Proficiency Challenges are sets of interesting, mixed-topic problems. It may take a lot of time to complete each set, so consider doing only one or two parts at a time.
PROFICIENCY CHALLENGE 13

1. Sevi writes the numbers 1 – 10 on ten different cards. He shuffles the cards and places the cards face-down in a row as shown below.

   

He turns over the first card and it’s a 6.

   a. Write “6” on the first card. Would you expect the second card to be higher or lower than a 6? Explain.

   b. The second card was a 10. Would you expect the third card to be higher or lower? Explain.

   c. The third card is a 9. Sevi says that there’s 100% chance that the fourth number will be less than a 9. Do you agree with Sevi’s conjecture? Explain.

   d. The fourth card is flipped. Sevi says that the fifth card has an equal likelihood of being higher or lower than the number on the fourth card. What is the number on the fourth card?

2. Examine each of the claims below. For each claim, determine if it is accurate and then explain your reasoning thoroughly.

   a. Claim 1: It’s either going to rain tomorrow or it’s not. Therefore, the probability of rain is one half.

   b. Claim 2: If a family has 5 girls and are expecting a 6th child, the 6th child is more likely to be a boy than a girl.

   c. Claim 3: If you roll a 6-sided cube four times and write down the numbers in order, you are more likely to roll a 3, 4, 2, 1 than you are to roll a 5, 5, 5, 5.

3. A movie studio shows a trailer for the new summer blockbuster to a group of 50 people selected randomly. The people were asked, “Based on the trailer, are you likely to go see the movie when it is released?” Twenty-two out of the 50 people said “yes”.

   If about 40,000,000 different people see the trailer on TV, about how many people can the studio expect will see the film? What might be some variables to consider that could affect the accuracy of this estimate?
PROFICIENCY CHALLENGE 13 (Continued)

4. Use the box plots below for the problems on this page.

![Box Plots A and B](image)

a. Create a data set of 20 values that could be represented by box plot A.

b. Charlotte examines the box plots above and thinks that there must be a mistake. “Box plot B can’t be possible!”

- What is unusual about box plot B?
- Create a data set of 20 values to show Charlotte that box plot B could exist.

c. Explain why the IQR for the data for box plot B is NOT a good measure of spread.

d. Create ONE context that would work for BOTH your data sets. In other words, create your own word problem that would involve the data sets you created.

In your description, be sure to include:

- The attribute (or “thing”) being measured and the units of measurement.
- A description of the overall pattern (shape) of both data sets and what that pattern says about the nature of the attribute represented.
- A measure of center that you think would be most appropriate for each data set, and a comparison of these measures of center.
- A measure of spread (variability) that can be analyzed along with your chosen measure of center, and a comparison of these measures of spread.
- A statistical question that could be answered by this data.
- An answer to your statistical question.
PROFICIENCY CHALLENGE 14

1. Which rectangle is more like a square: a rectangle that has a length of 4 cm and a width of 5 cm or a rectangle that has a length of 14 cm and a width of 15 cm? Explain your reasoning.

2. Determine if each of these statements is true or false.
   a. The sum of the measures of two supplementary angles is 180°.
   b. Two adjacent angles are complementary.
   c. Adjacent angles are also vertical angles.
   d. If the measure of an angle is $b$, then the measure of the angle complementary to it is $(90 - b)^\circ$.

   For each statement that you determined is false, either provide one concrete example for which the statement is true, or state that it can never be true. Clearly label any diagrams used.

3. Fill in the blanks with whole numbers 0 through 9 to create four coordinates that form a quadrilateral with the largest area possible. You may use a number only once.

   (___, ___) (___, ___) (___, ___) (___, ___)

4. A car dealership was trying to sell a used car that no one wanted to buy. First, they tried to sell the car for 10% off of the original price. That didn’t work. Second, they tried to sell the car for 20% off of the first sale price. Still no one bought it. Finally, they offered the car at 25% off of the second sale price. Someone bought the car at this final sale price of $5400. What was the original price of the car?

5. For this problem use the fact that 1 year is 365 days.
   a. How many minutes are there in a year?

      The speed of light is 186,282 miles per second. That means that light could travel around the Earth about 7.5 times in a second!

      A light year is how far light can travel in a year. Light years are often used to measure things that are really far away like other stars and galaxies in the universe.

   b. How many miles does light travel in a year?

