

USING LINEAR FUNCTIONS TO MODEL MATH IDEAS

Presented by
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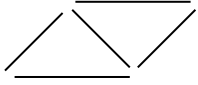
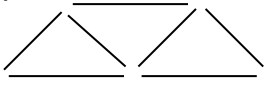
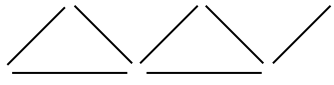
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TRIANGLES

(From Linear Functions 1.2)

A Geometric Pattern	Table		
1. Build and draw the first several steps suggested by this pattern.	Step # (Input)	Arithmetic	Number of toothpicks (Output)
Step 1 	1	3 + 2 or 3 + 1(2)	5
Step 2 	2	3 + 2 + 2 or 3 + 2(2)	
Step 3 	3		
Step 4	4		
Step 5	5		
	⋮		
	30		

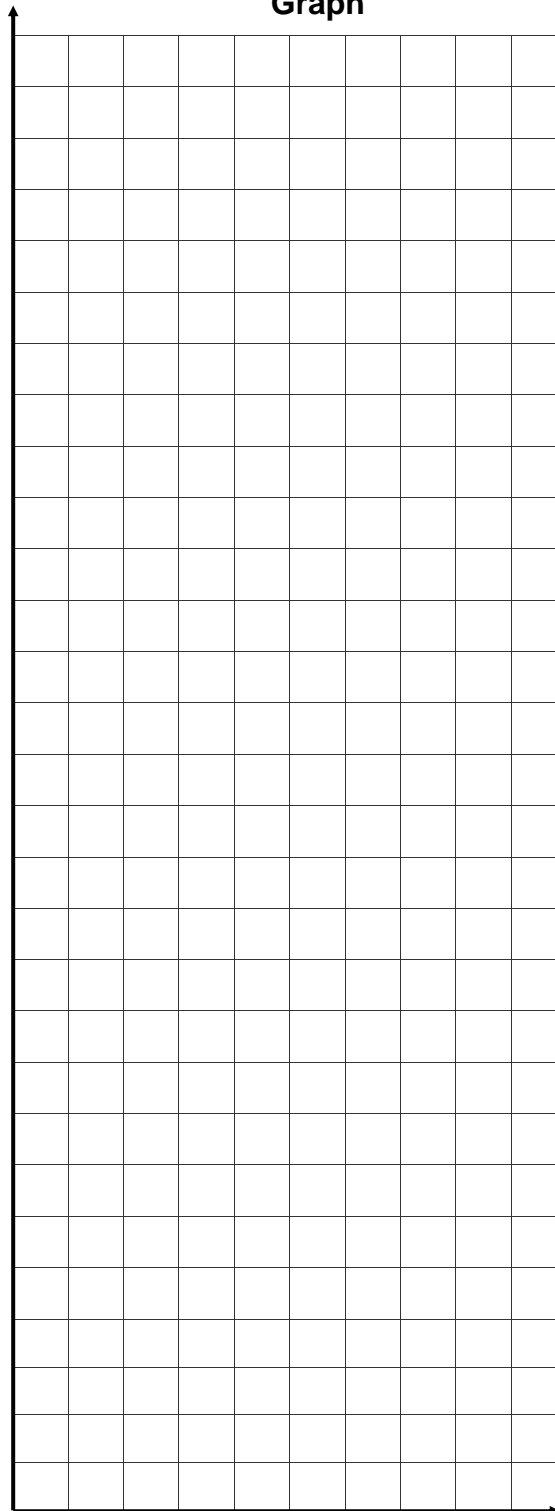
TRIANGLES (continued)

Transfer the data from the previous page into the table below.

