PAINT MIXTURES

A Middle School Lesson About Ratios

LESSON PLAN: PART 1

Whole Class / Partners

Student Page
Paint Mixtures

Reproducible
Paint Mixture Cards

Materials
- Scissors
- Strips of paper (optional)

- Distribute one set of cards to each pair and ask students to cut them up.
- Students organize the cards from "least blue" to "most blue" on their table. As students work, circulate and ask for explanations. Encourage them to rewrite representations on the cards in different ways to help them justify their ordering decisions.
- If desired, give each partner a strip of paper, and ask them to record their order. Post them for discussion. (The correct order is H, A = D, C, B = E, F = G).
- Compare ordering and discuss. Choose a discrepancy and ask students to discuss the ordering of the pair with their partners. Then record their explanations on the board. Encourage students to challenge each other's reasoning. For example:

  Student 1: "I think D and F represent the same blueness because they are both $\frac{2}{3}$ blue paint."

  Student 2: " I agree that F is $\frac{2}{3}$ blue paint. But D is $\frac{2}{3}$ white paint, so it is only $\frac{1}{3}$ blue paint.

LESSON PLAN: PART 2

Whole Class / Partners

Student Page
Paint Mixtures

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Paint Mixture Cards

- Students discuss and critique the reasoning of the four students' statements. Then they compare the "blueness" of mixture G to mixtures with different units of measure. Possible true statements for problem 3 are:

  (1) G is more blue than A because though they have the same amount of blue paint, A has much more white paint.

  (2) G is more blue because it is $\frac{2}{3}$ cups of blue paint and A is only $\frac{1}{3}$ cups of blue paint.

  (3) If the cups in G are doubled (use 2 as the multiplier), there are 4 cups of blue paint for every 2 cups white paint. A has 2 cups blue for every 4 cups white.

- Challenge students with questions related to changing of units of measure. Generalize comparisons of specific units (such as cups) to any equal units of measure (called "parts").

  (Problem 7) ***Last period some students said that this mixture is less blue because ounces are less than cups. Were they correct?*** No. As long as the units are consistent (cups : cups, ounces : ounces, or in general parts : parts), the ratios are equivalent.

  (Problem 9) ***Will this paint mixture look the same as G?*** No. G has twice as much blue as white. 2 ounces of blue is much less than 1 gallon of white, so it will look much whiter. This is no longer the same ratio of parts to parts.

PAINT MIXTURES

Your teacher will give you cards that indicate ways to combine cups of BLUE paint and cups of WHITE paint to form a paint mixture. Discuss the questions below with your partners and record your thinking.

1. Arrange the cards from least "blue" to most "blue" and record the letters on the cards here. Note that some of the mixtures may be "equally blue," or equivalent.

   _____ _____ _____ _____ _____ _____ _____ _____

   least   most

2. Choose a pair of cards and explain how you know which is more blue.

Critique the reasoning of each student.

3. Jody said, "Mixture A and mixture G will be the same because they both have the same number of cups of blue."

4. Ed said, "Mixture E and mixture C will be the same because mixture E has one more cup of white than blue, and mixture C has one more cup of white than blue."

5. Cary said, "Mixture C is more blue than mixture G because it has more cups of blue."

6. Jon said, "For any mixture, if I double the number of cups of blue and double the number of cups of white, the color will be the same."

Determine which the following are more, less, or equally blue when compared to mixture G (2 cups blue : 1 cup white). Explain your answers.

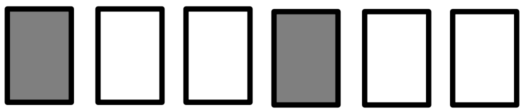
7. The mixture is changed to 2 **ounces** blue : 1 **ounce** white.

8. The mixture is changed to 2 **gallons** blue : 1 **gallon** white.

9. The mixture is changed to 2 **ounces** blue : 1 **gallon** white

PAINT MIXTURE CARDS

A.

