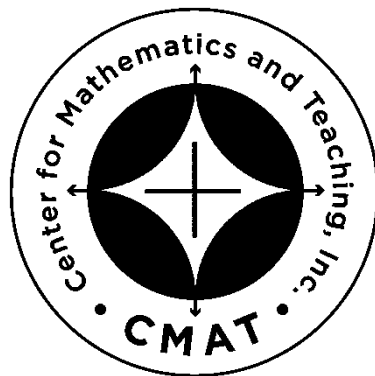


# Discovering the Better Buy

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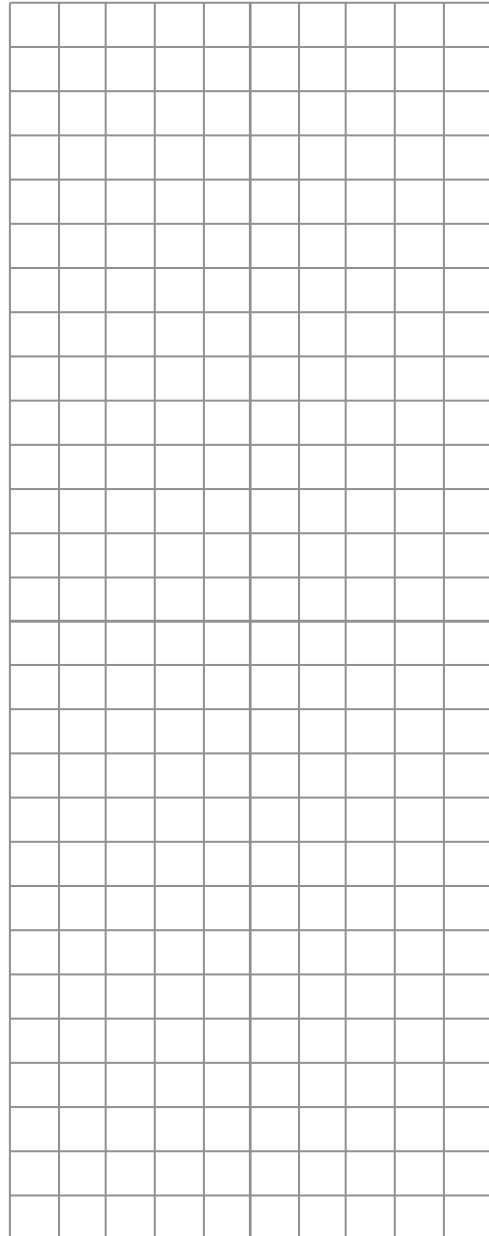
California Mathematics Council  
Palm Springs, CA  
November 6, 2015  
Session #154  
**Evaluation poll code 3027**

# TORTILLAS

FLAT 'N ROUND  
3 tortillas for \$0.60

WRAP IT UP  
4 tortillas for \$0.76

What are some ways we could represent this information?



# BAGELS

*MathLinks: Grade 6*

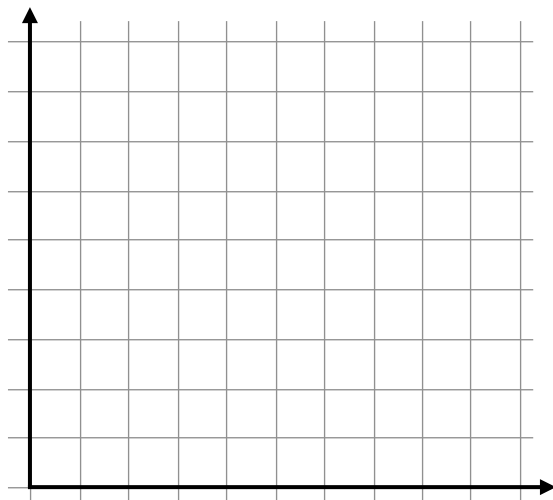
SHMEAR 'N THINGS  
4 bagels for \$3.00

HOLE-Y BREAD  
5 bagels for \$4.00

Assume you can buy any number of bagels at this rate. Complete the tables and answer the questions below.

