PROFICIENCY CHALLENGE 5 ANSWER KEY

AMV = “Answers May Vary”

1	If both *a* and *b* are positive integers, all expressions have positive values. Explanations may vary. If both *a* and *b* are negative integers, all expressions have positive values. The product or quotient of two positive or two negative integers is positive.

2	5, -5, -2

3	a	AMV. Examples include: $\frac{16}{-2}, \frac{8}{-2}, \frac{9}{-3}$
	b	AMV. Examples include: $\frac{-2}{-3}, \frac{-2}{-7}, \frac{8}{500}$
	c	AMV. Examples include: $\frac{8}{-7}, \frac{9}{-7}$
	d	$\frac{-7}{-2}$
	e	$\frac{500}{-2}$

4

(Note: This is a calculus-level problem involving limits. While challenging to 7th grade students, it is accessible to all. As your students find success with it, build their confidence by telling them they’re doing calculus!!)

Table may vary.

x	2	3	4	5	6	7	8	9	1
y	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{1}$

As *x* becomes greater and greater, the value of *y* gets smaller and smaller.

Disagree; Jason is not correct. $\frac{1}{x}$ will reach almost zero but not ever be zero.

Agree; Christina is correct. When 1 is partitioned into infinitely many parts, the value of each part will always be greater than zero.

PROFICIENCY CHALLENGE 6 ANSWER KEY

AMV = "Answers May Vary"

1	1; 8; $2\frac{1}{8}$

2	Wanda's mistake was that tripling the sum should be multiplying the sum by 3, not cubing the sum. AMV. A numerical expression that matches William's statement: $\frac{(2+6)\times 3}{12}$. AMV. A verbal expression that matches Wanda's expression: Add 2 and 6, cube the sum (or raise the sum to the third power), and then divide by 12.

3	$(7 - 3)^2 + (10 - 2^3) = 14$

4	Lance and Jerome will not make it to the next gas station. 27.4 miles per gallon x 4.8 gallons = 131.52 miles. Since 131.52 < 132.7, they're probably going to be walking the last mile or so.

5	AMV depending on student definitions of fairness. If the money is divided up on an hourly rate, then the following reasoning applies: Curly worked 12 hours. In total they worked $22\frac{1}{6}$ hours. \$330 divided by $22\frac{1}{6}$ is about \$14.89. Based on this hourly rate, Mo gets about \$97, Larry gets about \$55, and Curly gets about \$179. Exact answers may very based on rounding. It is possible that students come up with different definitions of fairness and use their own logical reasoning to guide their mathematical decisions.

6	$(-1)^5$ → Negative; product of an odd number of negative integers is negative. $(-1)^6$ → Positive; product of an even number of negative integers is positive. $(-1)^{100}$ → Positive; product of an even number of negative integers is positive. $(-1)^{99999}$ → Negative; product of an odd number of negative integers is negative. When n is even, $(-1)^n$ is positive. When n is odd, $(-1)^n$ is negative.

7	C (2, -0.4) $F\left(-1\frac{1}{5}, -1\frac{3}{5}\right)$ Rectangle ABCD has the greater area. Area of rectangle $ABCD$ = 1.6 square units. Area of rectangle $EFGH$ = $1\frac{11}{25}$ (or 1.44) square units.

PROFICIENCY CHALLENGE 7 ANSWER KEY

1	Statement 1: Incorrect. It is possible that there is a 50% chance of rain (or not) tomorrow, but the logic in the sentence isn't correct. Just because there are two outcomes, it doesn't necessitate that both outcomes are equally likely. For example, when flipping a coin, the two outcomes (heads or tails) are equally likely so the probability of each is 0.5. However, a student could say that there are two outcomes for today: I arrive at school on time or I don't. But since the outcome (hopefully) is not equally likely, then the probability of each is not 0.5. Statement 2: Incorrect; the gender of the fourth child does not depend on the gender of the three other children. Statement 3: Correct; P(sum = 2) = $\frac{1}{36}$; P(sum = 3) = $\frac{2}{36}$ Statement 4: Incorrect. As with statement 1 above, outcomes are not always equally likely. Just because three outcomes exist, it does not mean that there's a $\frac{1}{3}$ chance that each will happen. For example, a student could say that there are three outcomes for today: I arrive on time, I arrive late, or I don't arrive at all. But since the outcome (hopefully) is not equally likely, then the probability of each is not 33.3%

2	a	Yes, it is likely, because there is only a $\frac{1}{16}$ chance of circling the same number-letter combination. In theory, she should be collecting \$16 for every \$10 she pays to winners.
	b	Janice should expect to lose money since she would be collecting only \$16 for every \$20 she can expect to pay to winners.

PROFICIENCY CHALLENGE 8 ANSWER KEY

AMV = “Answers May Vary”

1	AMV. As an example: If $x = 0$, $\frac{x}{6} + \frac{2}{3} = \frac{2}{3}$ but $x + 4 = 4$, and $\frac{2}{3} \neq 4$.

2	$2[4 + 2 \bullet 3 + 4] = 28$ $2[4 + 2(3 + 4)] = 36$ $2[(4 + 2)(3 + 4)] = 84$ No. They are all necessary.

3	$2(2 + 3x)$ $3x + 2 + 3x + 2$ $2(x + x + x + 1 + 1)$

4	Perimeter of rectangle 1: $4x + 24$ Perimeter of rectangle 2: $4x + 12$ Perimeter of rectangle 3: $4x + 24$ Yes; rectangles 1 and 3 have equivalent perimeters because the expressions are equivalent.

5	William’s expression: $2(\ell + w)$ Matthew’s expression: $2\ell + 2w$ The distributive property tells us that these expressions are equivalent.

6	The maximum width of each picture is 2 feet.