CCSS MIDDLE SCHOOL PROPORTIONAL REASONING: IT'S A BIG DEAL

Presented by

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Adapted from *MathLinks*

EXAMPLE FOR SESSION EVALUATIONS

38102 is the sample. DO NOT use it for this session

Please use poll code 44648

GOALS FOR THIS PRESENTATION

1. Introduce and/or clarify some of the new CCSS content standards in the domain of ratios and proportional relationships.

2. Discuss interpretations of these standards, and the vocabulary contained within, based upon "the progressions." *

3. Provide examples of some problems that lead to the mathematics in these standards

4. Discuss coherence and connections through the middle grades in this topic area, including grade 8 algebra and geometry.

5. Connect to the Standards for Mathematical Practice

* Progressions for the Common Core State Standards in Mathematics (referred to by many as "*the progressions*") (Draft, 6-7 Progression on Ratios and Proportional Relationships, by the Common Core Standards Writing Team) http://ime.math.arizona.edu/progressions/

MATHEMATICAL CONTENT STANDARDS
A Selection from Grades 6-8

6.RP.2	Understand the concept of a **unit rate** *a*/*b* associated with a ratio *a:b* with *b* ≠0, and **use rate language in the context of a ratio relationship**. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*
6.RP.3a	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about **tables of equivalent ratios, tape diagrams, double number line diagrams, or equations**: Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
7.RP.A.2a	Recognize and represent proportional relationships between quantities. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin
7.RP.A.2b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.A.2c	Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*
7.RP.A.2d	Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving*
8.EE.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

MATHEMATICAL CONTENT STANDARDS
(Continued)

The Standards for Mathematical Content outline the skills and understandings students will learn by grade level or course, and are organized around domains and coherent clusters.

<table>
<tr><td>K</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td colspan="9">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td></tr>
<tr><td colspan="6">Numbers and Operations in Base Ten</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td></tr>
<tr><td>Counting and Cardinality</td><td colspan="2"></td><td colspan="3">Number and Operations: Fractions</td><td colspan="2">Ratios and Proportional Relationships</td><td>Functions</td></tr>
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THE CCSS SHIFTS
Seeking increased FOCUS, COHERENCE, and RIGOR

Focus	Coherence	Rigor
• Strongly emphasized at appropriate grade levels for deep understanding. • Fewer topics and standards per grade level allow for greater depth and focus; avoids the rush to "cover" content, and is counter to the "mile-wide-inch-deep" approach.	• Thinking <u>across</u> grade levels, and linking to major topics <u>within</u> grade levels. • Important ideas can be linked across any one grade level (horizontally), and also from one grade level to the next (vertically).	• Pursue, with equal intensity in major topics: (1) conceptual understanding; (2) procedural skill and fluency; (3) problem solving and application. • The recent culture in mathematics education was to concentrate mainly on procedural skill and fluency (think standardized testing).

INTERPRETING STANDARDS AND VOCABULARY

6th grade:

- Deemphasize fraction notation for ratios and proportional relationships. Instead, use ratio language such as *for each*, *for every*, *per*.
- Use tables as a means to show ratios and proportional relationships.
- De-emphasize “proportion” as a tool for solving problems. Instead use ratio tables, tape diagrams, and double number lines as visual tools.
- A ratio is a comparison of quantities. Ratios have associated rates, which includes a numerical value with units.

  Example: If you run 6 miles in 54 minutes, it takes you 9 minutes for every 1 mile. The 9 $\frac{\text{minutes}}{\text{mile}}$ is your rate, and the unit rate is the numerical part, namely 9. The unit rate highlights the “for every 1.” This may seem odd to us, but it seems to have implications later on. Stay tuned.

In 7th grade:

- Continue using the above tools. Proportion seems to be an appropriate tool in 7th grade, however, it has been suggested that this word adds an unnecessary layer of vocabulary, since proportions are just equations.