1.

SHMEAR 'N THINGS		HOLE-Y BREAD	
# of bagels (x)	cost (y)	# of bagels (x)	cost (y)
4		5	
8		10	
12		15	
16		20	
20		25	



2. Which shop has the better buy? Use entries in the tables above to explain your reasoning.

3. Label and scale the grid. Graph the data using two different colors. Explain how to use the graph above to tell which bagel is the better buy.

4. Find the unit price for bagels at both shops. Use these numbers to explain which shop has the better buy.

5. Write an equation to represent the cost (dependent variable), given the number of bagels (independent variable), for:

a. Shmear 'n Things:  $y = \underline{\hspace{2cm}}$

b. Hole-y Bread:  $y = \underline{\hspace{2cm}}$

# BAGELS

*MathLinks: Grade 7*

SHMEAR 'N THINGS  
4 bagels for \$3.00

HOLE-Y BREAD  
5 bagels for \$4.00

1. Complete the tables below. Assume each shop will sell you any number of bagels at the rates shown above.

SHMEAR 'N THINGS	
# of bagels (x)	cost in dollars (y)
4	
8	
12	
16	
20	

HOLE-Y BREAD	
# of bagels (x)	cost in dollars(y)
5	
10	
15	
20	
25	

Two variables (quantities that vary) are proportional if one is a constant multiple of the other. One method of testing for a proportional relationship is to write and compare ratios. If the values of the ratios are all equal, then the quantities are proportional.

2. Write the values of some ratios for each shop in the table below using the data tables above.

SHMEAR 'N THINGS	$\frac{\text{cost in dollars}}{\text{\# of bagels}}$	$\frac{3}{4}$	$\frac{6}{8}$	$\frac{\square}{12}$		
	Simplify	$\frac{3}{4}$				
	Unit price (in dollars/bagel)	0.75				
HOLE-Y BREAD	$\frac{\text{cost in dollars}}{\text{\# of bagels}}$	$\frac{4}{5}$	$\frac{\square}{10}$			
	Simplify	$\frac{4}{5}$				
	Unit price					

3. Which shop has the better buy? \_\_\_\_\_  
Circle entries in your table to justify your answer and explain in words.

# TESTING FOR PROPORTIONAL RELATIONSHIPS

## MathLinks: Grade 7

Each table below shows a relationship between quantities that vary. Write, simplify, and compare the values of the ratios (unit rates). Then state whether the pairs in each table represent a proportional relationship between the quantities.

1. The number of meals Venya served to the homeless and their cost.

	<b>Monday</b>	<b>Tuesday</b>	<b>Wednesday</b>	<b>Thursday</b>	<b>Friday</b>
# of meals	45	60	20	100	55
cost	\$135	\$180	\$60	\$300	\$165
unit rate					
Proportional?					

