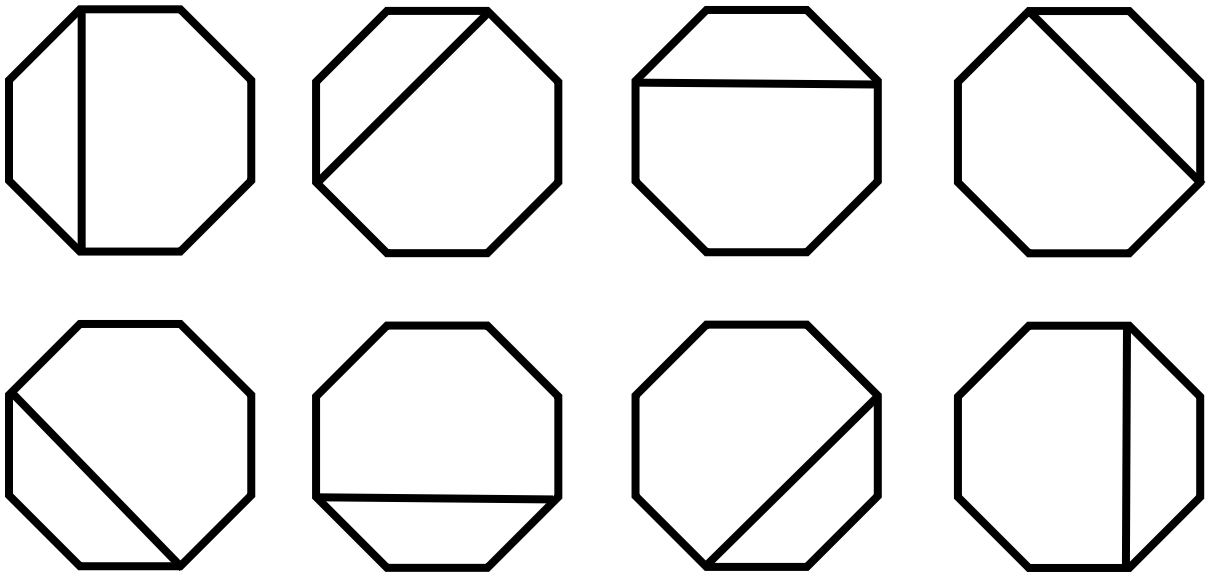
`



Octagon Shape-Up Solutions

Draw one line to make 1 trapezoid and 1 hexagon. [Page 34]

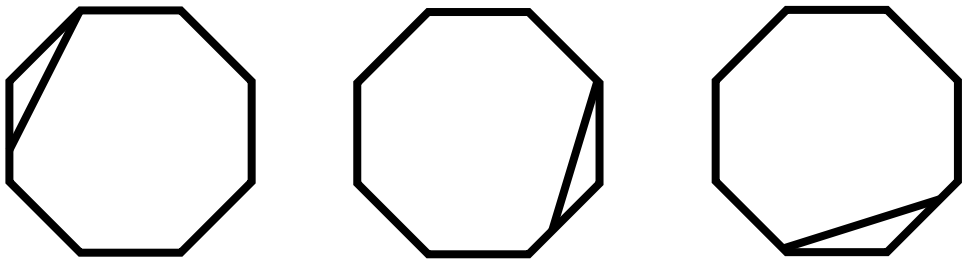
[There are exactly 8 solutions.]



Octagon Shape-Up Solutions

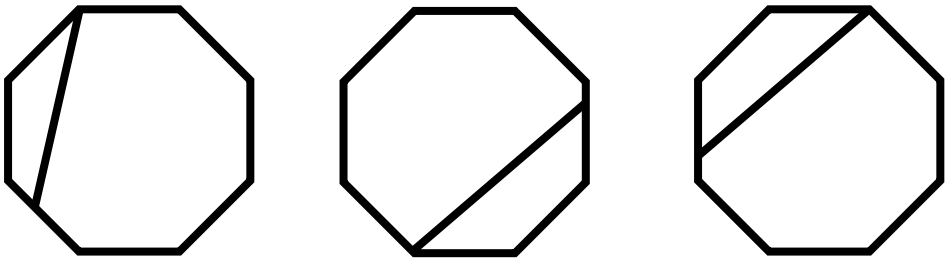
Draw one line to make 1 triangle and 1 octagon. [Page 35]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 1 quadrilateral and 1 heptagon. [Page 35]

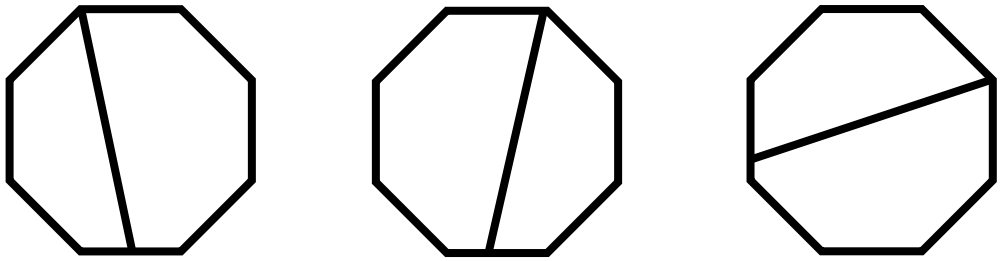
[There are an infinite number of solutions. Here are three solutions.]



Octagon Shape-Up Solutions

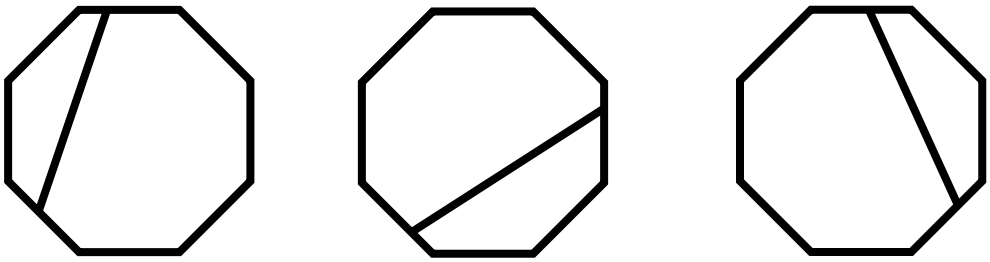
Draw one line to make 1 pentagon and 1 hexagon. [Page 36]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 1 pentagon and 1 heptagon. [Page 36]

[There are an infinite number of solutions. Here are three solutions.]



Draw one line to make 1 trapezoid and 1 octagon. [Page 36]

[There are an infinite number of solutions. Here are three solutions.]