Let’s see how some of this might play out…

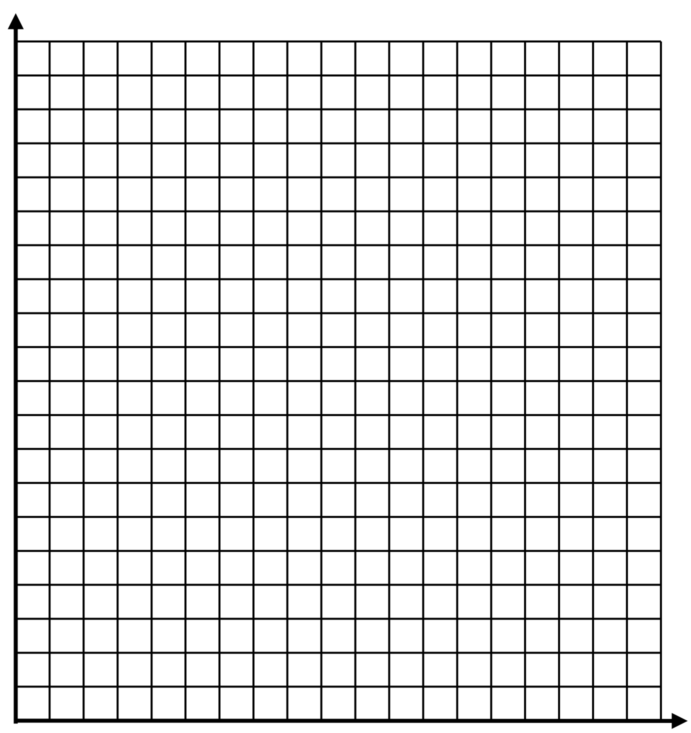
Adapted from *MathLinks*

BETTER BUY

Which of these croissant shops offers the better buy?
Use a strategy of your choice.

MOON'S
5 for $3.00

CURVEY'S
8 for $4.50



Adapted from *MathLinks*

TESTING FOR PROPORTIONAL RELATIONSHIPS

(Grade 6-7)

Two ratios are <u>equivalent</u> if one ratio pair is obtained from the other by multiplying by the same positive number. Equivalent ratios are in a <u>proportional relationship</u>.

Compare the ratios in each table and state whether they represent a proportional relationship. Graph each ratio pair. Be sure to label and scale the grids appropriately.

1. The number of bags of feathers Jaime used to make pillows.

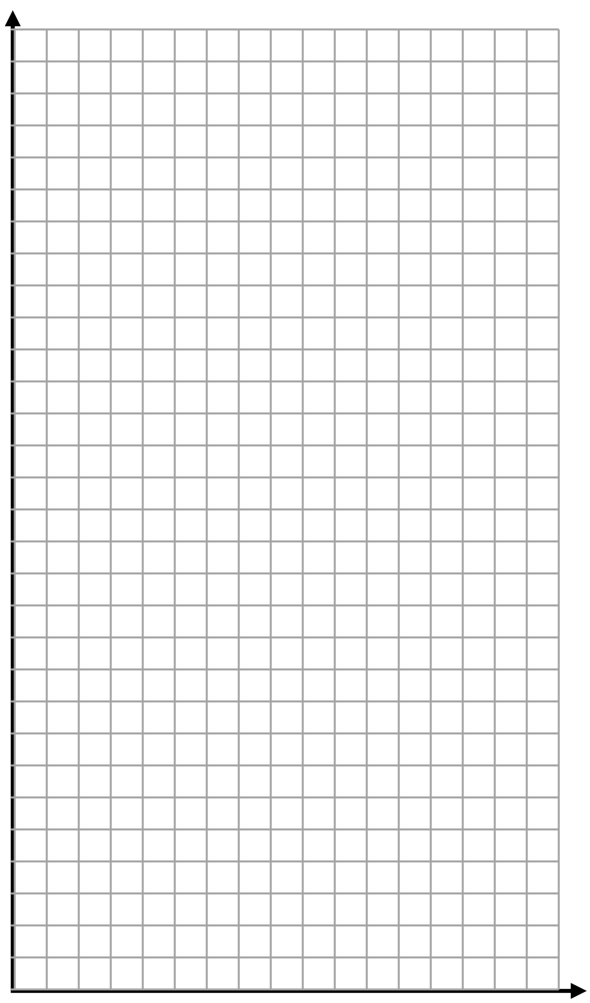
# of bags	9	24	3	18	4.5
# of pillows	6	16	2	12	3

