MIDDLE SCHOOL RATIO AND PROPORTIONAL RELATIONSHIPS:

HOW THINGS HAVE CHANGED!

Presented by

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Adapted from *MathLinks* Grades 6, 7, 8

PROPORTIONAL REASONING: GRADE 6

*In Grade 6, instructional time should focus on four critical areas, the first of which is connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems.

Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

6.RP.1

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.2

Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*

6.RP.3

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of

6.RP.3a

Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the

6.RP.3b

Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

6.RP.3c

Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

6.RP.3d

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when

*From the Common Core St

PROPORTIONAL REASONING: GRADE 7

*In Grade 7, instructional time should focus on four critical areas, the first of which is developing understanding of and applying proportional relationships.

Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

7.RP.A.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as*

7.RP.A.2
Recognize and represent proportional relationships between quantities.

7.RP.A.2a
Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line

7.RP.A.2b

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional

7.RP.A.2c

Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t =*

7.RP.A.2d Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit

7.RP.A.3
Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

PROPORTIONAL REASONING: GRADE 8

*In Grade 8, instructional time should focus on three critical areas, the first of which is formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations**.**

Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m.

8.EE.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving*

8.EE.6

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

8.F.2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it

8.G.4.

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.

FROM THE COMMON CORE STATE STANDARDS
http://www.corestandards.org/

In Grade 6, instructional time should focus on ***four critical areas***: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

In Grade 7, instructional time should focus on ***four critical areas***: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

STANDARDS FOR MATHEMATICAL CONTENT

OVERVIEW

The Standards for Mathematical Content outline the skills and understandings students will learn by grade level or course, and are organized around domains and coherent clusters.

<table>
<tr><td>K</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td colspan="9">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td></tr>
<tr><td colspan="6">Numbers and Operations in Base Ten</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td></tr>
<tr><td>Counting and Cardinality</td><td colspan="2"></td><td colspan="3">Number and Operations: Fractions</td><td colspan="2">Ratios and Proportional Relationships</td><td>Functions</td></tr>
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THE CCSS SHIFTS

Seeking increased FOCUS, COHERENCE, and RIGOR

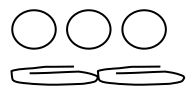
Focus	**Coherence**	**Rigor**
• Strongly emphasized at appropriate grade levels for deep understanding. • Fewer topics and standards per grade level allow for greater depth and focus; avoids the rush to “cover” content, and is counter to the “mile-wide-inch-deep” approach.	• Thinking <u>across</u> grade levels, and linking to major topics <u>within</u> grade levels. • Important ideas can be linked across any one grade level (horizontally), and also from one grade level to the next (vertically).	• Pursue, with equal intensity in major topics: (1) conceptual understanding; (2) procedural skill and fluency; (3) problem solving and application. • The recent culture in mathematics education was to concentrate mainly on procedural skill and fluency (think standardized testing).

SOME VOCABULARY*

A <u>ratio</u> associates two or more quantities.

The ratio of a to b can be denoted by $a : b$ (read "a to b"). Also stress the language *for each*, *for every*, *per*, etc.

The ratio of coins to paperclips in the picture is 3 to 2.



The <u>value of this ratio</u> can be written as the quotient $\frac{3}{2}$.

This value is connected to the concept of rate. Fractions are downplayed when discussing ratio.

The original ratio of coins to paperclips (3 to 2) and the new ratio of coins to paperclips are <u>equivalent ratios</u> because each measurement in the original ratio pair can be multiplied by the same number to arrive at the new ratio pair (6 to 4). We say that equivalent ratios are in a <u>proportional relationship</u>.

A ratio has an associated <u>rate</u>, which includes a numerical value with the units.

Example: If you drive 40 miles in 2 hours, then the ratio 40 miles for every 2 hours can be written $\frac{40}{2}$ miles for every 1 hour, or 20 miles for every 1 hour. This is the rate.