      Light from the sun takes about 8 minutes to reach the Earth.

   c. How many miles away is the sun from the Earth?
PROFICIENCY CHALLENGE 15

1. Find one possible value of a radius for a circle that has an area between 28 and 32 square inches.

2. Consider the quotient $\frac{h}{k}$ where $h$ is a positive integer and $k$ is a negative integer.
   For each of the statements below, determine if the quotient changes or remains unchanged. Then explain your reasoning.
   - Both $h$ and $k$ are divided by the same nonzero integer.
   - Take the opposite of both $h$ and $k$.
   - The same nonzero integer is subtracted from both $h$ and $k$.
   - Both $h$ and $k$ are multiplied by -2.

3. Create a data set of 10 numbers that meets the following requirements.
   - The numbers in the data set are integers.
   - The median is -3.
   - The median is not a value in the data set.
   - The data set has two modes.

4. Using only the whole numbers from 1-20, how many different ratios can you create that are equivalent to 40 : 50?

5. This activity is about creating a cylinder using a sheet of paper. Soda cans and rolls of paper towels are good examples of cylinders in the real world.
   Find two regular sheets of paper that measure 8.5 in by 11 in. Take one sheet of paper and tape the 8.5 in ends of the paper together. You should have a cylinder-like figure that is “short and squat with open tops and bottoms.
   Take the other sheet of paper and tape the 11 in ends of the paper together. You should have a different cylinder-like figure that is “tall and skinny” with open tops and bottoms.
   Which of the figures you created do you think has a larger volume? Explain your reasoning.
1. Use the digits 1-9 to create the dimensions of a rectangular prism with the largest volume. Each dimension is a two-digit number. You may use a digit only once.

An example that follows these conditions is $12 \times 34 \times 56$.

Use the same directions to find the dimensions of a rectangular prism with the largest surface area.

2. Use the rectangular prism shown as a building block to create a new rectangular prism with a surface area of between 100 and 110 square units.

   a. Draw your new prism showing the appropriate number of these building blocks.
   
   b. Show that the surface area is between 100 and 110 square units.
   
   c. Find the volume of your prism.

3. The figure to the right shows a square and four overlapping semicircles using each side of the square as a diameter.

   What fraction of the square is shaded?

4. Abbott’s Pizza Company sells pizza at the following prices (note that a pizza is measured by its diameter):

   - a small 12” pizza is $8
   - a medium 14” pizza is $12
   - a large 16” pizza is $16

   Which pizza size is the “best buy”?

5. A file cabinet measures 6 feet by 3 feet by 1.5 feet. The average square sticky note measures 3 inches by 3 inches.

   How many sticky notes would it take to cover the sides and the top of the file cabinet?
PROFICIENCY CHALLENGE 16 (Continued)

   a. What is the perimeter of his bedroom on his drawing?
   b. What is the actual perimeter of his bedroom?
   
   c. What is the area of his bedroom on his drawing?
   d. What is the actual area of his bedroom?

   Greg wonders what happens if he uses a different scale for the drawing of his bedroom. He thinks that if he creates a scale drawing with scale 40 in : 1 in, his diagram will be four times larger than the first scale drawing.

   e. Before moving on, do you think Greg’s conjecture is correct?
   f. What is the perimeter of his bedroom on his new drawing?
   g. How does it compare to the perimeter on first drawing?
   
   h. What is the area of his bedroom on his new drawing?
   i. How does it compare to the area on first drawing?
   
   j. Explain how a change of scale affects perimeter versus how it affects area

7. The amount of paint needed to cover a surface is proportional to the area of the surface. If 3 quarts of paint are needed to paint a square with a side of 7 feet, how many quarts are needed to paint a square that is 10 feet 6 inches long?

8. The 7th graders at Amherst Middle School were helping to improve a playground at the nearby elementary school.

   For safety reasons, city regulations require that the sand underneath the swings must be at least 15 inches deep. The sand under the small and the large swing sets was only 12 inches deep when they started. The rectangular area under a small swing set measures 9 feet by 12 feet.

   a. How many cubic feet of sand will they need to add?

   It took 40 bags of sand to make the depth 15 inches under the small swing set. The area under the larger swing set is 1.5 times as long and 1.5 times as wide as the area under the smaller swing set.

   b. How many bags of sand will the students need to make the larger swing set safe?