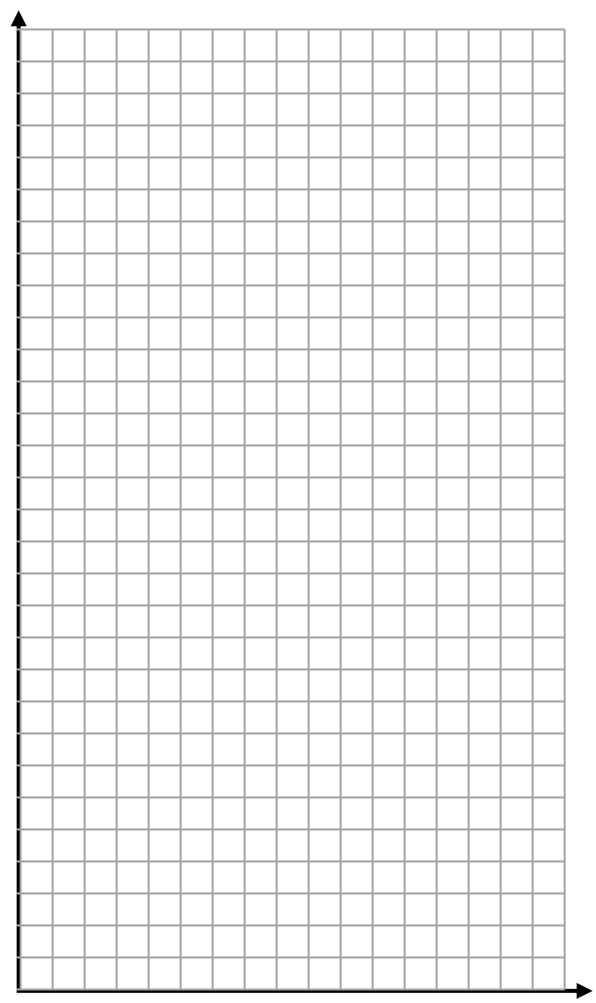
Proportional relationship?
Why?

2. The number of tables LaTonya rented for a party and their cost.

# of tables	1	2	3	4	5
cost	$20	$25	$30	$35	$40

Proportional relationship?
Why?





Adapted from *MathLinks*

BAGELS

(Grade 6)

SHMEAR ‘N THINGS	HOLE-Y BREAD
4 bagels for $3.00	5 bagels for $4.00

1. Complete the tables and answer the questions. Assume a proportional relationship between the number of bagels (the independent variable) and the cost (the dependent variable).

SHMEAR ‘N THINGS

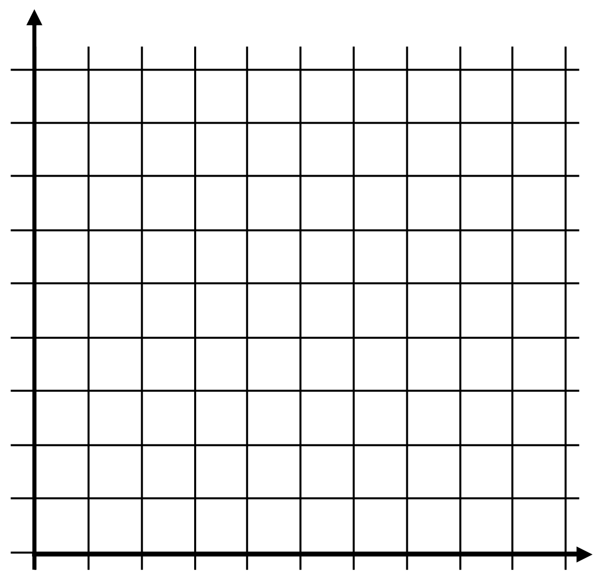
# of bagels (*x*)	cost (*y*)
4	
8	
12	
16	
20	

HOLE-Y BREAD

# of bagels (*x*)	cost (*y*)
5	
10	
15	
20	
25	

2. Which shop has the better buy? Use entries in the tables to explain your reasoning.

3. Label and scale the grid. Graph the data using two different colors. Explain how to use the graph to tell which is the better buy.