Table

Step # (Input)	Number of toothpicks (Output)

Graph



2. Label the horizontal and vertical axes and graph the data points.

3. Recursive Rule:
 Start with _____ toothpicks, and then
 _____ each time

4. Explicit Rule: Explain what to do to the input number at each step to get the corresponding output number.

5. How many toothpicks are in step #50?

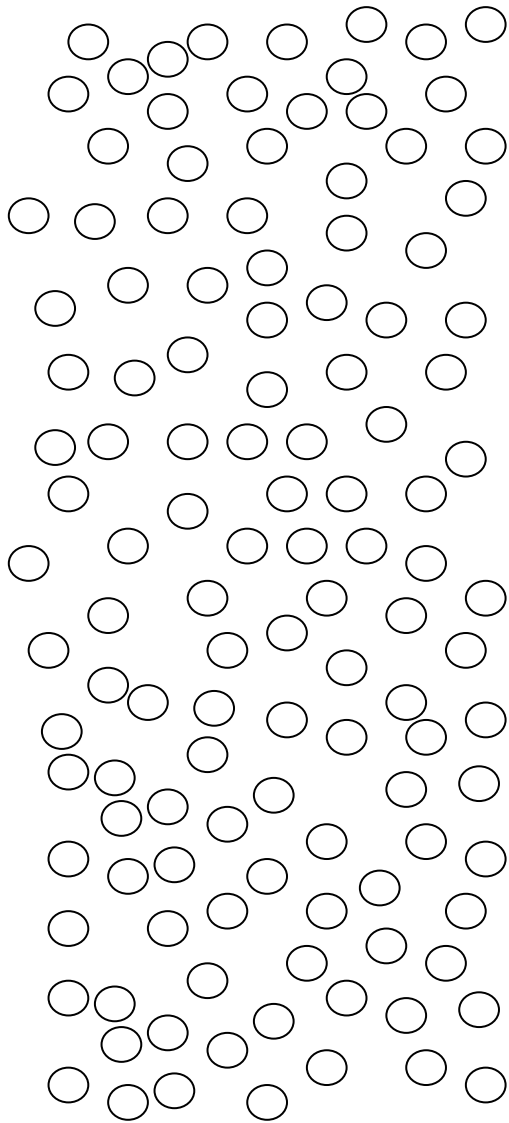
6. In what step number are there exactly 59 toothpicks? _____

7. What is the output for step #0? _____

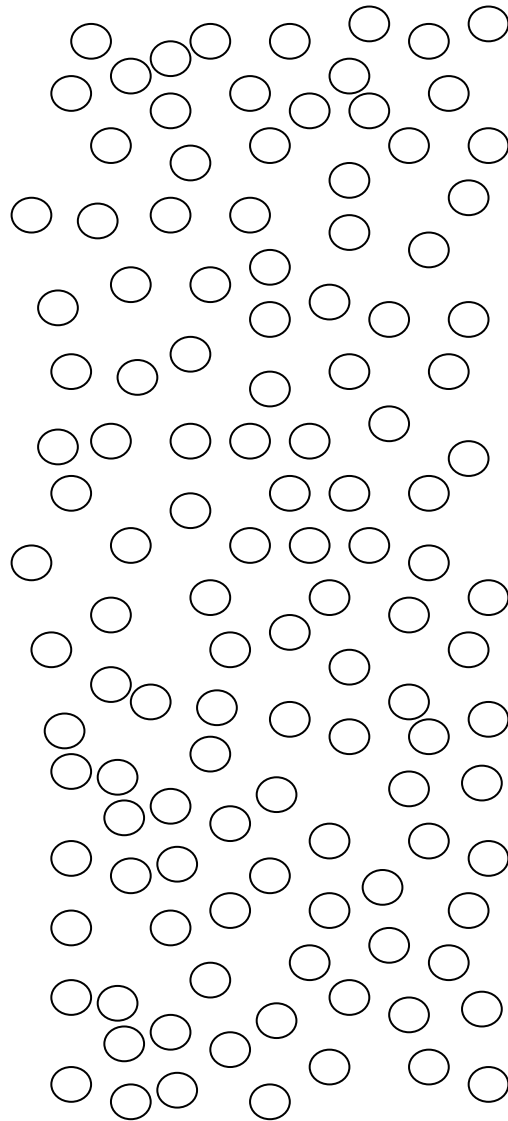
LEFTY-RIGHTY EXPERIMENT

(From Linear Functions 1.3)

1. Use your left hand to write as many X's as you can inside the circles. Wait for your teacher's signal to start. Stop when your teacher says time is over.



