

# MathLinks

## **PROFICIENCY CHALLENGES GRADE 8 SETS 13-16**

Proficiency Challenges are sets of interesting, mixed-topic problems. It may take a lot of time to complete each set, so consider doing only one or two parts at a time.

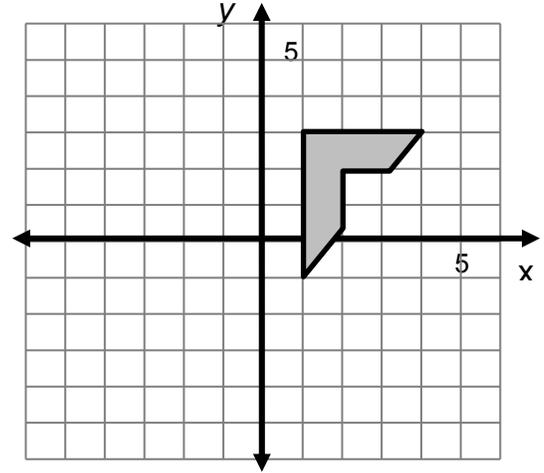
## PROFICIENCY CHALLENGE 13

Complete each problem on your own paper. Show all work. Use graph paper if needed.

1. Draw two images of the figure on the right under the following transformations:

- a. A reflection over the  $y$ -axis.
- b. A translation according to the rule

$$(x, y) \rightarrow (x + 1, y - 3).$$



2.  $\overline{AB}$  has endpoints  $A(-3, 6)$  and  $B(-3, -4)$ . This segment undergoes a sequence of transformations, and its resulting image is  $\overline{A'B'}$ . Here is the sequence of transformations:

- Translate according to the rule  $(x, y) \rightarrow (x + 2, y - 4)$ ;
- Reflected across the  $y$ -axis;
- Rotate  $90^\circ$  counterclockwise (left).

How long is  $\overline{A'B'}$ ? Explain how you figured it out.

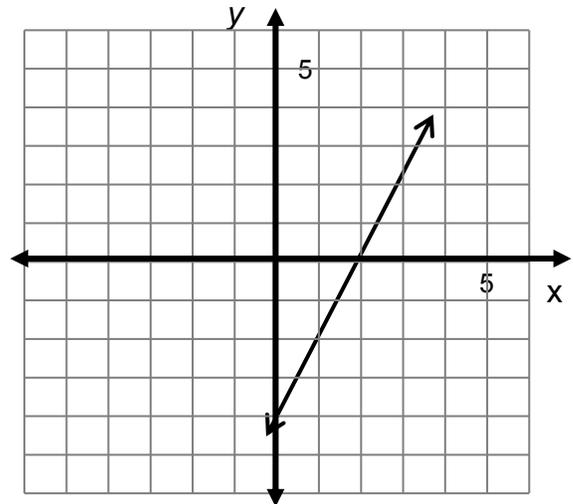
3. Choose all functions whose rate of change that is less than the rate of change of the function graphed on the right.

$$y = 3x - 1$$

$$y = \frac{1}{2}x + 4$$

$$y = 2x + 2$$

$$y = \frac{1}{3}x - 3$$



## PROFICIENCY CHALLENGE 14

Complete each problem on your own paper. Show all work. Use graph paper if needed.

1. Examine the graph to the right.

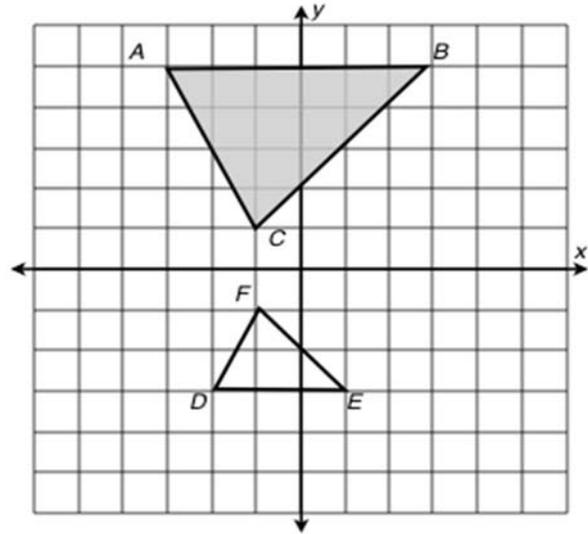
a. Describe a sequence of transformations that maps  $\triangle ABC$  onto  $\triangle DEF$ .

b. Christina thinks that  $\triangle ABC$  and  $\triangle DEF$  are congruent.

Is Christina correct? Justify your answer.

c. Marcelle thinks that  $\triangle ABC$  and  $\triangle DEF$  are similar.