A <u>unit rate</u> is the numerical part of the rate (to highlight the "for every 1.")

Example: For the rate above, $\frac{40}{2}$ miles for every hours is equivalent to 20 miles per 1 hour, so the unit rate is $\frac{40}{2}$, or more simply 20.

*Language from Progressions for the Common Core State Standards in Mathematics (referred to by many as "*the progressions*") (Draft, 6-7 Progression on Ratios and Proportional Relationships, by the Common Core Standards Writing Team) http://ime.math.arizona.edu/progressions/

SELECTED STANDARDS FOR MATHEMATICAL CONTENT Grade 6-8

6.RP.1	Understand the concept of a ratio and use **ratio language to describe a ratio relationship between two quantities**. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*
6.RP.2	Understand the concept of a **unit rate** *a*/*b* associated with a ratio *a*:*b* with $b \neq 0$, and **use rate language in the context of a ratio relationship**. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”*
6.RP.3a	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about **tables of equivalent ratios, tape diagrams, double number line diagrams, or equations**: Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
6.RP.3b	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about **tables of equivalent ratios, tape diagrams, double number line diagrams, or equations**: Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

***Standards for Mathematical Practice: see pages 15-17**

SELECTED STANDARDS
FOR MATHEMATICAL CONTENT
Grades 6-8 (Continued)

7.RP.2a	Recognize and represent proportional relationships between quantities: Decide whether **two quantities are in a proportional relationship,** e.g., by testing for equivalent ratios in **a table** or **graphing on a coordinate plane** and observing whether the graph is a straight line through the origin.
7.RP.2b	Recognize and represent proportional relationships between quantities: Identify the **constant of proportionality (unit rate**) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.2c	Recognize and represent proportional relationships between quantities: Represent proportional relationships **by equations**. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.
7.RP.2d	Recognize and represent proportional relationships between quantities: Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with **special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.**
8.EE.5	Graph proportional relationships, interpreting the **unit rate as the slope of the graph.**
8.F.2	**Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions**). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Adapted from *MathLinks* Grades 6, 7, 8

RATIO TABLES AND TAPE DIAGRAMS

(Grade 6)

A ratio table is a table whose entries are equivalent ratios. A ratio table illustrates a proportional relationship.

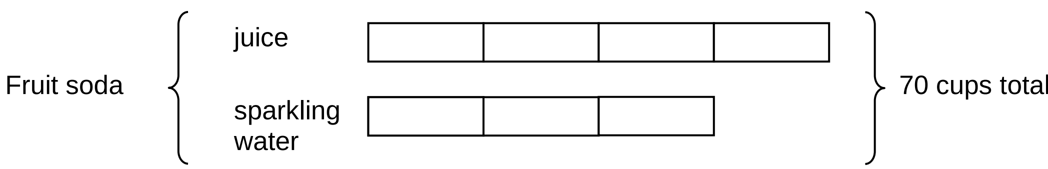
Milo likes to make fruit soda when he has people over to his house. He has determined that the juice to sparkling water ratio should be 4 : 3.

1. Fill in the ratio table for this information

Cups juice	4			200	2
Cups sparkling water		12			
Total cups			14		

A tape diagram for two quantities is two strips that visually depict the number of parts in each quantity. Tape diagrams are typically used when the quantities have the same units.

2. Milo estimates that he will want 70 cups in all. How much juice and how much sparkling water will he need?



3. Kendra makes tie-dyed shirts. For the orange dye she uses red and yellow in a ratio of 3 : 2. How many quarts of red and yellow dye will she need if she wants to make 80 quarts of dye?