2. Use your right hand to write as many X's as you can inside the circles. Wait for your teacher's signal to start. Stop when your teacher says time is over.



3. Count the number of circles that have an X on your **lefty** side. _____
4. Count the number of circles that have an X on your **righty** side. _____

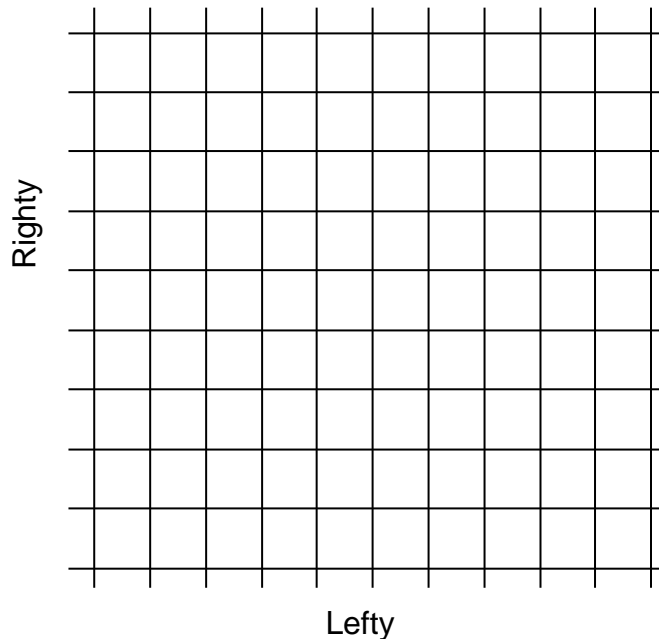
RECORDING AND GRAPHING DATA

1. Record the class data in the table below.

	# of circles with X's on the lefty side (x)	# of circles with X's on the righty side (y)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

	# of circles with X's on the lefty side (x)	# of circles with X's on the righty side (y)
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

2. First graph the line $y = x$. Then graph all coordinate pairs from the table on the previous page. Use an appropriate scale.



EXPERIMENT QUESTIONS

Using your graph, complete the questions below.

1. How many data points are **on** the line $y = x$? _____ What do these data points mean in the context of the experiment?
2. How many data points are **above** the line $y = x$? _____ What do these data points mean in the context of the experiment?
3. How many data points are **below** the line $y = x$? _____ What do these data points mean in the context of the experiment?
4. Where do most of the data points lie, above or below the line $y = x$? _____
What does this tell us about the class' ability to cross off the circles?
5. An outlier of a data set is a data value that is unusually small or unusually large relative to the overall pattern of values in the data set.

Do you see any potential outliers in your lefty-righty data set? _____ What does this tell us in the context of the experiment?