2. The number of bags of feathers Jaime used to make pillows.

# of bags	9	24	3	18	4.5
# of pillows	6	16	2	12	3
unit rate					
Proportional?					

3. The number of tables LaTonya rented for a party and their cost.

# of tables	1	2	3	4	5
cost	\$20	\$25	\$30	\$35	\$40
unit rate					
Proportional?					

4. The size of each room in Satoshi's house and the number of plants in each room.

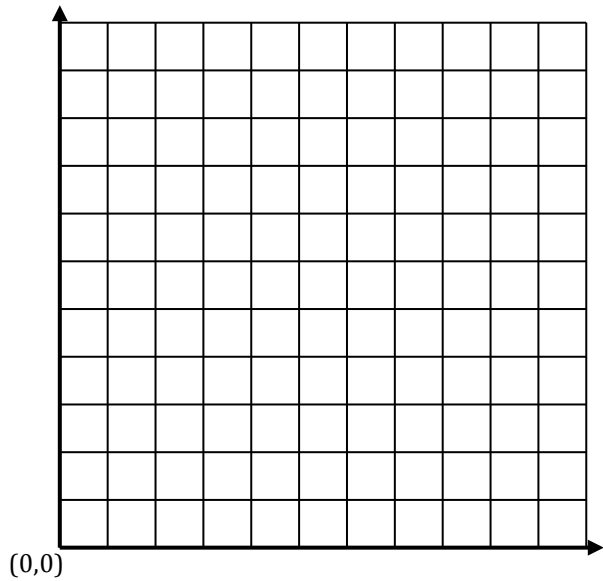
	<b>bedroom</b>	<b>living room</b>	<b>den</b>	<b>kitchen</b>	<b>patio</b>
square feet	100	200	150	125	250
# of plants	2	5	6	5	10
unit rate					
Proportional?					

# TESTING FOR PROPORTIONAL RELATIONSHIPS (Continued)

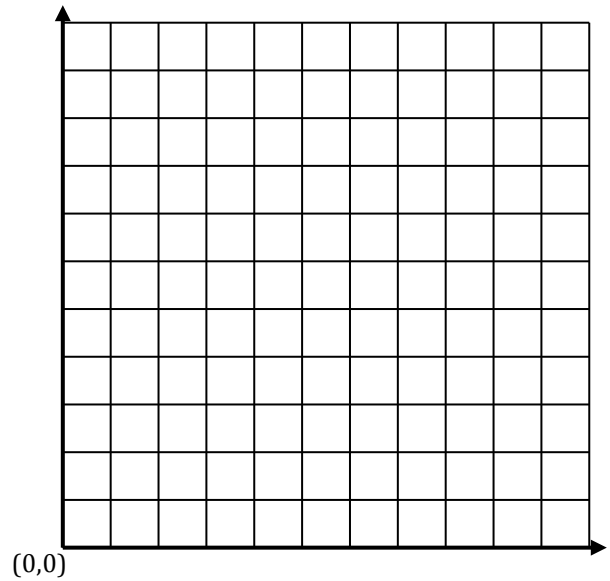
*MathLinks: Grade 7*

Graph the data from each of the tables on the previous page. Be sure to label and scale the axes appropriately.

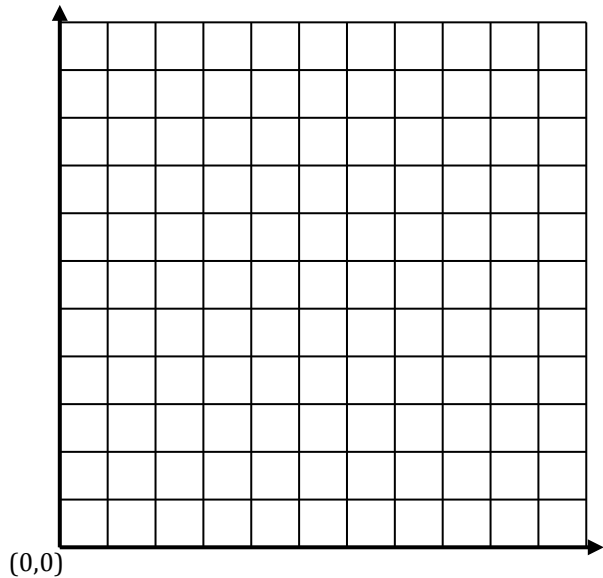
5. The number of meals Venya served to the homeless and their cost.



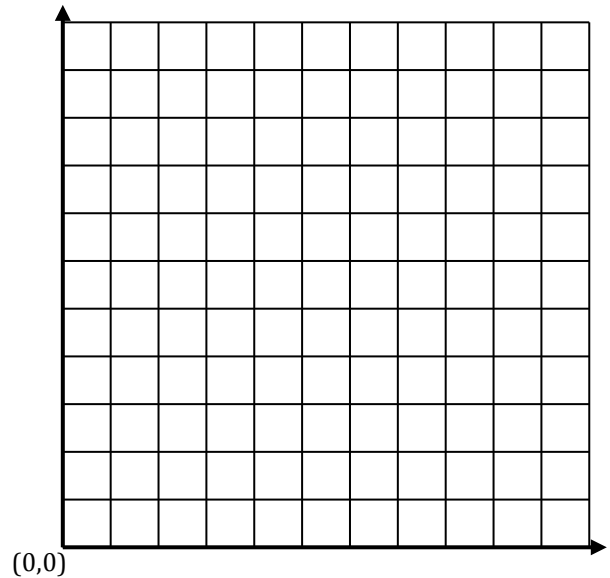
6. The number of bags of feathers Jaime used to make pillows