Adapted from *MathLinks* Grades 6, 7, 8

RATIO TABLES AND DOUBLE NUMBER LINES

(Grade 6)

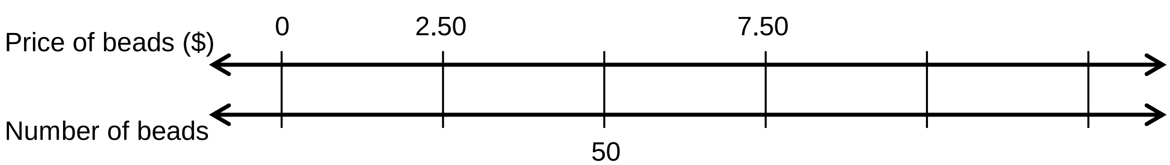
1. Why is this a ratio table?

   What is the unit rate in price per bead at Bead Barn?

	Bag A	**Bag B**	**Bag C**	**Bag D**	**Bag E**
Price of beads	$5.00	$2.50	$10.00	$7.50	$12.50
Number of beads	50	25	100	75	125

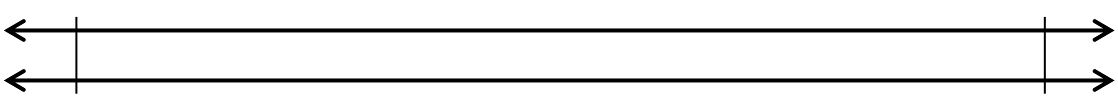
A double number line is a diagram made up of two parallel number lines that visually depict the relative sizes of two quantities. Double number lines are often used when the two quantities have different units.

2. Complete the double number line for the table above.



3. Alberto jogs 5 yards every 2 seconds. Complete the ratio table and make a double number line diagram. Hint: a few entries may not fit easily, so make a case for which you may want to leave out.

yards	5			20			1	2
seconds		4	6		0	1		



Where does Alberto's unit rate in yards per second appear in the table?

Use the unit rate to find the number of yards he can jog in one minute.

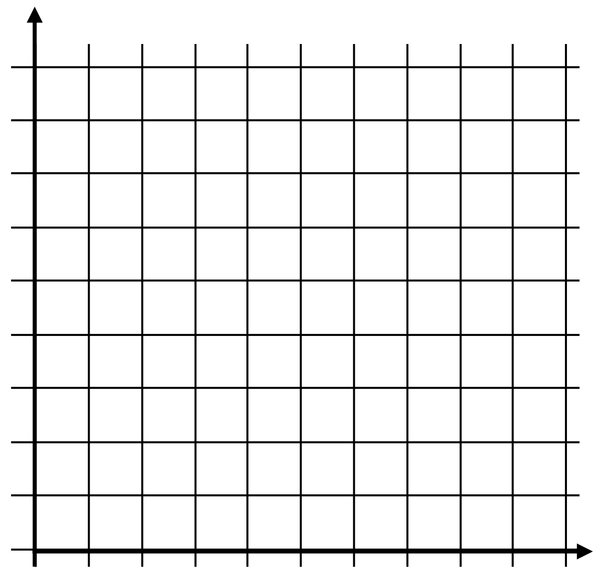
Adapted from *MathLinks* Grades 6, 7, 8

BETTER BUY

Which of these bagel shops offers the better buy?

<u>SHMEAR 'N THINGS</u>
4 bagels for $3.00

<u>HOLE-Y BREAD</u>
5 bagels for $4.00



Adapted from *MathLinks* Grades 6, 7, 8

TESTING FOR PROPORTIONAL RELATIONSHIPS

(Grade 6-7)