4. Find the unit rates for bagels at both shops. Use these numbers to explain which shop has the better buy.

5. Write an equation to represent the cost, given the number of bagels, for:

   a. Shmear ‘n Things ________________ b. Hole-y Bread _____________________

TORTILLAS

(Grade 7)

FLAT 'N ROUND
3 tortillas sell for $0.60

WRAP IT UP
4 tortillas sell for $0.40

1. Complete the tables. Assume each shop will sell any number of tortillas at the rate shown.

FLAT 'N ROUND	
# of tortillas (x)	cost (y)
0	
3	
6	

WRAP IT UP	
# of tortillas (x)	cost (y)
0	
4	
8	

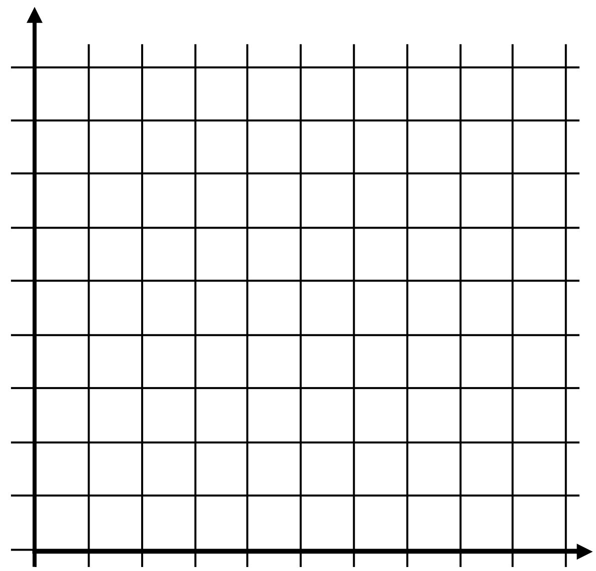
2. Find the unit price at each tortilla store.

3. Write equations to relate the number of tortillas to the cost.

FLAT 'N ROUND y = _______________

WRAP IT UP y = _______________

4. Label and scale the grid. Graph the data using two different colors.



5. Identify the y-coordinate when $x = 1$

FLAT 'N ROUND (1, ____)

WRAP IT UP (1, ____)

6. Explain the meaning of these coordinate pairs in this context.

The equations above are linear functions in the form $y = mx$. This is called a direct proportion equation because y is directly proportional to (is a constant multiple of) x. The number m is called the constant of proportionality.

7. How are the coordinates in problem 5 related to the linear function in the form $y = mx$?

PITA BREAD

(Grade 8)

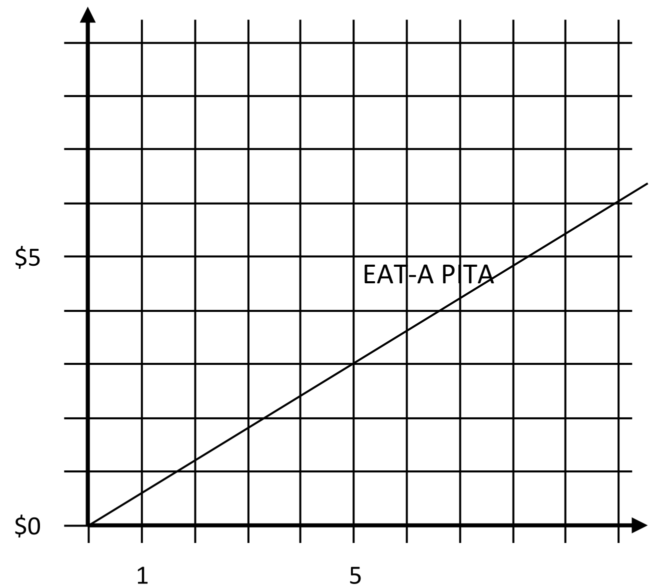
PAPA'S PITA
6 pitas for $____

EAT-A PITA
10 pitas for $____

1. Complete the tables and graphs. The graph for EAT-A PITA is provided. A partial table for PAPA'S PITA is provided. Use tables and graphs to extend the pricing information above. Assume proportional relationships between the number of pitas and cost.

PAPA'S PITA	
# of pitas (*x*)	cost (*y*)
2	$1.00

EAT-A PITA	
# of pitas (*x*)	cost (*y*)



2. Which shop has the better buy? Use entries in the tables or graphs to explain your reasoning.

3. Write equations to relate the number of pitas to cost.

PAPA'S PITA *y* = ______________

EAT-A PITA *y* = ______________

How can you determine unit rates from these equations?

Which shows the greater rate of increase?

4. Identify the coordinates when *x* = 1.

PAPA'S PITA (1, ____)

EAT-A PITA (1, ____)

What do these *y*-coordinates represent in the context of the problem?