7. The number of tables LaTonya rented for a party and their cost.



8. The size of each room in Satoshi's house and the number of plants in each room.



9. What do you notice about graphs that represent proportional relationships, compared to those that do not?

# TORTILLAS

MathLinks: Grade 7

FLAT 'N ROUND  
3 tortillas for \$0.60

WRAP IT UP  
4 tortillas for \$0.76

Complete the tables below. Assume each shop will sell any number of tortillas at the rates shown.

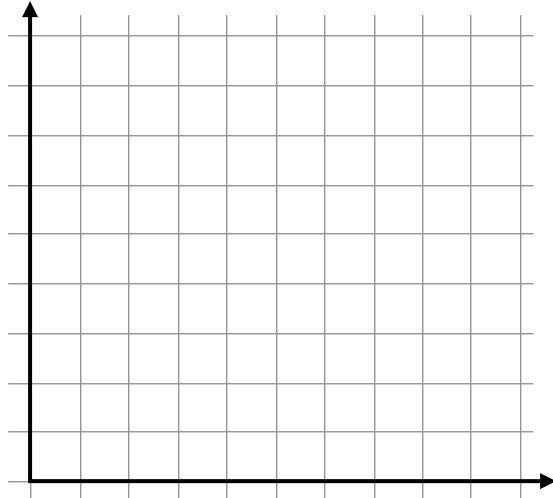
FLAT 'N ROUND		WRAP IT UP	
# of tortillas (x)	cost (y)	# of tortillas (x)	cost (y)
3		4	
6		8	

- Find the unit price at each tortilla store.
- Write equations to relate the number of tortillas to the cost.

FLAT 'N ROUND  $y =$  \_\_\_\_\_

WRAP IT UP  $y =$  \_\_\_\_\_

- Label and scale the grid. Graph the data using two different colors.



- Identify the y-coordinate when  $x = 1$ .

FLAT 'N ROUND (1, \_\_\_\_\_)

WRAP IT UP (1, \_\_\_\_\_)

- Explain the meaning of these coordinate pairs in this context.

The equations above are in the form  $y = mx$  and represent proportional relationships. For each respective equation,  $y$  is proportional to  $x$ . The number  $m$  is called the constant of proportionality.

- How are the coordinates for the ordered pairs in problem 5 related to the equation in the form  $y = mx$ ?

# BAGELS

MathLinks: Grade 8

## SHMEAR 'N THINGS

4 bagels for \$3.00

## HOLE-Y BREAD

5 bagels for \$4.00

1. Complete the tables. Assume a proportional relationship between the number of bagels and the cost.

SHMEAR 'N THINGS	
# of bagels (x)	cost (y)
4	
8	
12	
16	
20	

HOLE-Y BREAD	
# of bagels (x)	cost (y)
5	
10	
15	
20	
25	

2. Which shop has the better buy? Use entries in the tables to explain your reasoning.
3. Write equations to relate the number of bagels to cost.