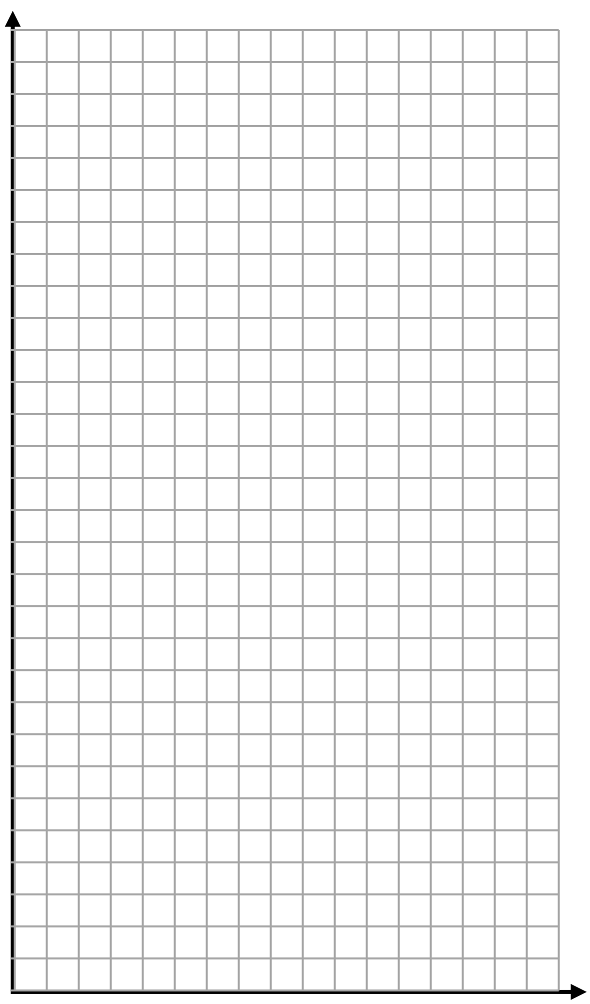
Each table below shows a relationship between quantities. Compare the ratios. State whether the pairs in each table represent a proportional relationship, and defend your answer. Graph each ratio pair. Be sure to label and scale the grids appropriately.

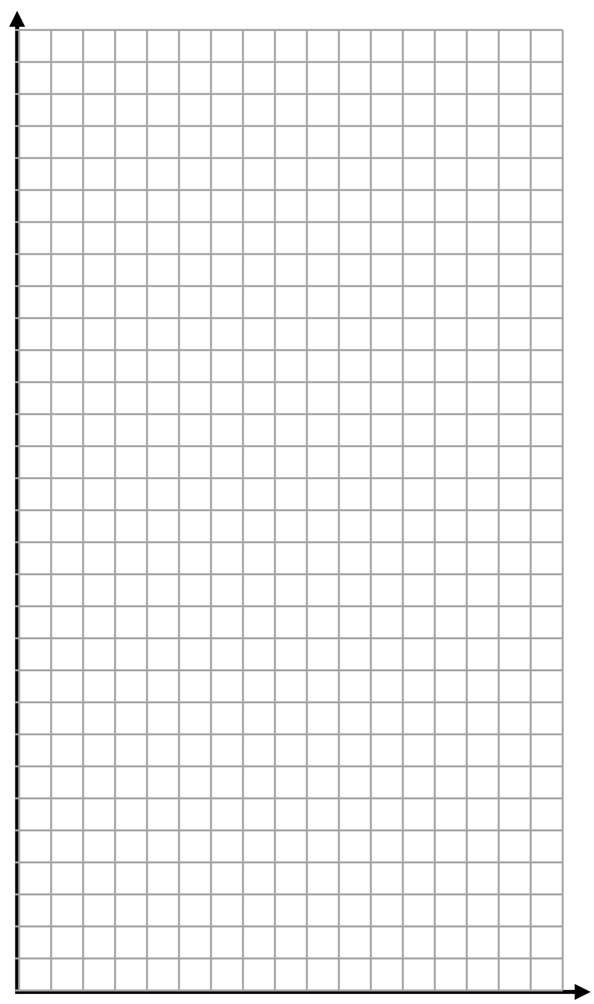
1. The number of bags of feathers Jaime used to make pillows.