5. Identify the coordinates when *x* = 0.

PAPA'S PITA (0, ____)

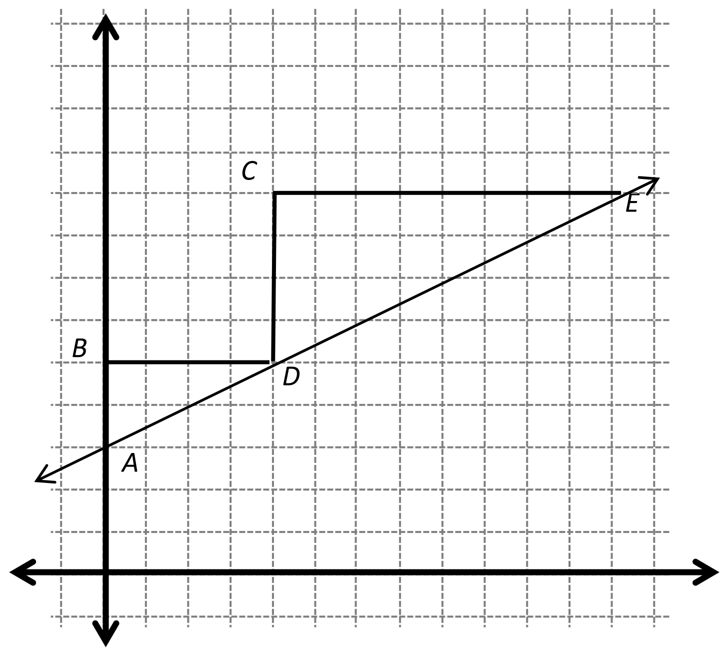
EAT-A PITA (0, ____)

What do these *y*-coordinates represent in the context of the problem?

SLOPE
(Grade 8)

We will now connect our knowledge of similar triangles to the concept of slope.

First we will establish that $\Delta ABD \sim \Delta DCE$ in the diagram to the right.



1. Why is $\overline{BD} \parallel \overline{CE}$?

2. $\angle ABD \cong \angle$ _______ Explain

3. $\angle BAD \cong \angle$ _______ Explain

4. $\Delta ABD \sim \Delta$ _______ Explain

5. Mark congruent angles for ΔABD and ΔDCE on the diagram.

6. In similar triangles, corresponding side lengths are _________________.

   Therefore the ratios of corresponding legs will be _________________.

7. Find ratios of corresponding legs within the similar triangles.

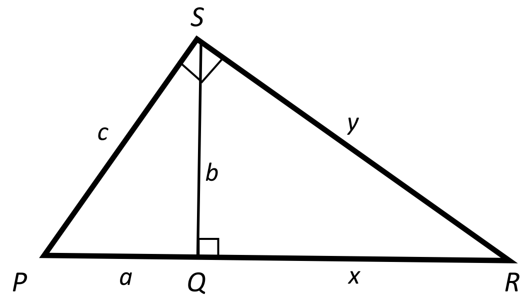
   $\frac{|AB|}{|BD|} =$ $\qquad$ $\frac{|CD|}{|CE|} =$

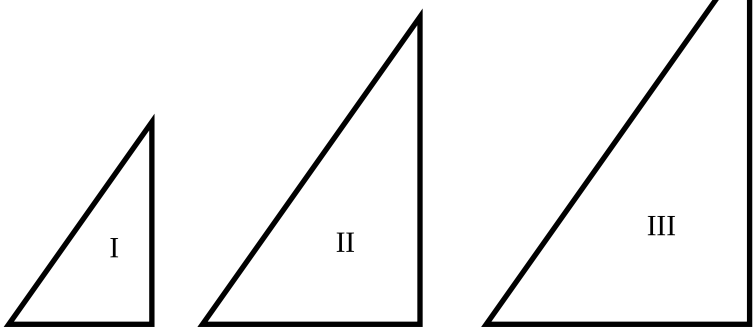
8. Find the equation of $\overleftrightarrow{AE}$ in slope-intercept form. Circle the slope in your equation.

9. How are the results from problems 7 and 8 related?

SIMILIARITY DISCOVERY

1. Within the diagram at the right are three right triangles. Label angles and lengths of the triangles as they sit side by side so that corresponding segments are easily identified.





2. Establish that the triangles are similar using the AA Similarity Criterion.

   Δ I ~ Δ III because ∠______ ≅ ∠______ and ∠______ ≅ ∠______ .

   Δ II ~ Δ III because ∠______ ≅ ∠______ and ∠______ ≅ ∠______ .

   Δ I ~ Δ II because ________________________________ .