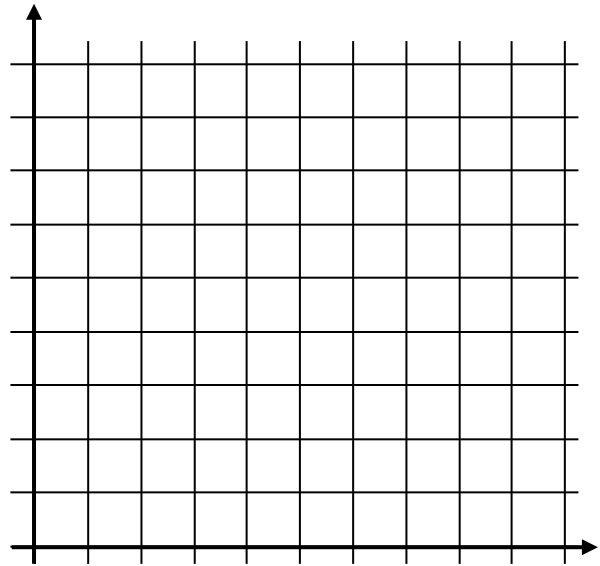
SHMEAR 'N THINGS  $y = \underline{\hspace{2cm}}$

HOLE-Y BREAD  $y = \underline{\hspace{2cm}}$

The linear functions you wrote above are both in the form  $y = mx$ . This is called a direct proportion equation because  $y$  is directly proportional to (is a multiple of)  $x$ .

4. How is the direct proportion equation different from the linear function  $y = mx + b$ ?

5. Title, label, and scale the grid. Graph the data using two different colors.



6. Explain which graph illustrates a slower rise in price.

7. Identify the coordinates when  $x = 1$

SHMEAR 'N THINGS (1,    )

HOLE-Y BREAD (1,    )

8. What do these  $y$ -coordinates represent in the context of the problem?



# TORTILLAS

*MathLinks: Grade 8*

**FLAT 'N ROUND**  
3 tortillas for \$0.60

**WRAP IT UP**  
4 tortillas for \$1.00

1. Complete the tables. Assume a proportional relationship between the number of tortillas and cost.

FLAT 'N ROUND	
# of tortillas (x)	cost (y)
3	
6	

WRAP IT UP	
# of tortillas (x)	cost (y)
4	
8	

2. Which shop has the better buy? Use entries in the tables to explain your reasoning.

3. Write equations to relate the number of tortillas to cost.

FLAT 'N ROUND  $y =$  \_\_\_\_\_

WRAP IT UP  $y =$  \_\_\_\_\_

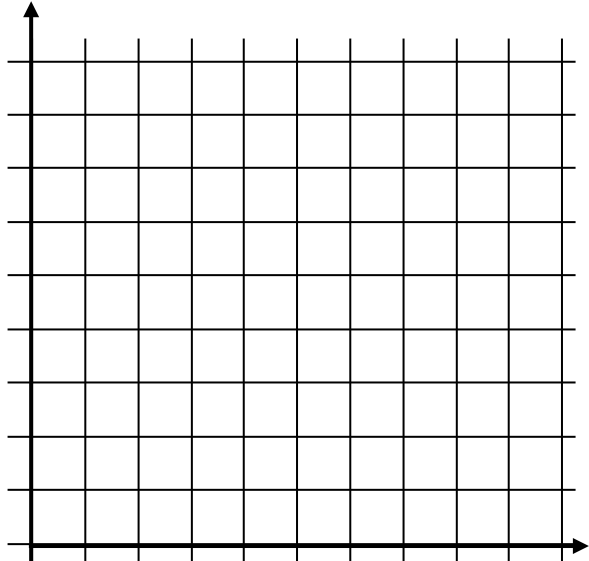
4. Identify the coordinates when  $x = 1$

FLAT 'N ROUND (1, \_\_\_)

WRAP IT UP (1, \_\_\_)

How are these coordinates related to the unit rate for one tortilla?

5. Title, label, and scale the grid. Graph the data using two different colors.



6. Explain which graph illustrates a slower rise in price.

In the linear function  $y = mx + b$ ,  $b$  represents the  $y$ -intercept.

7. Write coordinates for the  $y$ -intercepts for each function.

FLAT 'N ROUND (0, \_\_\_)

WRAP IT UP (0, \_\_\_)

What do these  $y$ -coordinates represent in the context of the problem?