# of bags	9	24	3	18	4.5
# of pillows	6	16	2	12	3
Proportional relationship? Defend:					

2. The number of tables LaTonya rented for a party and their cost.

# of tables	1	2	3	4	5
cost	$20	$25	$30	$35	$40
Proportional relationship? Defend:					





Adapted from *MathLinks* Grades 6, 7, 8

BAGELS

(Grade 6)

<u>SHMEAR 'N THINGS</u>
4 bagels for $3.00

<u>HOLE-Y BREAD</u>
5 bagels for $4.00

1. Complete the tables and answer the questions. Assume a proportional relationship between the number of bagels (the independent variable) and the cost (the dependent variable).

SHMEAR 'N THINGS

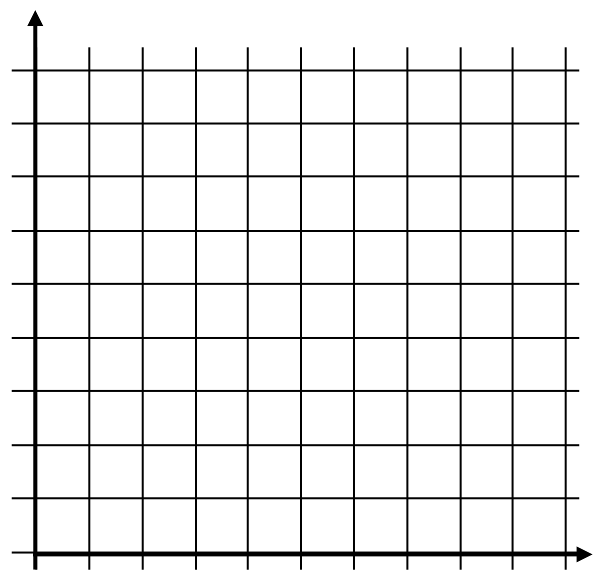
# of bagels (*x*)	**cost (*y*)**
4	
8	
12	
16	
20	

HOLE-Y BREAD

# of bagels (*x*)	**cost (*y*)**
5	
10	
15	
20	
25	

2. Which shop has the better buy? Use entries in the tables to explain your reasoning.

3. Label and scale the grid. Graph the data using two different colors. Explain how to use the graph to tell which is the better buy.



4. Find the unit rates for bagels at both shops. Use these numbers to explain which shop has the better buy.

5. Write an equation to represent the cost, given the number of bagels, for:

   a. Shmear 'n Things ________________ b. Hole-y Bread ____________________

Adapted from *MathLinks* Grades 6, 7, 8

TORTILLAS

(Grade 7)

FLAT 'N ROUND	WRAP IT UP
3 tortillas sell for $0.60	4 tortillas sell for $0.40

1. Complete the tables. Assume each shop will sell any number of tortillas at the rate shown.

FLAT 'N ROUND	
# of tortillas (x)	cost (y)
0	
3	
6	

WRAP IT UP	
# of tortillas (x)	cost (y)
0	
4	
8	

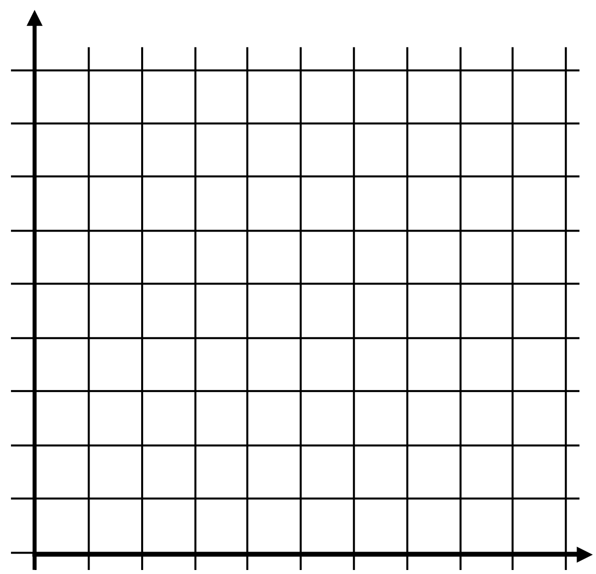
2. Find the unit price at each tortilla store.