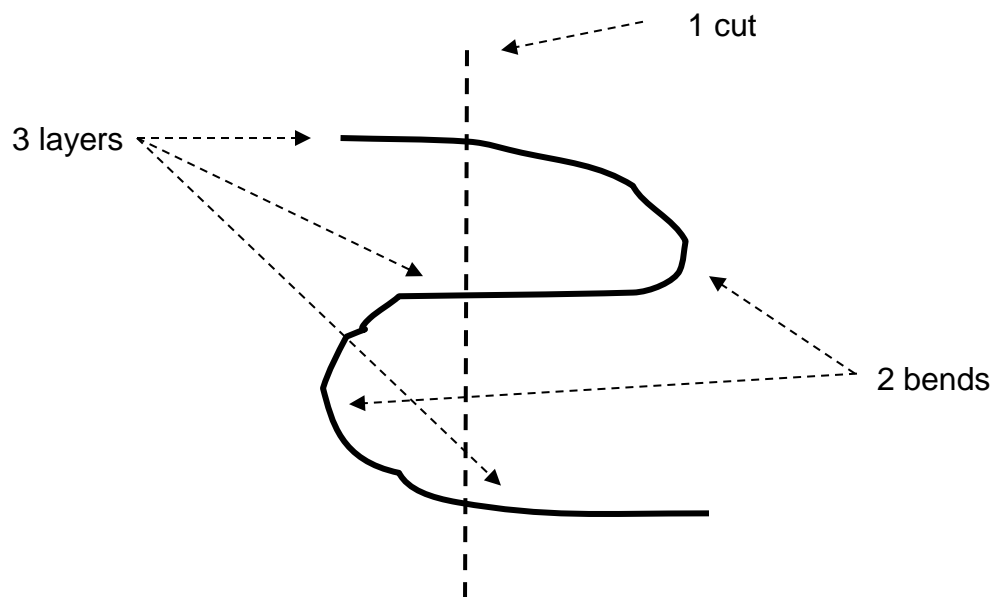
6. Clustering of data refers to a group of numbers where members of each group surround a particular number. Does there appear to be any clustering of the data points? _____ Explain that this means in the context of the experiment.

CUTTING THE ROPE

(From Linear Functions 4.3)

INTRODUCTION

- Start with one long piece of rope with 3 “layers” and 2 “bends.”
- If 1 vertical cut is made through all 3 layers as shown:




1. How many pieces of rope are there?
2. Draw a 2nd vertical line that cuts through each layer again.
How many pieces of rope are there now?

CUTTING THE ROPE: PICTURES, NUMBERS, AND SYMBOLS

Explore cutting the rope for different numbers of layers and cuts.

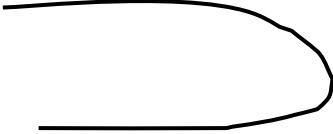
1. # of layers = 1



# of cuts(c)	# of pieces (p)

Rule for any number of cuts:
 $p =$

2. # of layers = 2



# of cuts(c)	# of pieces (p)

Rule for any number of cuts:
 $p =$

3. # of layers = 3

# of cuts (c)	# of pieces (p)

Rule for any number of cuts:
 $p =$

4. # of layers = 4

# of cuts (c)	# of pieces (p)

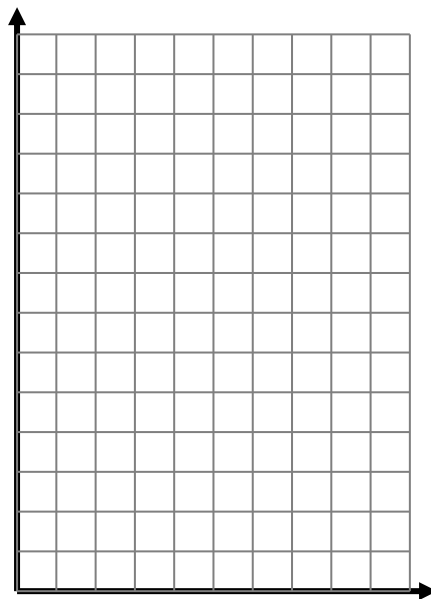
Rule for any number of cuts:
 $p =$

CUTTING THE ROPE: GRAPHS, SYMBOLS, AND WORDS

1. For each table, plot the points on the grid. Graph each set of points with a different color.
2. What is the same about each graph?

3. What is different?

4. What does the y -intercept represent for each graph in terms of the cutting rope experiment?



5. What does the slope tell us?

6. Looking at all four tables, write a rule that can be used to find the total number of pieces (p) for any number of layers (ℓ) and any number of cuts (c):

$$p = \underline{\hspace{2cm}}$$

THE AMUSEMENT PARK

(From Linear Functions 6.3)

You and your friends are going to the amusement park and everyone has to decide which payment option is best depending how much they like going on the rides.