When triangles are similar, their sides are proportional.

3. Write an equation that states that $\frac{\text{length of shorter leg}}{\text{length of longer leg}}$ if triangles I and II are proportional. This proportion tells us that ax = _____	4. Write an equation that states that $\frac{\text{length of hypotenuse}}{\text{length of shorter leg}}$ if triangles I and III are proportional. This proportion tells us that ax = ________

5. Substitute the value of *ax* from the problem three equation into the problem 4 equation.

   What did you just prove?

Adapted from *MathLinks*

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe a variety of processes and proficiencies that educators seek to develop in mathematically proficient and fluent students across all grade levels.

Many processes and proficiencies in these practice standards overlap, several may be used together on any given problem or task, and rarely would we expect students to use them all at once. We do expect that over time students will use them frequently. In addition, some will be used naturally within the context of solving particular problems, and others will only occur in an environment in which students are provided ample opportunities to collaborate and discuss.

One way to think about the practices is in groupings (graphic from CCSS-M author, Bill McCallum).

HABITS OF MIND

- MP1 Make sense of problems and persevere in solving them
- MP6 Attend to precision

REASONING AND EXPLAINING

- MP2 Reason abstractly and quantitatively
- MP3 Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS

- MP4 Model with mathematics
- MP5 Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING

- MP7 Look for and make use of structure
- MP8 Look for and make use of repeated reasoning

From CCSS-M:

"Students who lack understanding of a topic may rely on procedures too heavily...In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. Without understanding, a student may rely on procedures and may not represent problems coherently, justify conclusions, apply the mathematics to other situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an interview, or deviate from a known procedure to find a shortcut."

STANDARDS FOR MATHEMATICAL PRACTICE (Continued)

Habits of Mind

MP1 Make sense of problems and persevere in solving them	MP6 Attend to precision
○ Understand a problem and look for entry points ○ Consider given information, constraints, and relationships ○ Consider simpler or analogous problems ○ Make conjectures, monitor progress and alter their solution course as needed ○ Explain correspondences using multiple representations ○ Understand and analyze the approaches of others ○ Continually ask, "Does this make sense?"	○ Communicate mathematical ideas precisely ○ Use clear definitions ○ State meaning of symbols and use them properly ○ Attend to units of measures and labeling of axes ○ Calculate accurately and give solution with appropriate degree of accuracy

Reasoning and Explaining

MP2 Reason abstractly and quantitatively	MP3 Construct viable arguments and critique the reasoning of others
○ Attend to the meaning of quantities ○ Decontextualize a problem using symbols, and manipulate them as if they have a life of their own ○ Contextualize manipulations to create a coherent representation of a problem	○ Use assumptions, definitions, and established results to create arguments (deductive reasoning) ○ Make and test conjectures based on evidence (inductive reasoning) ○ Analyze situations by breaking them into cases ○ Use counterexamples to disprove a statement ○ Identify flaws in an argument ○ Listen to or read to arguments and ask useful questions to clarify reasoning

STANDARDS FOR MATHEMATICAL PRACTICE (Continued)

Modeling and Using Tools	
MP4 **Model with Mathematics**	**MP5** **Use appropriate tools strategically**
○ Apply mathematics to solve everyday problems ○ Make reasonable assumptions and approximations to simplify a situation ○ Identify important quantities in a situation ○ Use multiple representations to analyze relationships and draw conclusions ○ Interpret results in the context of the situation ○ Improve the mathematical approach (model) if it has not served its purpose	○ Select useful tools such as paper and pencil, graph paper ruler, calculator, concrete model, spreadsheet, or statistical software to solve problems ○ Use concrete models and technology tools to explore concepts ○ Recognize limitations of tools ○ Identify and use relevant external resources, such as the internet

Structure and Generalizing	
MP7 **Look for and make use of structure**	**MP8** **Look for and make use of repeated reasoning**
○ Identify patterns and apply them to solve problems ○ Recognize the structure of a symbolic representation and generalize it ○ See complicated objects as composed of chunks of simpler objects	○ Identify repeated calculations and patterns ○ Generalize procedures based on repeated patterns or calculations ○ Find shortcuts based on repeated patterns or calculations

Specific information about how the Mathematical Practices are integrated into *MathLinks* lessons is located in Teaching Note 1 (TN1) in every packet in the Teacher Guide, and in set-off boxes within the lessons.