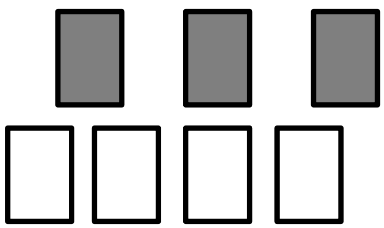


B.

Cups of Blue : Cups of White

4 : 5

C.

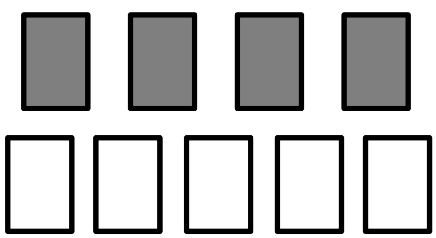


D.

Cups of White to Paint Mixture (total)

2 to 3

E.

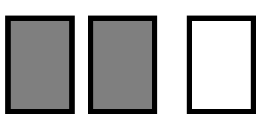


F.

For every 1 cup of paint,

there is $\frac{2}{3}$ cups Blue.

G.



H.

Cups Blue	**1**	**4**	**2**	**7**
Cups White	**3**	**12**	**6**	**21**

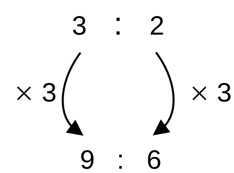
MATH BACKGROUND

(from *MathLinks*: Grade 6)

MN6: Ratios and Rates – Then and Now [11.1, 11.2, 11.3]

Many definitions in mathematics have metamorphosed over time. Originally, the definition of "rectangles" did not include "squares," but it has become standard to include squares as a subset of the rectangle family because it makes many properties easier to explain. To complicate the picture even more, some mathematical terms are defined differently in different textbooks and in different parts of the world. For example, some books define a trapezoid as a quadrilateral with exactly one pair of parallel sides, while others define it as a quadrilateral with at least one pair of parallel sides.

Here we observe changes in the definitions of ratio and rate, as a result of the Common Core State Standards in Mathematics (CCSS-M). In the first column are samples of definitions that have been used in the past.

Before CCSS-M (Some Examples)	Definitions Based on CCSS-M
A ratio is a comparison of two numbers by division. The ratio of a to b is denoted by $a : b$ (read "a to b"), or by $\frac{a}{b}$, where $b \neq 0$. Example: The ratio of 3 to 2 may be denoted by 3 : 2 or by $\frac{3}{2} = 1.5$.	A ratio is a pair of nonnegative numbers, not both zero, in a specific order. The ratio of a to b is denoted by $a : b$ (read "a to b," or "a for every b"). Example: If there are 3 coins and 2 paperclips in your pocket, then the ratio of coins to paperclips may be denoted 3 to 2 or 3 : 2.
	The value of a ratio $a : b$ is the number $\frac{a}{b}$, $b \neq 0$. Example: The value of the ratio of 3 : 2 is $\frac{3}{2} = 1.5$.
Two ratios are equivalent if they have the same value. Example: The ratios 3 : 2 and 9 : 6 are equivalent because $\frac{3}{2} = \frac{9}{6}$.	Two ratios are equivalent if each number in one ratio is a multiple of the corresponding number in the other ratio by the same positive number. Example: This arrow diagram shows that the ratios 3 : 2 and 9 : 6 are equivalent. 
A rate is a ratio in which the numbers have units attached to them. Example: $\frac{\text{20 miles}}{\text{10 minutes}}$ is a rate.	There is no formal definition of "rate." It is treated as a word in common language. Such phrases as "at that rate" or "at the same rate" are used. Example: Sally runs 10 miles in 50 minutes. If she runs the entire marathon at that rate, what will be her marathon time?
A unit rate is a rate for one unit of measure. Example: 80 miles per hour may be written $\frac{\text{80 miles}}{\text{1 hour}}$ or $80 \frac{\text{miles}}{\text{hour}}$.	The unit rate associated to a ratio $a : b$, where a and b have units attached, is the number $\frac{a}{b}$, with the units "a-units per b-unit" attached. Example: The ratio of 400 miles for every 8 hours corresponds to the unit rate 50 miles per hour.