Option 1: Unlimited rides all day for a one-time \$50 fee

Option 2: A one-time \$20 fee plus \$2 for each ride

Option 3: \$5 for each ride with no other fee

Identify the variables in the problem. Use the fourfold way (numbers, pictures, symbols, and words) to give a complete and clear explanation of the solution to the problem. Take into account different amusement park preferences for everyone in your explanation about which option is best under which circumstances.

Use this page to organize your notes and thinking.

Use your own paper or poster to present your work completely and clearly.

COMMON CORE STATE STANDARDS-MATHEMATICS

SOME STANDARDS FOR MATHEMATICAL CONTENT*

- 6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*
- 8.EE.8a Analyze and solve pairs of simultaneous linear equations: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- 8.EE.8c Analyze and solve pairs of simultaneous linear equations: Solve real-world and mathematical problems leading to two linear equations in two variables.
- 8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; ~~give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1)$, $(2,4)$ and $(3,9)$, which are not on a straight line.~~
- 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, ~~positive or negative association, linear association, and nonlinear association.~~
- 8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- 8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
- N-Q-2 Define appropriate quantities for the purpose of descriptive modeling.
- A-CED-2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-REI-10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

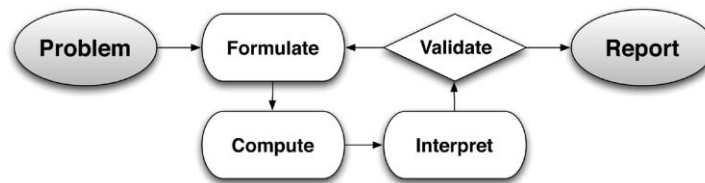
STANDARDS FOR MATHEMATICAL PRACTICE*

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

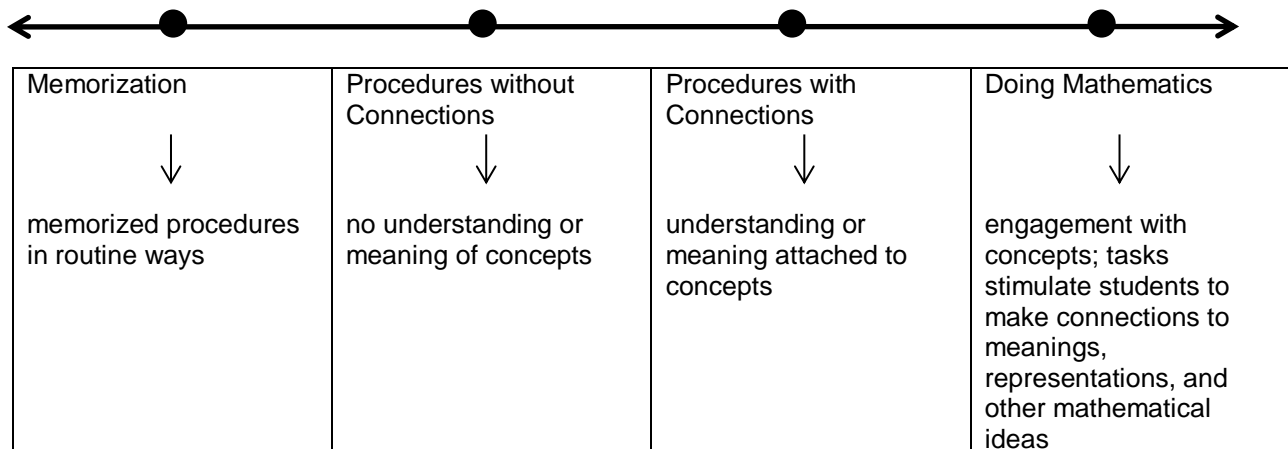
THE BASIC MODELING CYCLE*

Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations in order to understand them better. This understanding helps us to make better decisions.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



COGNITIVE DEMAND SPECTRUM



*From The Common Core State Standards for Mathematics