3. Write equations to relate the number of tortillas to the cost.

   FLAT 'N ROUND y = _______________

   WRAP IT UP y = _______________

4. Label and scale the grid. Graph the data using two different colors.



5. Identify the y-coordinate when $x = 1$

   FLAT 'N ROUND (1, ____)

   WRAP IT UP (1, ____)

6. Explain the meaning of these coordinate pairs in this context.

The equations above are linear functions in the form $y = mx$. This is called a direct proportion equation because y is directly proportional to (is a constant multiple of) x. The number m is called the constant of proportionality.

7. How are the coordinates in problem 5 related to the linear function in the form $y = mx$?

PITA BREAD

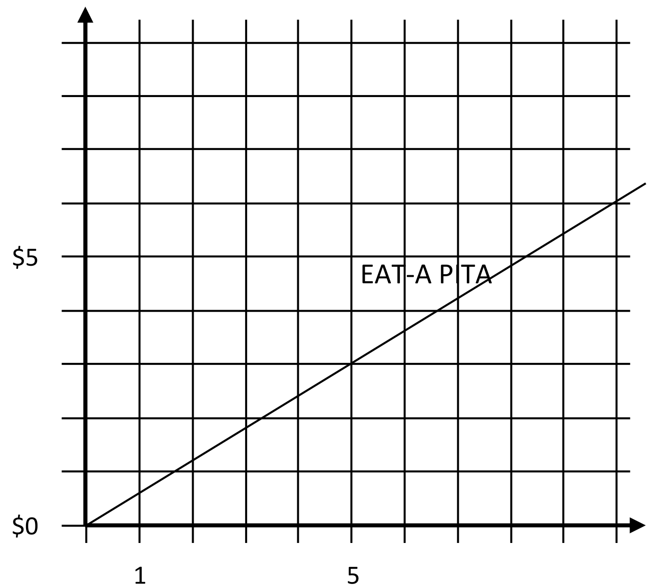
(Grade 8)

PAPA'S PITA	EAT-A PITA
6 pitas for $____	10 pitas for $____

1. Complete the tables and graphs. The graph for EAT-A PITA is provided. A partial table for PAPA'S PITA is provided. Use tables and graphs to extend the pricing information above. Assume proportional relationships between the number of pitas and cost.

PAPA'S PITA	
# of pitas (x)	cost (y)
2	$1.00

EAT-A PITA	
# of pitas (x)	cost (y)



2. Which shop has the better buy? Use entries in the tables or graphs to explain your reasoning.

3. Write equations to relate the number of pitas to cost.

   PAPA'S PITA y = ______________

   EAT-A PITA y = ______________

   How can you determine unit rates from these equations?

   Which shows the greater rate of increase?

4. Identify the coordinates when $x = 1$.

   PAPA'S PITA (1, ____)

   EAT-A PITA (1, ____)

   What do these y-coordinates represent in the context of the problem?

5. Identify the coordinates when $x = 0$.

   PAPA'S PITA (0, ____)

   EAT-A PITA (0, ____)

   What do these y-coordinates represent in the context of the problem?

APPLES, APPLES, APPLES

1. You want to buy 5 pounds (lbs.) of apples. You see ads at six different stores. Choose the best buy(s). Show all work.

<u>Store A</u>: apples cost \$2.00/lb.	<u>Store B</u>: apples come in a 5 lb bag for \$10.50
<u>Store C</u>: apples come in a 1.75 lb bag for \$3.60	<u>Store D</u>: apples come in a 7.5 lb bag for \$15.25
<u>Store E</u>: apples cost \$2.25 per pound with a coupon for \$0.12 off per half-pound.	<u>Store F</u>: apples cost \$2.25 per pound with a coupon for \$0.07 off per quarter-pound

Explain your answer in words.