Is Marcelle correct? If so, state the scale factor.



2. In a game played at family math night, players must spin a spinner and toss two number cubes. In order to win

- the spinner's arrow stops in a blue section AND
- both number cubes land with odd numbers facing up.

The person who created the game wants 20% of the players to win. Design a spinner for which the theoretical probability of winning reflects this person's goal.

3. Two sides of a right triangle have lengths  $\sqrt{12}$  units and  $\sqrt{13}$  units. There are two possible lengths for the third side.

What is the shorter possible side length for the third side?

What is the longer possible side length for the third side?

4. A right triangle has hypotenuse of 20 units long and each leg is a whole number of units long. Sketch the triangle and label the lengths of each leg. Explain your reasoning.

## PROFICIENCY CHALLENGE 15

Complete each problem on your own paper. Show all work. Use graph paper if needed.

- Warren and Julia are going to buy pizza for their friends. Their choices are to buy 4 small 6-inch diameter pizzas for \$3 each, or 1 large 12-inch diameter pizza for \$11. Both prices include tax and tip. The friends agree that their best choice is the one that gives them the most pizza for the money. What is the best choice? Explain your answer.
- A sphere and a cone have the same volume. Each figure has a radius of 6 inches. What is the height of the cone?
- Mila and Gila are given  $\triangle ABC$  and told to perform a sequence of three of the following transformations of the plane.

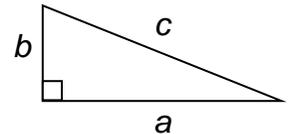
Translation

Rotation

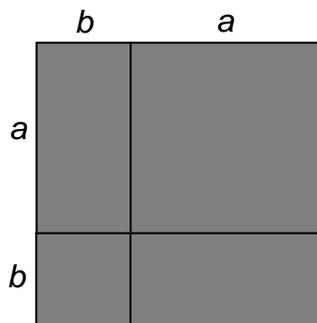
Reflection

Dilation

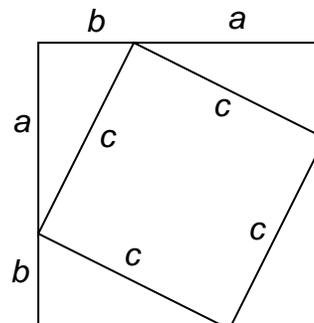
- Mila says that the image she obtains,  $\triangle DEF$ , is congruent to  $\triangle ABC$ . Explain in detail what sequence of transformations Mila could have performed. Include a sketch of the original triangle and its image.
  - Gila says that the image she obtains,  $\triangle TUV$ , is similar to  $\triangle ABC$ . Explain in detail what sequence of transformations Gila could have performed. Include a sketch of the original triangle and its image.
- Sketch the congruent squares as they appear below. They were created using lengths of the sides of the triangle pictured to the right.



- Write the areas inside the polygonal pieces in the two square figures.
- Write an equation that equates the sum of the areas of the shaded polygons with the sum of the areas of the unshaded polygons.
- Simplify the equation.
- Use words to state the meaning of this equation as it refers to the legs and the hypotenuse of the original triangle.
- What is this relationship called?



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## PROFICIENCY CHALLENGE 16

Complete each problem on your own paper. Show all work. Use graph paper if needed.

1. Barbara and Juan have different methods of finding cube roots.

- Barbara finds the cube root of  $x$  by finding a number so that the product of the number times itself, then times itself again is  $x$ . For example, the cube root of 27 is 3 because  $3 \cdot 3 \cdot 3 = 27$ .
- Juan finds the cube root of  $x$  by dividing  $x$  into three equal parts. For example, the cube root of 27 is 9 because  $9 + 9 + 9 = 27$ .

Which student's method is NOT correct? Explain why the method you selected is NOT correct.

2. Examine these numbers:

-64      -8      1      4      8      27      50      64      125      300

Create a table using the headings below and organize the numbers.

Numbers that are only perfect squares	Numbers that are only perfect cubes	Numbers that are both perfect squares and perfect cubes	Numbers that are neither perfect cubes nor perfect squares.
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3. Kemo is conducting a chemistry experiment. He mixes  $2.2 \times 10^{23}$  atoms of Element A with  $4.4 \times 10^{20}$  atoms of Element B. How many times as many atoms of Element A are in the mixture compared to Element B?

4. Show that the following numbers are rational by writing them as a ratio of two integers:

a. 0.25

b. 0.252525....

c. 0.99999....

5. Choose all classifications that describe the numbers  $\sqrt{2}$ ,  $\sqrt{\frac{1}{4}}$ , 13, and 14.

integer

irrational

rational

real

prime

composite