# PITA BREAD

*MathLinks: Grade 8*

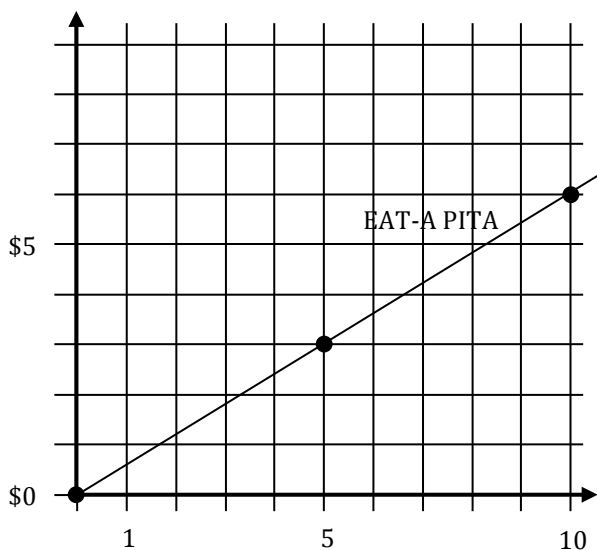
PAPA'S PITA  
6 pitas for \$\_\_\_

EAT-A PITA  
5 pitas for \$\_\_\_

- The graph for EAT-A PITA is provided below. How do you know by looking at the graph for EAT-A PITA that it depicts a proportional relationship?
- Complete the tables below and graph for PAPA'S PITA. A partial table for PAPA'S PITA is provided. Use tables and graphs to complete the pricing information above. Assume each shop will sell any number of pitas at the rates shown.

PAPA'S PITA	
# of pitas (x)	cost (y)
3	
6	

EAT-A PITA	
# of pitas (x)	cost (y)
4	
8	



- Find the unit prices at both pita stores.
- Write equations to relate the number of pitas to the cost.  
  
 PAPA'S PITA       $y = \underline{\hspace{2cm}}$   
  
 EAT-A PITA       $y = \underline{\hspace{2cm}}$
- Verify that the relationship between the number of pitas and the price paid at EAT-A PITA is proportional by creating ratios and checking for equivalent fractions.
- Does the graph for PAPA'S PITA go through the origin? What does that mean in this context?
- Which graph illustrates a slower rise in price? How do you know?

# PIZZA

**MathLinks: Grade 8**

DOOR-TO-DOOR PIZZA

\$\_\_\_ per pizza

\$5 for delivery

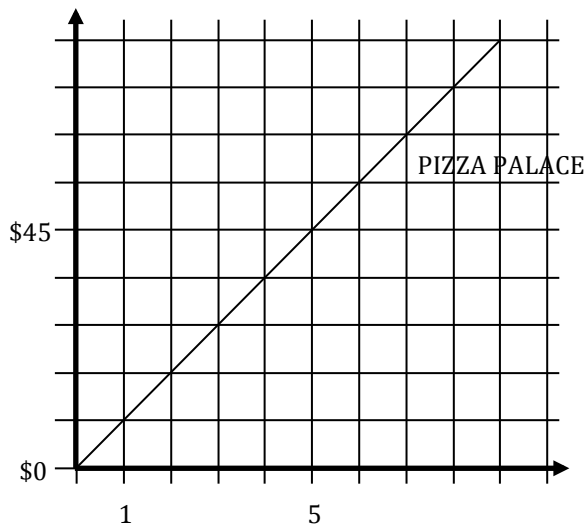
PIZZA PALACE

\$\_\_\_ per pizza

1. Complete the tables and graphs. The graph for PIZZA PALACE is provided. Use tables and graphs to complete the pricing information above.