2. Use your understanding of the concept of unit price to interpret each situation.

<u>Store G</u>: \$0.00 for 3 apples	<u>Store H</u>: \$3 for 0 apples

Adapted from *MathLinks* Grades 6, 7, 8

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe a variety of processes and proficiencies that educators seek to develop in mathematically proficient and fluent students across all grade levels.

Many processes and proficiencies in these practice standards overlap, several may be used together on any given problem or task, and rarely would we expect students to use them all at once. We do expect that over time students will use them frequently. In addition, some will be used naturally within the context of solving particular problems, and others will only occur in an environment in which students are provided ample opportunities to collaborate and discuss.

One way to think about the practices is in groupings (graphic from CCSS-M author, Bill McCallum).

HABITS OF MIND

MP1 Make sense of problems and persevere in solving them
MP6 Attend to precision

REASONING AND EXPLAINING

MP2 Reason abstractly and quantitatively
MP3 Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS

MP4 Model with mathematics
MP5 Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING

MP7 Look for and make use of structure
MP8 Look for and make use of repeated reasoning

From CCSS-M:

"Students who lack understanding of a topic may rely on procedures too heavily...In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. Without understanding, a student may rely on procedures and may not represent problems coherently, justify conclusions, apply the mathematics to other situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an interview, or deviate from a known procedure to find a shortcut."

STANDARDS FOR MATHEMATICAL PRACTICE (Continued)

Habits of Mind	
MP1 **Make sense of problems and persevere in solving them**	**MP6** **Attend to precision**
• Understand a problem and look for entry points • Consider given information, constraints, and relationships • Consider simpler or analogous problems • Make conjectures, monitor progress and alter their solution course as needed • Explain correspondences using multiple representations • Understand and analyze the approaches of others • Continually ask, “Does this make sense?”	• Communicate mathematical ideas precisely • Use clear definitions • State meaning of symbols and use them properly • Attend to units of measures and labeling of axes • Calculate accurately and give solution with appropriate degree of accuracy

Reasoning and Explaining	
MP2 **Reason abstractly and quantitatively**	**MP3** **Construct viable arguments and critique the reasoning of others**
• Attend to the meaning of quantities • Decontextualize a problem using symbols, and manipulate them as if they have a life of their own • Contextualize manipulations to create a coherent representation of a problem	• Use assumptions, definitions, and established results to create arguments (deductive reasoning) • Make and test conjectures based on evidence (inductive reasoning) • Analyze situations by breaking them into cases • Use counterexamples to disprove a statement • Identify flaws in an argument • Listen to or read to arguments and ask useful questions to clarify reasoning

Adapted from *MathLinks* Grades 6, 7, 8

STANDARDS FOR MATHEMATICAL PRACTICE (Continued)

Modeling and Using Tools	
MP4 **Model with Mathematics**	**MP5** **Use appropriate tools strategically**
• Apply mathematics to solve everyday problems • Make reasonable assumptions and approximations to simplify a situation • Identify important quantities in a situation • Use multiple representations to analyze relationships and draw conclusions • Interpret results in the context of the situation • Improve the mathematical approach (model) if it has not served its purpose	• Select useful tools such as paper and pencil, graph paper ruler, calculator, concrete model, spreadsheet, or statistical software to solve problems • Use concrete models and technology tools to explore concepts • Recognize limitations of tools • Identify and use relevant external resources, such as the internet

Structure and Generalizing	
MP7 **Look for and make use of structure**	**MP8** **Look for and make use of repeated reasoning**
• Identify patterns and apply them to solve problems • Recognize the structure of a symbolic representation and generalize it • See complicated objects as composed of chunks of simpler objects	• Identify repeated calculations and patterns • Generalize procedures based on repeated patterns or calculations • Find shortcuts based on repeated patterns or calculations

Adapted from *MathLinks* Grades 6, 7, 8