DOOR-TO-DOOR (include delivery)	
# of pizzas ( $x$ )	cost ( $y$ )
3	8
6	

PIZZA PALACE	
# of pizzas ( $x$ )	cost ( $y$ )
4	
8	



2. Using data from the table, explain/show which relationship is proportional.
  
3. Do both graphs represent proportional relationships? Why or why not?
  
4. Using the data from the tables, determine which shop offers the better buy. Explain your reasoning.
  
5. Identify the coordinates of the point where the two graphs intersect.
  
6. What does this point mean within the context of the problem?

# MATHEMATICAL CONTENT STANDARDS

## A Selection from Grades 6-8 that may be discussed during this presentation

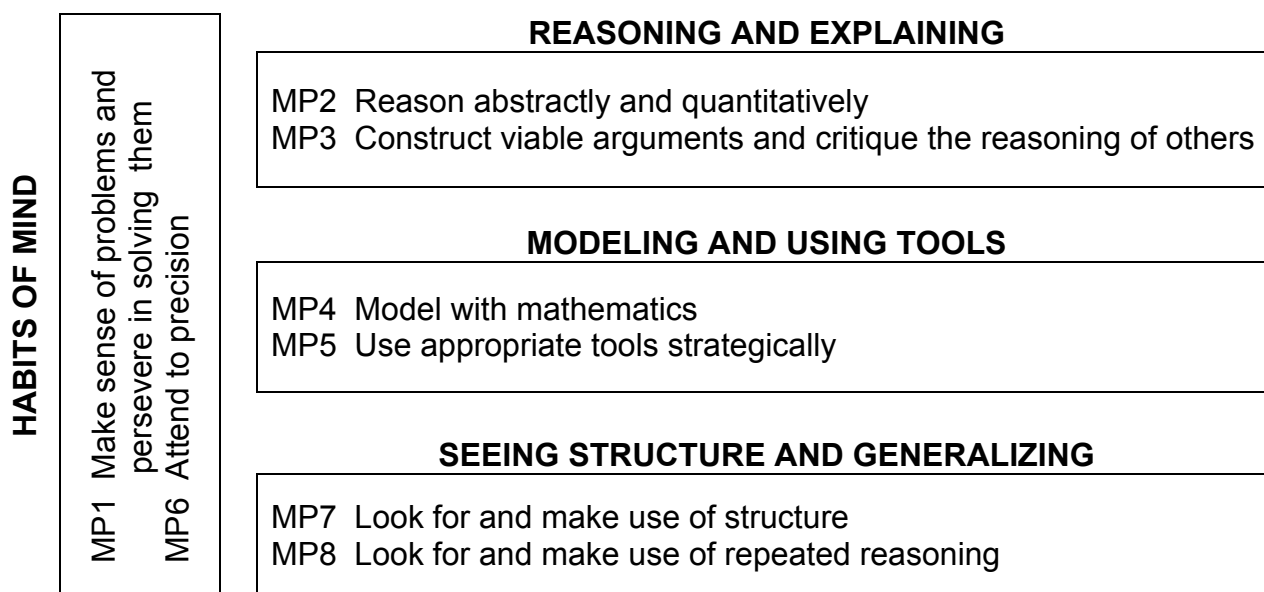
6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. <i>For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</i>
6.RP.2	Understand the concept of a unit rate $a/b$ associated with a ratio $a:b$ with $b \neq 0$ , and use rate language in the context of a ratio relationship. <i>For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is <math>3/4</math> cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”</i>
6.RP.3a	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
6.RP.3b	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i>
7.RP.A.2a	Recognize and represent proportional relationships between quantities. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin
7.RP.A.2b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.A.2c	Represent proportional relationships by equations. <i>For example, if total cost <math>t</math> is proportional to the number <math>n</math> of items purchased at a constant price <math>p</math>, the relationship between the total cost and the number of items can be expressed as <math>t = pn</math></i>
7.RP.A.2d	Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. <i>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</i>

# STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe a variety of processes and proficiencies that educators seek to develop in mathematically proficient and fluent students across all grade levels.

Many processes and proficiencies in these practice standards overlap, several may be used together on any given problem or task, and rarely would we expect students to use them all at once. We do expect that over time students will use them frequently. In addition, some will be used naturally within the context of solving particular problems, and others will only occur in an environment in which students are provided ample opportunities to collaborate and discuss.

One way to think about the practices is in groupings (graphic from CCSS-M author, Bill McCallum).



From CCSS-M:

“Students who lack understanding of a topic may rely on procedures too heavily...In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. Without understanding, a student may rely on procedures and may not represent problems coherently, justify conclusions, apply the mathematics to other situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an interview, or deviate from a known procedure to find a shortcut.”

# STANDARDS FOR MATHEMATICAL PRACTICE (Continued)

<u>Habits of Mind</u>	
<b>MP1</b> <b>Make sense of problems and persevere in solving them</b>	<b>MP6</b> <b>Attend to precision</b>
<ul style="list-style-type: none"> <li>○ Understand a problem and look for entry points</li> <li>○ Consider given information, constraints, and relationships</li> <li>○ Consider simpler or analogous problems</li> <li>○ Make conjectures, monitor progress and alter their solution course as needed</li> <li>○ Explain correspondences using multiple representations</li> <li>○ Understand and analyze the approaches of others</li> <li>○ Continually ask, “Does this make sense?”</li> </ul>	<ul style="list-style-type: none"> <li>○ Communicate mathematical ideas precisely</li> <li>○ Use clear definitions</li> <li>○ State meaning of symbols and use them properly</li> <li>○ Attend to units of measures and labeling of axes</li> <li>○ Calculate accurately and give solution with appropriate degree of accuracy</li> </ul>

<u>Reasoning and Explaining</u>	
<b>MP2</b> <b>Reason abstractly and quantitatively</b>	<b>MP3</b> <b>Construct viable arguments and critique the reasoning of others</b>
<ul style="list-style-type: none"> <li>○ Attend to the meaning of quantities</li> <li>○ Decontextualize a problem using symbols, and manipulate them as if they have a life of their own</li> <li>○ Contextualize manipulations to create a coherent representation of a problem</li> </ul>	<ul style="list-style-type: none"> <li>○ Use assumptions, definitions, and established results to create arguments (deductive reasoning)</li> <li>○ Make and test conjectures based on evidence (inductive reasoning)</li> <li>○ Analyze situations by breaking them into cases</li> <li>○ Use counterexamples to disprove a statement</li> <li>○ Identify flaws in an argument</li> <li>○ Listen to or read to arguments and ask useful questions to clarify reasoning</li> </ul>

# STANDARDS FOR MATHEMATICAL PRACTICE (Continued)

<u>Modeling and Using Tools</u>	
<b>MP4</b> <b>Model with Mathematics</b>	<b>MP5</b> <b>Use appropriate tools strategically</b>
<ul style="list-style-type: none"> <li>○ Apply mathematics to solve everyday problems</li> <li>○ Make reasonable assumptions and approximations to simplify a situation</li> <li>○ Identify important quantities in a situation</li> <li>○ Use multiple representations to analyze relationships and draw conclusions</li> <li>○ Interpret results in the context of the situation</li> <li>○ Improve the mathematical approach (model) if it has not served its purpose</li> </ul>	<ul style="list-style-type: none"> <li>○ Select useful tools such as paper and pencil, graph paper ruler, calculator, concrete model, spreadsheet, or statistical software to solve problems</li> <li>○ Use concrete models and technology tools to explore concepts</li> <li>○ Recognize limitations of tools</li> <li>○ Identify and use relevant external resources, such as the internet</li> </ul>

<u>Structure and Generalizing</u>	
<b>MP7</b> <b>Look for and make use of structure</b>	<b>MP8</b> <b>Look for and make use of repeated reasoning</b>
<ul style="list-style-type: none"> <li>○ Identify patterns and apply them to solve problems</li> <li>○ Recognize the structure of a symbolic representation and generalize it</li> <li>○ See complicated objects as composed of chunks of simpler objects</li> </ul>	<ul style="list-style-type: none"> <li>○ Identify repeated calculations and patterns</li> <li>○ Generalize procedures based on repeated patterns or calculations</li> <li>○ Find shortcuts based on repeated patterns or calculations</li> </ul>